PROCEEDINGS OF SCIENCE

Confining properties of two-color QCD at finite density

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We study the confining properties of QCD with two colors across the finite density phase transition. A disorder parameter detecting dual superconductivity of the QCD vacuum is used as a probe for the confinement/deconfinement phase transition.

XXIV International Symposium on Lattice Field Theory
July 23-28 2006
Tucson Arizona, US

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PoS(LAT2006)131

1. Introduction

Confinement of color is a well established property of strongly interacting matter at low temperatures and densities, even if not yet fully understood starting from the QCD first principles. The existence of a high temperature phase transition to a deconfined state of matter has been predicted by numerical simulations of lattice QCD and is presently under investigation in Heavy Ion experiments. In the present study we address the question regarding the fate of confining properties as the low temperature - high density phase transition is crossed: that is relevant in order to characterize the nature of matter in compact astrophysical objects and more in general to understand how deconfinement at high densities compares to what happens at high temperatures.

Numerical studies of QCD at finite density are notoriously difficult because of the sign problem, which makes usual importance sampling simulations unfeasible: for that reason we have studied the theory with 2 colors, where that problem is absent. No sensible differences are expected for the confining properties of the theory when going from $N_c = 2$ to $N_c = 3$, where N_c is the number of colors: for that reason we believe that our study could be relevant also for real QCD.

Indications about deconfinement at high density obtained so far have been based on the analysis of the Polyakov loop [1], which however is not a true order parameter for confinement in presence of dynamical fermions. Different order parameters can be constructed in the framework of specific mechanisms for color confinement. One successful mechanism is that based on dual superconductivity of the QCD vacuum [2, 3, 4], which relates confinement to the spontaneous breaking of a dual magnetic symmetry induced by the condensation of magnetic monopoles: in that context a disorder parameter can be developed [5, 6] which is the expectation value of a magnetically charge operator, $\langle \mathscr{M} \rangle$[1], and which has been successfully tested as an order parameter for color confinement both in the quenched theory [7, 8] and in presence of dynamical fermions [9, 10]. Similar parameters have been constructed elsewhere [11, 12, 13, 14].

Our plan is to study the behaviour of $\langle \mathscr{M} \rangle$ in the whole $T - \mu$ plane, in order to characterize the confining properties of the various phases in the QCD phase diagram. In Section 2 we will review the definition of the disorder parameter and present our strategy for its numerical study; some preliminary results concerning the theory with 8 flavors of staggered quarks will be presented in Section 3.

2. The disorder parameter $\langle \mathscr{M} \rangle$

$\mathscr{M}(\vec{x},t)$ is defined in the continuum as the operator which creates a magnetic charge in $\vec{x},t$ by shifting the quantum field by the monopole vector potential $\vec{b}_\perp(\vec{y}-\vec{x})$

$$\mathscr{M}(\vec{x},t) = \exp\left[i \int d\vec{y}\, \vec{E}_{\perp\, diag}(\vec{y},t) \vec{b}_\perp(\vec{y}-\vec{x}) \right] \tag{2.1}$$

Its expectation value, when discretized on the lattice, appears in the following form

$$\langle \mathscr{M} \rangle = \tilde{Z}/Z; \tag{2.2}$$

[1] We change the usual notation for the disorder operator, $\langle \mu \rangle$, in order to avoid confusion with the notation for the chemical potential.

Z is the usual QCD partition function

$$Z = \int (\mathscr{D}U)\, \det M(\mu)\, e^{-\beta S} \tag{2.3}$$

where M is the fermionic matrix and S is the pure gauge action, while the partition function $\tilde{Z}$ is obtained from Z by a change in the pure gauge action $S \to \tilde{S}$ which adds the monopole field to the temporal plaquettes at timeslice t.

Being expressed as the ratio of two different partition functions, the numerical study of $\langle \mathscr{M} \rangle$ is a highly non trivial task: while numerical methods have been recently developed which permit a direct determination of $\langle \mathscr{M} \rangle$ [15], we will not use them in the present study since they involve the combination of several different Monte Carlo simulations, which in presence of dynamical fermions could be unpractical. We will instead study, as usual, susceptibilities of the disorder parameter, from which the behaviour of $\langle \mathscr{M} \rangle$ at the phase transition can be inferred.

For instance, being interested in $\langle \mathscr{M} \rangle$ as a function of β, as for the $\mu = 0$ phase transition, one usually measures [5, 6, 7]

$$\rho = \frac{d}{d\beta} \ln \langle \mathscr{M} \rangle = \frac{d}{d\beta} \ln \tilde{Z} - \frac{d}{d\beta} \ln Z = \langle S \rangle_S - \langle \tilde{S} \rangle_{\tilde{S}} \tag{2.4}$$

where the subscript means the pure gauge action used for Monte Carlo sampling. The disorder parameter can be reconstructed from the susceptibility ρ, exploiting the fact that $\langle \mathscr{M} \rangle = 1$ at $\beta = 0$

$$\langle \mathscr{M} \rangle(\beta) = \exp\left(\int_0^\beta \rho(\beta') \mathrm{d}\beta' \right) . \tag{2.5}$$

In particular $\rho \simeq 0$ in the confined phase means $\langle \mathscr{M} \rangle \neq 0$, a sharp negative peak of ρ at the phase transition implies a sudden drop of $\langle \mathscr{M} \rangle$ and ρ diverging in the thermodynamical limit in the deconfined phase means that $\langle \mathscr{M} \rangle$ is exactly zero beyond the phase transition.

At finite temperature and density we are interested in studying the behaviour of $\langle \mathscr{M} \rangle$ in the two parameter space (β, μ), where μ is the chemical potential. For that reason we introduce the new susceptibility

$$\rho_D \equiv \frac{d}{d\mu} \ln \langle \mathscr{M} \rangle = \frac{d}{d\mu} \ln \tilde{Z} - \frac{d}{d\mu} \ln Z = \langle N_f \rangle_{\tilde{S}} - \langle N_f \rangle_S \tag{2.6}$$

where N_f is the quark number operator. The dependence of $\langle \mathscr{M} \rangle$ on the chemical potential μ can then be reconstructed as follows:

$$\langle \mathscr{M} \rangle(\beta, \mu) = \langle \mathscr{M} \rangle(\beta, 0) \exp\left(\int_0^\mu \rho_D(\mu') \mathrm{d}\mu' \right)$$

so that, if the starting point at $\mu = 0$ is in the confined phase ($\langle \mathscr{M} \rangle(\beta, 0) \neq 0$), the behaviour expected for $\rho_D(\mu)$ in correspondence of a possible finite density deconfinement transition will be the same showed by ρ across the finite temperature transition.

3. Numerical results

We have used staggered fermions corresponding to $N_f = 8$ degenerate continuum flavors, with bare mass $am = 0.07$. Different lattices have been considered, with a fixed temporal extent $L_t = 6$

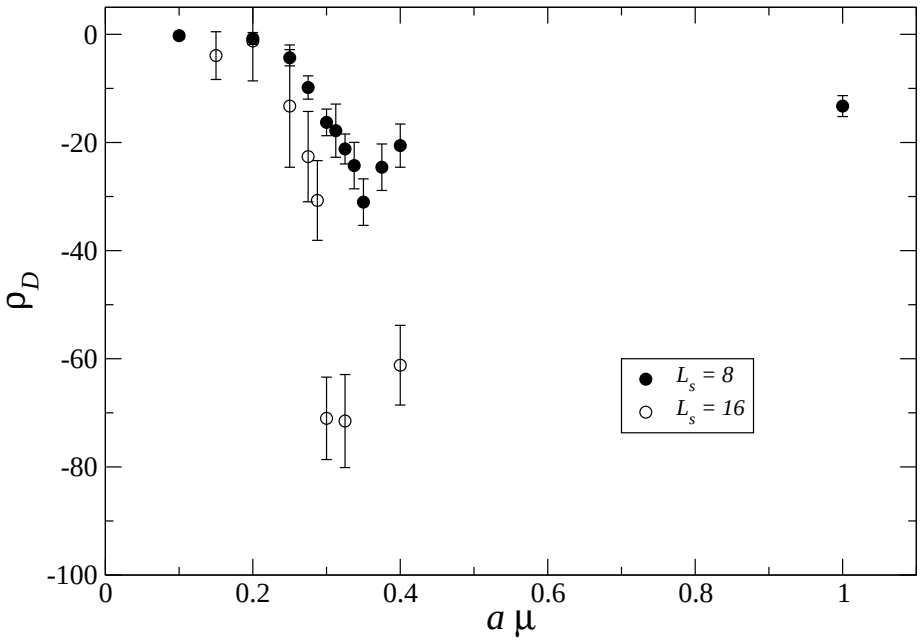

Figure 1: Results obtained for ρ_D

and a variable spatial size (only $L_s = 8, 16$ so far) in order to make a finite size scaling analysis of the phase transition. The critical value of β at $\mu = 0$ is $\beta_c \simeq 1.59$ [16]: we have made simulations at different values of the chemical potential μ and at a fixed value of $\beta = 1.5 < \beta_c$, *i.e.* starting from the confined phase at $\mu = 0$. An exact HMC algorithm has been used and standard actions both in the gluonic and in the fermionic sector: we have used trajectories of unit length and a step size variable from 10^{-2} to $4 \cdot 10^{-3}$: the reduced time step was necessary in order to keep a reasonable acceptance rate around and above the phase transition, as a consequence of the presence of small eigenvalues of the fermion matrix. The same problem was at the origin of a severe slowing down around the critical value of μ, which made the availability of the recently installed apeNEXT facility in Rome essential in order to carry out simulations on the larger lattice ($L_s = 16$). Simulations on the smaller lattice $L_s = 8$ have been carried out both on the APEmille facility in Pisa and on the PC farm of INFN in Genova, using there a code adapted from the publicly available version by the MILC collaboration.

Our results for ρ_D as a function of μ are shown in Fig. 1. ρ_D shows a clear peak at a critical value of the chemical potential $\mu_c \simeq 0.3$. The peak deepens as the lattice volume is increased, suggesting the presence of a true phase transition at which $\langle \mathscr{M} \rangle$ drops to zero and confinement (dual superconductivity) disappears. The position of the peak is coincident with that of other susceptibilities and we show in Fig. 2 the case of the Polyakov loop: our data are still noisy and do not show any clear size dependence, which however is not expected since the Polyakov loop is not an order parameter for the phase transition.

A more detailed analysis of the disorder parameter around the phase transition can be carried

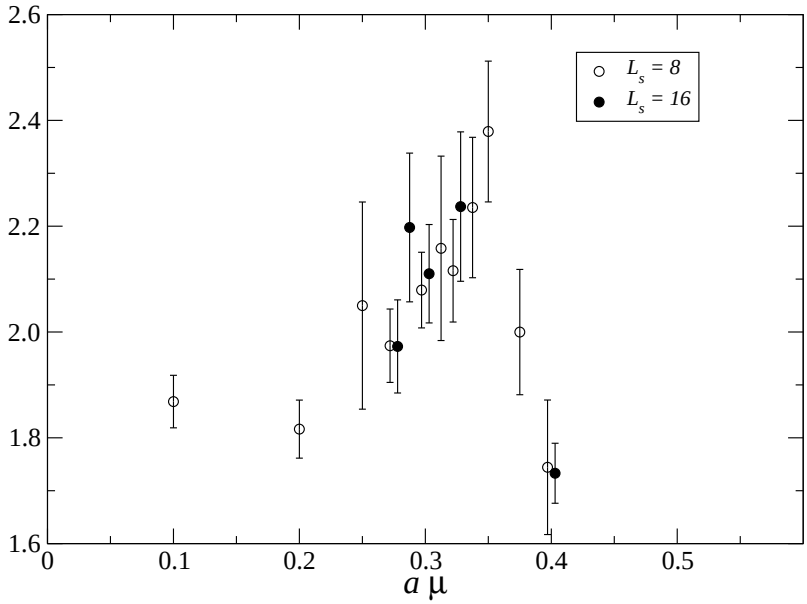

Figure 2: Results obtained for the suscepibility of the Polyakov loop.

out as follows. At fixed T we can assume the following finite size scaling behaviour for $\langle \mathscr{M} \rangle$ as a function of μ.

$$\langle \mathscr{M} \rangle = L_s^{-\beta/\nu} \Phi((\mu - \mu_c) L_s^{1/\nu})$$

from which it is easy to derive

$$\rho_D = L_s^{1/\nu} \phi((\mu - \mu_c) L_s^{1/\nu}) \tag{3.1}$$

Our data show a nice scaling with $\nu \sim 0.66$, as shown in Fig. 3, which is compatible with a second order phase transition in the universality class of the $3d$ Ising model.

4. Conclusions

We have made use of the disorder parameter $\langle \mathscr{M} \rangle$ detecting dual superconductivity of the vacuum to inspect the confining properties of QCD with 2 colors at finite temperature and density. The study has been carried our with $N_f = 8$ flavors of staggered fermions. In order to analyse $\langle \mathscr{M} \rangle$ as a function of the chemical potential μ we have introduced a new susceptibility $\rho_D = d/d\mu \ln\langle \mathscr{M} \rangle$, showing the presence of a phase transition at finite density where confinement (dual superconductivity) disappears; the transition is coincident with those signalled by other quantities such as the Polyakov loop. A preliminary finite size scaling analysis of the disorder parameter is compatible with a phase transition in the universality class of the $3d$ Ising model.

After these preliminary results, we plan in the future to make a more extensive study of the disorder parameter in order to characterize the whole phase diagram of the theory with two colors,

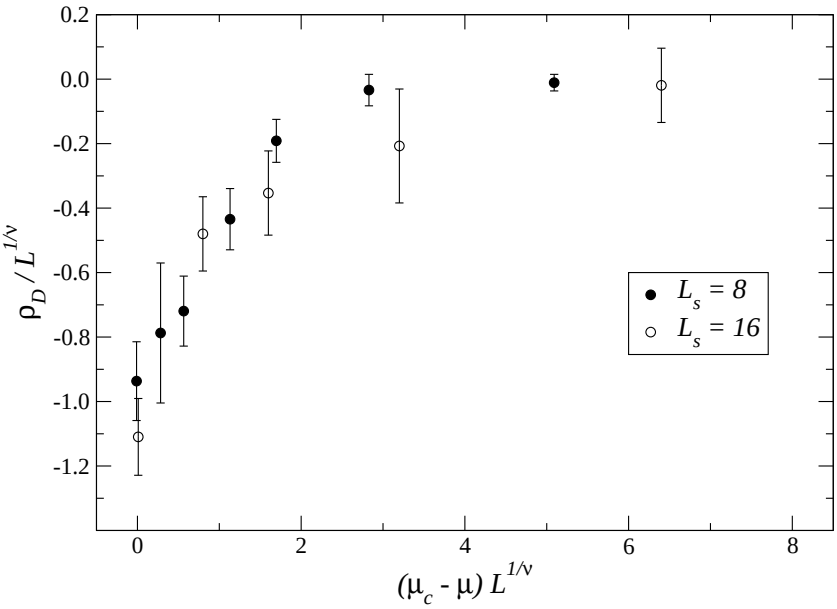

Figure 3: Finite size scaling analysis for ρ_D

with a particular interest in the region of low temperatures and high densities. To that aim, a combined study of both susceptibilities, ρ and ρ_D, could be particularly useful: indeed the knowledge of $\vec{\nabla}\ln\langle\mathscr{M}\rangle$, with $\vec{\nabla} = (d/d\beta, d/d\mu)$, in the whole $\beta - \mu$ plane, could give information not only on the location but also on the direction of the critical line, thus permitting a more careful study of the phase diagram.

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