

## Light quark electromagnetic structure of baryons

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Fascinating aspects of the light quark-mass behavior of baryon electromagnetic form factors are highlighted. Using FLIC fermions on  $20^3 \times 40$  quenched  $\mathcal{O}(a^2)$ -improved gauge fields, we explore charge radii and magnetic moments at pion masses as light as 300 MeV. Of particular interest is chiral curvature of proton charge radii and magnetic moments, the environmental dependence of strange quark properties in hyperons, and the remarkable signature of quenched chiral-nonanalytic behavior in the magnetic moment of  $\Delta$  baryon resonances.

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## 1. Introduction

With the advent of new improved fermion actions, it is now possible to explore the light quark-mass regime of hadron electromagnetic form factors on large physical volumes with unprecedented accuracy [1, 2]. In this report, we highlight a few of the most fascinating aspects of the light quark-mass behavior of baryon electromagnetic structure. For a complete discussion of these results and the associated lattice techniques used to obtain them, we refer the interested reader to Ref. [1]. We also note that these calculations have formed the foundation for precise determinations of strange-quark contributions to proton electromagnetic form factors [3, 4, 5].

## 2. Lattice Techniques

The electromagnetic form factors are obtained using the three-point function techniques established by Leinweber, *et al.* in Refs. [6, 7, 8] and updated for smeared sources in Ref. [1]. Our quenched gauge fields are generated with the  $\mathcal{O}(a^2)$  mean-field improved Luscher-Weisz plaquette plus rectangle gauge action [9] using the plaquette measure for the mean link. 400 quenched gauge field configurations on  $20^3 \times 40$  lattices with lattice spacing  $a = 0.128$  fm are generated via the Cabibbo-Marinari pseudo-heat-bath algorithm [10] using a parallel algorithm with appropriate link partitioning [11].

We use the fat-link irrelevant clover (FLIC) Dirac operator [12] which provides a new form of nonperturbative  $\mathcal{O}(a)$  improvement [13]. The improved chiral properties of FLIC fermions allow efficient access to the light quark-mass regime [14], making them ideal for dynamical fermion simulations now underway [15].

Of particular interest, is our use of an  $\mathcal{O}(a)$ -improved FLIC conserved vector current [1]. We follow the technique proposed by Martinelli *et al.* [16]. The standard conserved vector current for Wilson-type fermions is derived via the Noether procedure

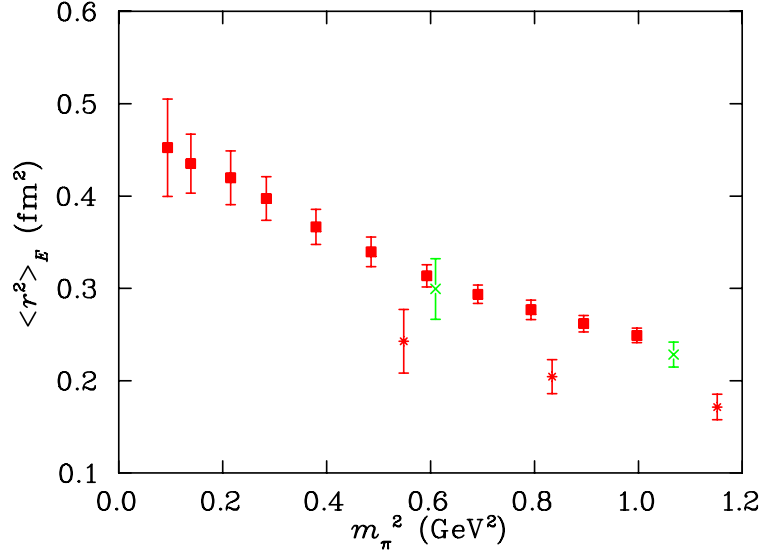
$$j_\mu^C \equiv \frac{1}{4} [\bar{\Psi}(x) (\gamma_\mu - r) U_\mu(x) \Psi(x + \hat{\mu}) + \bar{\Psi}(x + \hat{\mu}) (\gamma_\mu + r) U_\mu^\dagger(x) \Psi(x) + (x \rightarrow x - \hat{\mu})]. \quad (2.1)$$

The  $\mathcal{O}(a)$ -improvement term is also derived from the fermion action and is constructed in the form of a total four-divergence, preserving charge conservation. The  $\mathcal{O}(a)$ -improved conserved vector current is

$$j_\mu^{CI} \equiv j_\mu^C(x) + \frac{r}{2} C_{CVC} a \sum_\rho \partial_\rho (\bar{\Psi}(x) \sigma_{\rho\mu} \Psi(x)), \quad (2.2)$$

where  $C_{CVC}$  is the improvement coefficient for the conserved vector current.

The terms proportional to the Wilson parameter  $r$  in Eq. (2.1) and the four-divergence in Eq. (2.2) have their origin in the irrelevant operators of the fermion action and vanish in the continuum limit. Non-perturbative improvement is achieved by constructing these terms with fat-links. Perturbative corrections are small for fat-links and the use of the tree-level value for  $C_{CVC} = 1$  together with small mean-field improvement corrections ensures that  $\mathcal{O}(a)$  artifacts are accurately removed from the vector current. This is only possible when the current is constructed with fat-links. Otherwise,  $C_{CVC}$  needs to be appropriately tuned to ensure all  $\mathcal{O}(a)$  artifacts are removed.



**Figure 1:** The proton charge radius is compared with previous state of the art lattice simulation results in quenched QCD. The solid squares indicate current lattice QCD results with FLIC fermions. The stars indicate the lattice results of [6] while the crosses indicate the results of [18], both of which use the standard Wilson actions for the gauge and fermion fields.

### 3. Highlights

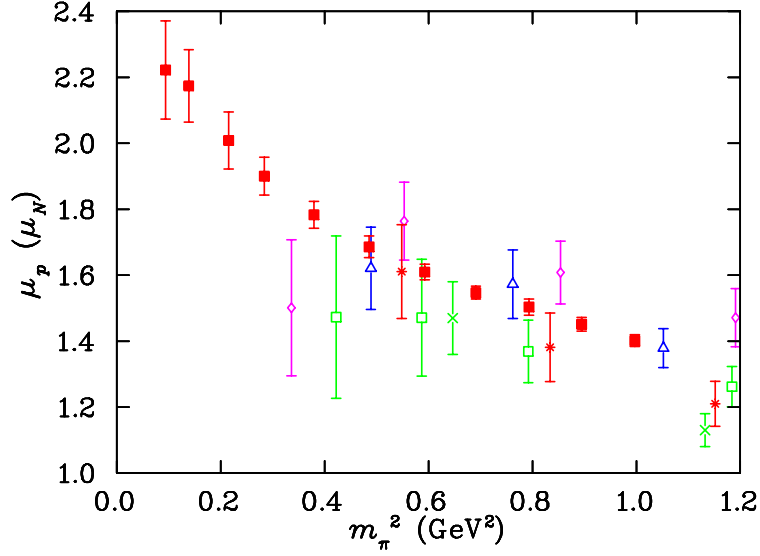
A chief aim of the CSSM Lattice Collaboration has been to reveal the electromagnetic structure of baryons near the chiral regime, search for evidence of chiral nonanalytic curvature and examine the extent to which the observed features are in accord with quenched chiral effective field theory (Q $\chi$ PT) [17]. In the following we discuss some of the more interesting aspects of hadron structure revealed in the light quark-mass regime.

We begin with an examination of the proton's charge radius and magnetic moment. In both cases, quenched chiral nonanalytic curvature is predicted to act to increase the magnitude of these observables; logarithmically for radii and as  $m_q^{1/2}$  for magnetic moments. Figures 1 and 2 illustrate our FLIC fermion results in the context of other three-point function based lattice calculations. In both cases, curvature acting to increase the magnitude of the results is observed.

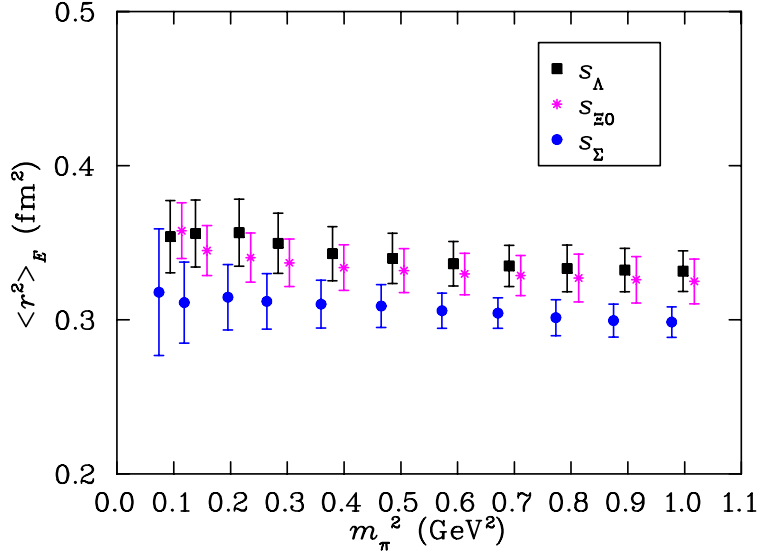
Of particular interest is the environment sensitivity of quark sector contributions to baryon electromagnetic properties, where some rather interesting physics has been discovered. Environment sensitivity is easily observed in the strange quark sector contributions to hyperon properties. As the strange quark mass is held fixed while the light quark masses are varied, any variation of the strange quark contribution is a pure environmental effect.

Of course such effects are predicted in chiral effective field theory where the mass of the Kaon changes as the light quark masses are varied. As the Kaon mass varies, the distribution of strange quarks and their contributions to the baryon magnetic moment will change, even when the strange quark mass is held constant.

Figures 3 and 4 present results for charge distribution radii and magnetic moments of strange quarks in hyperons respectively. These quark sector contributions are presented for a single quark with unit charge. The strange quark in  $\Lambda$ ,  $s_\Lambda$ , in an environment of two light quarks, displays the most significant dependence on the light-quark sector. It is interesting that the coupling of  $\Lambda$  to



**Figure 2:** The proton magnetic moment in nuclear magnetons is compared with a variety of lattice simulations using three-point function techniques. The solid squares indicate our current lattice QCD results with FLIC fermions. The stars indicate the early lattice results of Ref. [6]. The crosses (only one point) indicate the results of Ref. [18]. The open symbols describe the QCDSF collaboration results [19]. Open squares indicate results with  $\beta = 6.0$ , open triangles indicate those with  $\beta = 6.2$  while the open diamonds indicate their results with  $\beta = 6.4$ .



**Figure 3:** Electric charge distribution radii of strange quarks including  $s_\Lambda$ ,  $s_{\Xi^0}$  and  $s_\Sigma$ . The data for  $s_{\Xi^0}$  and  $s_\Lambda$  are plotted at shifted  $m_\pi^2$  values for clarity.

the energetically favoured  $KN$  channel is large in  $Q\chi$ PT [17]. Moreover, the sign of the chiral coefficient is such that the virtual transitions act to enhance the charge distribution and magnetic moment as the chiral limit is approached. We have also confirmed a nontrivial role for light-quark contributions to the Lambda magnetic moment [1, 6]. In simple quark models the light quarks are in an isospin and spin singlet configuration and do not contribute.

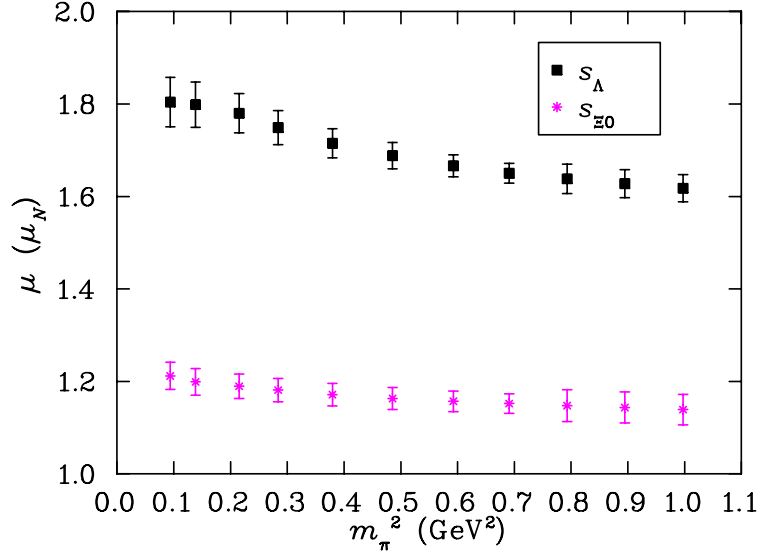


Figure 4: Magnetic moments of  $s_\Lambda$  and  $s_{Z0}$  as a function of quark mass.

#### 4. Quenched Chiral Artifacts

While in many cases, the quenched approximation preserves the qualitative features of full QCD, albeit with suppressed chiral coefficients, there are some cases where the sickness of the quenched approximation is fatal. Perhaps the best known example is the  $a_0$  meson correlator. At sufficiently light quark masses, decays to the negative-metric double-hairpin  $\pi\eta'$  channel changes the sign of the two-point function.

Figure 5 displays this classic signature obtained with FLIC fermions at our second lightest quark mass where  $m_\pi = 372(6)$  MeV. Whereas the correlator begins positive, it changes sign as the lowest-lying negative-metric  $\pi\eta'$  decay channel saturates the correlator.

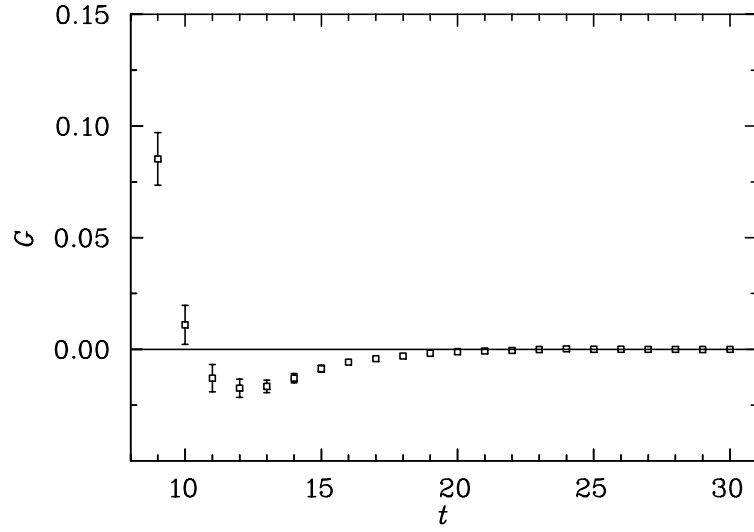
A similarly dramatic signature, discovered by the CSSM Lattice Collaboration in 2003 [20], is displayed in the magnetic moments of  $\Delta$  baryons. Figure 6 compares the magnetic moment of the  $\Delta^+$  with that calculated for the proton on the same lattice. The simplest quark model predicts that the proton and the  $\Delta^+$  have equal magnetic moments. Upon including hyperfine interactions the  $\Delta^+$  moment is expected to be larger and this is observed in Fig. 6 at large quark masses.

However at light quark masses, one must turn attention to the meson dressings of the baryons. The presence of the  $\Delta \rightarrow N\pi$  decay channel is particularly important for the quark-mass dependence of  $\Delta$  properties in general [20]. Rapid curvature associated with nonanalytic behavior is shifted to larger pion masses near the  $N$ - $\Delta$  mass splitting,  $m_\pi \sim M_\Delta - M_N$ .

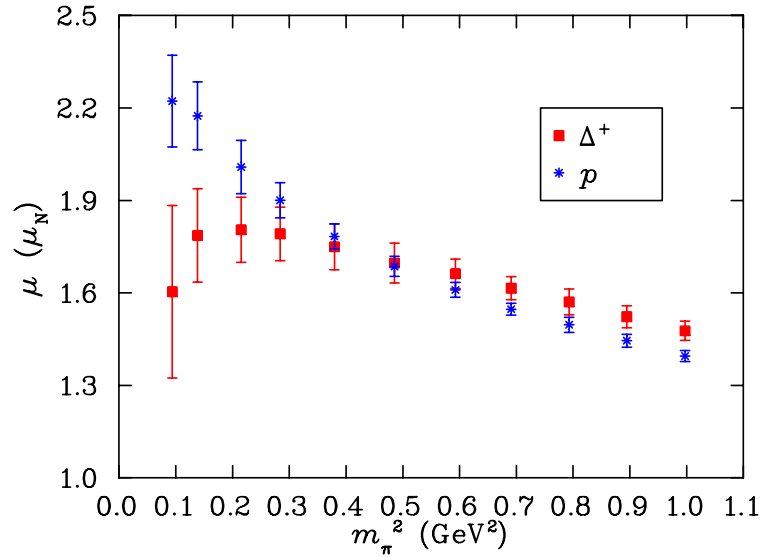
In full QCD, pion-loop contributions are expected to enhance the magnitude of both the proton and  $\Delta^+$  magnetic moments [21]. However a significant down turn is observed for the  $\Delta^+$  magnetic moment as the opening of the  $N\pi$  decay channel is approached.

As described in Ref. [20], this feature is in accord with the expectations of  $Q\chi$ PT. Quenched-QCD decay-channel contributions to  $\Delta$  properties come with a sign opposite to that of full QCD. This artifact provides an unmistakable signature of the quenched meson cloud.

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**Figure 5:** The signature of the quenched decay of the  $a_0$  meson to the negative-metric  $\pi\eta'$  channel. The correlator becomes negative as the lowest-lying negative-metric  $\pi\eta'$  decay channel saturates the correlator.



**Figure 6:** Magnetic moments of the  $\Delta^+$  and the proton at quark masses where the  $\Delta$  is stable by energy conservation.

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