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Searching for the elusive critical endpoint at finite temperature and isospin density

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We consider 3-flavour lattice QCD with a finite chemical potential μ_I for isospin, close to the finite temperature transition from hadronic matter to a quark-gluon plasma. In this region one can argue that the position and probably the nature of this transition mimic those at finite quark-number chemical potential μ. The quark mass is chosen to be close to the critical mass at zero chemical potentials. Since the Binder cumulants used to determine the nature of this transition in HMD(R) simulations are very sensitive to the updating increment dt, we have switched to the newer exact RHMC algorithm for our simulations. Preliminary results indicate that there is no critical endpoint in the small μ_I regime, at least none connected with the critical point at zero chemical potentials.

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1. Introduction

Direct simulations of QCD at finite baryon/quark number density are made difficult if not impossible because at finite quark-number chemical potential μ the fermion determinant is complex. At small chemical potentials, close to the finite temperature transition, various methods have been devised to circumvent this difficulty, series expansions in μ [1, 2, 3], analytic continuation from imaginary μ [4, 5, 6, 7, 8, 9], reweighting methods [10, 11] and canonical ensemble techniques [12, 13].

We adopt a different strategy, and simulate using the magnitude of the fermion determinant and ignoring the phase [14, 15]. This can be thought of as considering all quarks to be in isodoublets and introducing a finite chemical potential μ_I for isospin. In the region of small μ/μ_I, where the phase is expected to be less important one can argue that the finite μ and μ_I transitions might be identical. Since our fermion determinant is positive (or at least non-negative), we can use standard hybrid molecular-dynamics HMD(R) simulations [16]. However, for this algorithm, the Binder cumulants used to determine the nature of the finite temperature transition turn out to be strongly dependent on the updating increment dt. For this reason we now simulate using the rational hybrid monte-carlo (RHMC) algorithm [17, 18], which is exact in the sense of having no dt dependence for observables.

In the low chemical potential domain, the most interesting feature expected in the phase diagram is the critical endpoint, where the finite temperature transition changes from a crossover to a first-order transition as chemical potential is increased. The critical endpoint is expected to lie in the universality class of the 3-dimensional Ising model. For 3 flavours it had been expected that the critical point at zero chemical potentials, where the transition changes from a first order transition to a crossover as mass is increased, would move to higher masses as the chemical potential increases, thereby becoming the critical endpoint. Our preliminary results indicate that this does not happen.

In section 2 we give the fermion action and make a few comments on the RHMC implementation. Section 3 gives our preliminary results. Our conclusions occupy section 4.

2. QCD at finite isospin density and the RHMC

The pseudo-fermion action for QCD at finite μ_I, used for the implementation of the RHMC algorithm is

$$S_{pf} = p_\psi^\dagger \mathscr{M}^{-N_f/8} p_\psi \tag{2.1}$$

where p_ψ is the momentum conjugate to the pseudo-fermion field ψ.

$$\mathscr{M} = [\not{D}(\frac{1}{2}\mu_I) + m]^\dagger [\not{D}(\frac{1}{2}\mu_I) + m] + \lambda^2 \tag{2.2}$$

is the quadratic Dirac operator, and we set $\lambda = 0$ for our $\mu_I < m_\pi$ simulations.

To implement the RHMC method we need to know positive upper and lower bounds to the spectrum of $\mathscr{M}$. 25 exceeds the upper bound for the μ_I range of interest. We use a speculative lower bound of 10^{-4} since the actual lower bound of the spectrum is unknown. This is justified by varying the choice of lower bounds and comparing the results [19]. For $N_f = 3$ we use a

$(20,20)$ rational approximation to $\mathscr{M}^{(\pm 3/16)}$ at the ends of each trajectory, and a $(10,10)$ rational approximation to $\mathscr{M}^{(-3/8)}$ for the updating.

3. Simulations and Results

We are simulating lattice QCD with staggered fermions and $N_f = 3$ at quark masses close to m_c, the critical mass for $\mu = \mu_I = 0$ on $8^3 \times 4$, $12^3 \times 4$ and $16^3 \times 4$ lattices. $m = 0.02$, 0.025, 0.03, 0.035, and $\mu_I = 0.0, 0.2, 0.3$. For our $12^3 \times 4$ simulations we use runs of 300,000 trajectories at each of 4 β values close to β_c, for each m and μ_I. We mostly use $dt = 0.05$ for which length-1 trajectories give acceptances of $\sim 70\%$ for the RHMC algorithm.

To determine the nature of the transition, we use 4-th order Binder cumulants [20] for the chiral condensate. For any observable X this cumulant is defined by

$$B_4(X) = \frac{\langle (X - \langle X \rangle)^4 \rangle}{\langle (X - \langle X \rangle)^2 \rangle^2} \tag{3.1}$$

where the Xs are lattice averaged quantities. For infinite volumes, $B_4 = 3$ for a crossover, $B_4 = 1$ for a first-order transition and $B_4 = 1.604(1)$ for the 3-dimensional Ising model. Thus, if there is a critical endpoint we would expect B_4 to decrease with increasing μ_I, passing through a value close to the Ising value at the critical μ_I.

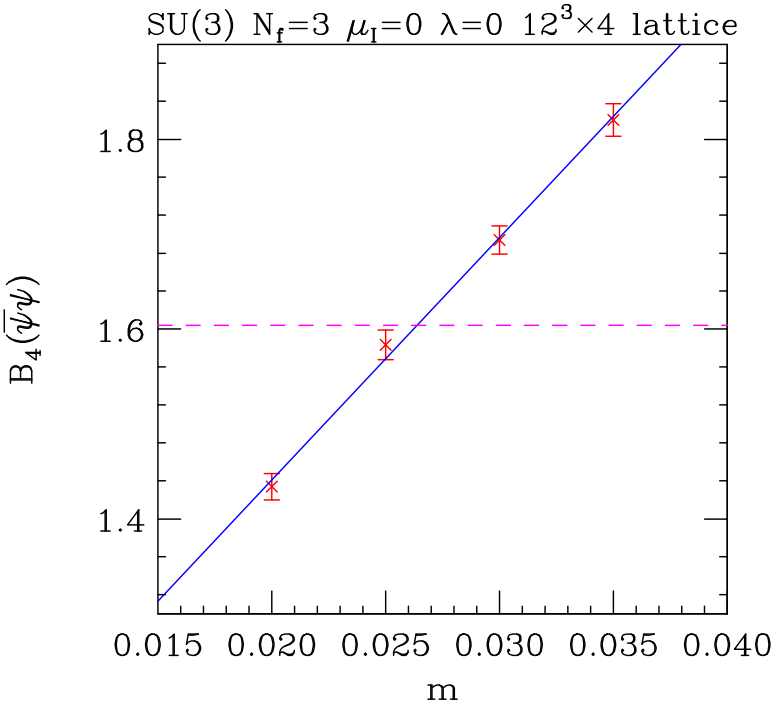

Figure 1: Binder cumulant at $T = T_c$ as a function of mass at $\mu_I = \mu = 0$.

Figure 1 shows our preliminary measurements of the Binder cumulant for the chiral condensate as a function of mass at $\mu_I = \mu = 0$ from our $12^3 \times 4$ simulations. Taking the point where the straight-line fit passes through the Ising value as our estimate for the critical mass yields $m_c =$

0.0264(3). Each of the points in this graph were obtained by averaging the Binder cumulants taken from several β values close to the transition, and extrapolated to β_c which minimizes these cumulants, using Ferrenberg-Swendsen rewieghting [21].

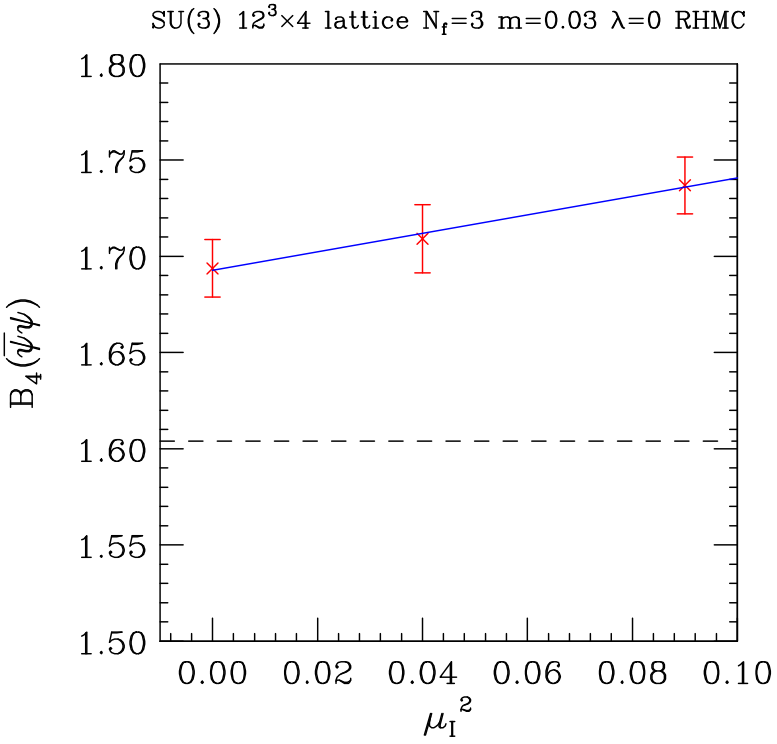

Figure 2: Binder cumulant at $T = T_c$ as a function of μ_I^2 at $m = 0.03$.

The μ_I dependence of this Binder cumulant at $\beta_c(\mu_I)$ is shown in figure 2, for $m = 0.03$, a little above m_c. It is clear that, rather than decreasing with increasing μ_I, it actually increases slowly. Since β_c and hence T_c decrease with increasing μ_I, in physical units m is actually decreasing with increasing μ_I meaning that at fixed physical m the rise would be even more pronounced. The behaviour at $m = 0.035$ is very similar.

Figure 3 shows the dependence of the transition β, β_c, on μ_I. As mentioned above, β_c and hence the transition temperature T_c fall (slowly) with increasing μ_I as expected. The fits shown to this preliminary 'data' are:

$$\begin{aligned} \beta_c &= 5.15326(10) - 0.173(2)\mu_I^2 \qquad m = 0.035 \\ \beta_c &= 5.14386(\ 8) - 0.172(1)\mu_I^2 \qquad m = 0.030 \\ \beta_c &= 5.13426(12) - 0.179(4)\mu_I^2 \qquad m = 0.025 \end{aligned} \tag{3.2}$$

which is in reasonable agreement with the results of de Forcrand and Philipsen for the μ dependence of the transition temperature, obtained from analytic continuation from imaginary μ if we make the identification $\mu_I = 2\mu$.

Figure 4 shows the dt dependence of the Binder cumulants at the transition for $m = 0.035$, $\mu_I = 0.2$ in the HMD(R) simulations. The exact RHMC result, which has no dt dependence, is

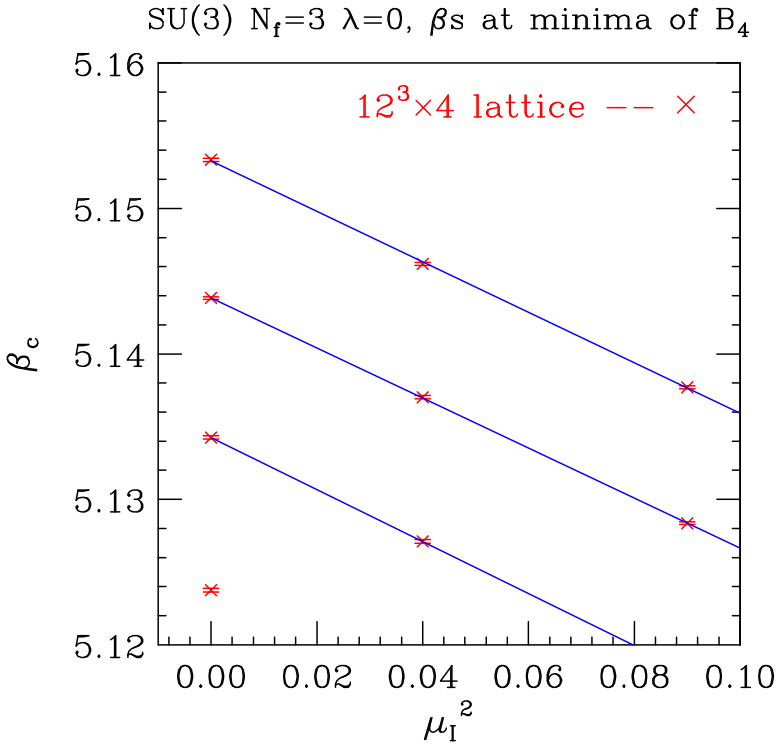

Figure 3: β_c as functions of μ_I^2. From top to bottom, $m = 0.035$, $m = 0.03$, $m = 0.025$, $m = 0.02$.

plotted on this graph at $dt = 0$. It is clear that the RHMC result is consistent with the $dt \to 0$ limit of the HMD(R) results. The actual value of dt used in the RHMC simulations was $dt = 0.05$, the value of dt for the rightmost point on this graph, showing one advantage of using this new algorithm.

4. Conclusions

We simulate lattice QCD with 3 flavours of staggered quarks with a small chemical potential $\mu_I < m_\pi$ for isospin, in the neighbourhood of the finite temperature transition from hadronic matter to a quark-gluon plasma. Fourth order Binder cumulants are used to probe the nature of this transition and search for the critical endpoint for masses slightly above the critical mass for zero chemical potentials. Earlier simulations using the HMD(R) algorithm were plagued by large finite dt errors [15]. We now use the RHMC algorithm which is exact in the sense of having no finite dt errors.

We measure the critical mass to be $m_c = 0.0264(3)$ for $N_t = 4$, in agreement with the recent results of de Forcrand and Philipsen [5], but considerably below earlier measurements which found values close to $m_c = 0.033$ [22, 23, 24]. These higher values were due to using the HMD(R) algorithm with dt large enough to produce large systematic errors.

For masses greater than m_c we found that the Binder cumulant for the chiral condensate increases with increasing μ_I and thus shows no evidence for a critical endpoint, contrary to earlier expectations. This also agrees with the observations of de Forcrand and Philipsen [5] for finite μ,

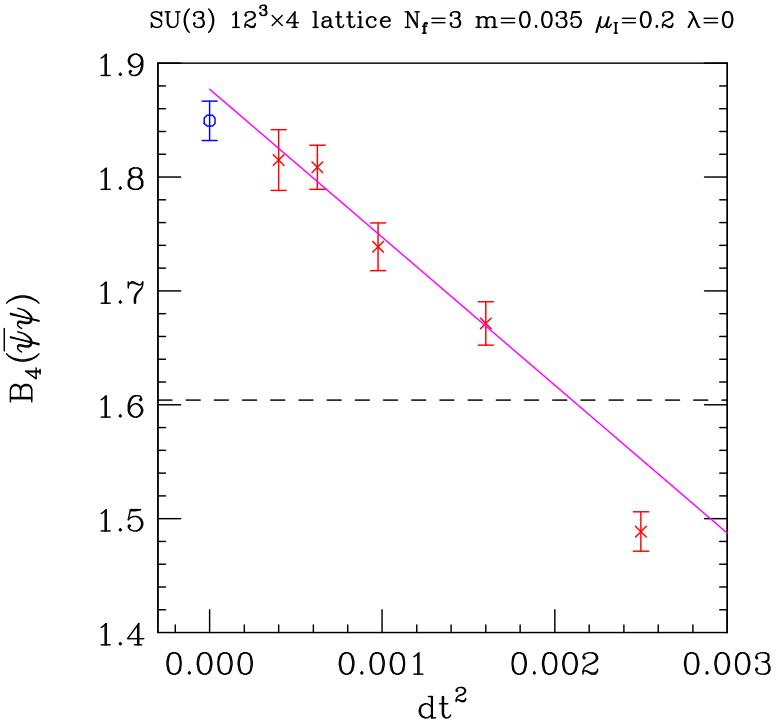

Figure 4: Binder cumulants for $m = 0.035$, $\mu_I = 0.2$ for HMD(R) simulations as a function of dt^2 compared with that from RHMC simulations.

emphasizing the similarities between finite μ and finite μ_I for small μ,μ_I, near the finite temperature transition. On these relatively small lattices ($12^3 \times 4$), we really should minimize the Binder cumulant of linear combinations of the chiral condensate, the plaquette and the isospin density to obtain the desired eigenfield of the renormalization group equations, to draw reliable conclusions [22] [1].

We end with the observation that we have used RHMC simulations where we do not know a positive lower bound for the spectrum of the quadratic Dirac operator. This is done by choosing a speculative lower bound and justifying our choice a postiori. We refer the reader to our recent paper on this subject [19].

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References

[1] C. R. Allton *et al.*, Phys. Rev. D **66**, 074507 (2002) [arXiv:hep-lat/0204010].

[1] We thank Frithjof Karsch for reminding us of this fact

[2] C. R. Allton *et al.*, Phys. Rev. D **71**, 054508 (2005) [arXiv:hep-lat/0501030].

[3] R. V. Gavai and S. Gupta, Phys. Rev. D **68**, 034506 (2003) [arXiv:hep-lat/0303013].

[4] P. de Forcrand and O. Philipsen, Nucl. Phys. B **642**, 290 (2002) [arXiv:hep-lat/0205016].

[5] P. de Forcrand and O. Philipsen, arXiv:hep-lat/0607017.

[6] V. Azcoiti, G. Di Carlo, A. Galante and V. Laliena, JHEP **0412**, 010 (2004) [arXiv:hep-lat/0409157].

[7] V. Azcoiti, G. Di Carlo, A. Galante and V. Laliena, Nucl. Phys. B **723**, 77 (2005) [arXiv:hep-lat/0503010].

[8] M. D'Elia and M. P. Lombardo, Phys. Rev. D **67**, 014505 (2003) [arXiv:hep-lat/0209146].

[9] M. D'Elia and M. P. Lombardo, Phys. Rev. D **70**, 074509 (2004) [arXiv:hep-lat/0406012].

[10] Z. Fodor and S. D. Katz, Phys. Lett. B **534**, 87 (2002) [arXiv:hep-lat/0104001].

[11] Z. Fodor and S. D. Katz, JHEP **0404**, 050 (2004) [arXiv:hep-lat/0402006].

[12] S. Kratochvila and P. de Forcrand, PoS **LAT2005**, 167 (2006) [arXiv:hep-lat/0509143].

[13] A. Alexandru, M. Faber, I. Horvath and K. F. Liu, Phys. Rev. D **72**, 114513 (2005) [arXiv:hep-lat/0507020].

[14] J. B. Kogut and D. K. Sinclair, Phys. Rev. D **66**, 034505 (2002) [arXiv:hep-lat/0202028].

[15] J. B. Kogut and D. K. Sinclair, arXiv:hep-lat/0509095.

[16] S. A. Gottlieb, W. Liu, D. Toussaint, R. L. Renken and R. L. Sugar, Phys. Rev. D **35**, 2531 (1987).

[17] A. D. Kennedy, I. Horvath and S. Sint, Nucl. Phys. Proc. Suppl. **73**, 834 (1999) [arXiv:hep-lat/9809092].

[18] M. A. Clark, P. de Forcrand and A. D. Kennedy, PoS **LAT2005**, 115 (2006) [arXiv:hep-lat/0510004].

[19] J. B. Kogut and D. K. Sinclair, arXiv:hep-lat/0608017.

[20] K. Binder, Z. Phys. B **43**, 119 (1981).

[21] A. M. Ferrenberg and R. H. Swendsen, Phys. Rev. Lett. **61**, 2635 (1988).

[22] F. Karsch, E. Laermann and C. Schmidt, Phys. Lett. B **520**, 41 (2001) [arXiv:hep-lat/0107020].

[23] N. H. Christ and X. Liao, Nucl. Phys. Proc. Suppl. **119**, 514 (2003).

[24] P. de Forcrand and O. Philipsen, Nucl. Phys. B **673**, 170 (2003) [arXiv:hep-lat/0307020].