Update on the physics of light pseudoscalar mesons

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We present an update of the MILC investigation of the properties of light pseudoscalar mesons using three flavors of improved staggered quarks. Results are presented for the π and K leptonic decay constants, the CKM matrix element V_{us}, the up, down and strange quark masses, and the coefficients of the $\mathcal{O}(p^4)$ chiral lagrangian. We have new data for lattice spacing $a \approx 0.15$ fm with several values of the light quark mass down to one-tenth the strange quark mass, higher statistics for $a \approx 0.09$ fm with the light quark mass equal to one-tenth the strange quark mass, and initial results for our smallest lattice spacing, $a \approx 0.06$ fm with light quark mass two-fifths of the strange quark mass.

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1. Introduction

We present an update of our long term investigation of properties of π and K mesons [1, 2, 3] with three flavors of improved staggered quarks [4]. Study of the $\pi - K$ system is interesting because it enables: 1) a sensitive check of algorithms and methods, including the use of the fourth root of the determinant for dynamical staggered quarks, by comparing lattice results for f_π to the well-determined experimental value; 2) a precise extraction of the CKM matrix element V_{us} from f_K or f_K/f_π, competitive with the world-average from alternative methods; 3) a determination of the light quark masses and their ratios with high precision; 4) a calculation of several of the physical coefficients of the $\mathscr{O}(p^4)$ chiral Lagrangian, the Gasser-Leutwyler L_i; 5) a test of the applicability of staggered chiral perturbation theory (S$\mathcal{X}$PT) [5, 6, 7] for describing lattice data; and 6) a determination of the extra, unphysical parameters that enter S$\mathcal{X}$PT. This allows, for example, use of heavy-light S$\mathcal{X}$PT [8] for computations of decay constants and form factors in D and B systems [9] without introducing large additional uncertainties.

2. Gauge Configurations and Chiral Fits

The MILC Collaboration has generated an extensive set of gauge configurations with three flavors of improved staggered quarks: $m'_u = m'_d \equiv \hat{m}'$, and m'_s. (Primes indicate simulation values; corresponding masses without primes are the physical values). In this work we report results for ensembles with lattices spacings $a \approx 0.15$, 0.12, 0.09 and 0.06 fm, which are referred to as coarser, coarse, fine and super-fine lattices, respectively. Coarser, coarse and fine lattices are available with a range of light quark masses, the lowest pion mass being $m_\pi \approx 240$ MeV. A run on the super-fine lattices with $m_\pi \approx 430$ MeV is half-finished, and lighter-mass runs are being started. The simulation strange quark masses m'_s are in the range $0.70 m_s \lesssim m'_s \lesssim 1.2 m_s$. The physical volumes of the lattices range from $\approx (2.4 \text{ fm})^3$ to $\approx (3.4 \text{ fm})^3$, with the large volumes being used for ensembles with the lightest quark masses. The lattice spacing is kept fixed within each ensemble (coarser, coarse, fine and super-fine) as the light quark mass is varied, using the length r_1 [10, 11] from the static quark potential to set the scale. The absolute scale is set from the Υ 2S-1S splitting determined by the HPQCD Collaboration [12, 13] on most of our lattices. From their results we find $r_1 = 0.318(7)$ fm.

For Goldstone masses and decay constants, we have extensive partially quenched data, typically all combinations of 8 or 9 valence quark masses between $0.1 m'_s$ and m'_s. Goldstone quantities have the smallest statistical errors, so we concentrate on them. We fit decay constants and masses together including all correlations, and we fit different lattices spacings together. The confidence level of the joint fit is 0.99. Statistical errors are very small: 0.1% to 0.8% for squared masses, and 0.1% to 1.4% for decay constants.

In order to obtain good fits to S$\mathcal{X}$PT forms, we need to place upper limits on quark masses. We consider two data sets:

Subset I: In this subset the valence Goldstone pion masses are restricted to be $\lesssim 350$ MeV, and the other pion masses to be $\lesssim 550$ MeV for the coarse ensemble, $\lesssim 460$ MeV for the fine ensemble, and $\lesssim 400$ MeV for the super-fine ensemble. The largest sea quark masses and the coarser ensemble are excluded, leaving a total of 122 data points. We expect errors of NLO S$\mathcal{X}$PT to be of order 0.8%, implying that NNLO terms are needed. NNLO S$\mathcal{X}$PT logs are unknown; however,

for higher masses, where NNLO terms are important, such logs should be smoothly varying and well approximated by NNLO analytic terms. The NNLO fit has 20 unconstrained parameters. An additional 6 tightly constrained parameters allow for variation of physical LO and NLO parameters with the lattice spacing ($\sim \alpha_S a^2 \Lambda_{QCD}^2 \approx 2\%$), giving a total of 26 parameters. This subset is used to determine the L_i, and the systematic errors on other quantities.

Subset II: In this subset the valence Goldstone pion masses are restricted to be $\lesssim 570$ MeV, and the other pion masses to be $\lesssim 780$ MeV for the coarser ensemble, $\lesssim 725$ MeV for the coarse ensemble, $\lesssim 630$ MeV for the fine ensemble, and $\lesssim 590$ MeV for the super-fine ensemble. All sea quark masses and the coarser ensemble are included, leaving 978 data points. Even NNLO fits break down for this subset. We therefore add 18 NNNLO analytic terms and 10 tightly constrained variations of NNLO parameters with a. We fix (within errors) LO and NLO terms from the fit to subset I. We are then left with 28 unconstrained parameters and 26 tightly constrained ones. This data set is not chiral, but it is used to interpolate around the physical value of m_s. The central values of decay constants and quark masses are determined from it.

We emphasize that the necessity for using high order SχPT fits with large numbers of parameters arises from the very small statistical errors in our data. If we did not care about the confidence level, we could perform a NLO SχPT fit with only 12 parameters. It changes f_π by 4%, f_K by 1%, and m_s by 3% on Subset I; however, it has $\chi^2/d.o.f. = 9.5$ for 110 $d.o.f.$. Similarly, the NLO SχPT fit with taste-violating parameters given as input and the lattice-spacing dependence of physical parameters set to zero has 6 parameters. It changes f_π by 2%, f_K by 1% and m_s by 0.5% for Subset I; but it has $\chi^2/d.o.f. = 40.5$ for 110 $d.o.f.$. Furthermore, such fits can change the low energy constants by 3 or 4 σ. It is important to note that if the physics is not correct, one will not be able to fit the data even with ~ 40 parameters. This was illustrated by a series of fits in Ref. [1]. There, comparable fits to the continuum χPT form had 36 parameters, but $\chi^2/d.o.f. = 8.8$ for 204 $d.o.f.$, for a confidence level of 10^{-250}. Fits with all chiral logs and finite volume corrections omitted from the fit function (analytic function only) were poor, providing good evidence for chiral logs. Finally, separate linear fits for m_π^2 or f_π as a function of the average valence quark mass were tried. For m_π^2 there were 6 parameters and $\chi^2/d.o.f. \approx 20$ for 234 $d.o.f.$, while for f_π there were 10 parameters, and $\chi^2/d.o.f. \approx 25$ for 230 $d.o.f.$.

3. Taste Symmetry and Chiral Logs

Violations of taste symmetry for staggered quarks are most apparent in the pion sector. Figure 1 shows the splitting between the square of Goldstone pion mass and the square of the masses of pions with other tastes for the coarse, fine and super-fine ensembles with $\hat{m}' = 0.4 m_s'$. The splittings are plotted as a function of $a^2 \alpha_s^2$, the variable in which they are expected to be linear at small lattice spacings. Figure 2 shows the square of the pion masses as a function of the light quark mass. The degeneracies predicted by Lee and Sharpe in Ref. [5] are clearly visible. These two figures provide strong evidence that taste symmetry violations are well understood from SχPT, and that they decrease as expected with decreasing lattice spacing.

The data is accurate enough and at small enough quark masses so that the effects of chiral logs are evident. This is illustrated in Fig. 3 where the square of the pion mass divided by the sum of the valence quark masses $m_x + m_y$ is plotted as a function of $m_x + m_y$ in units of r_1. The mass
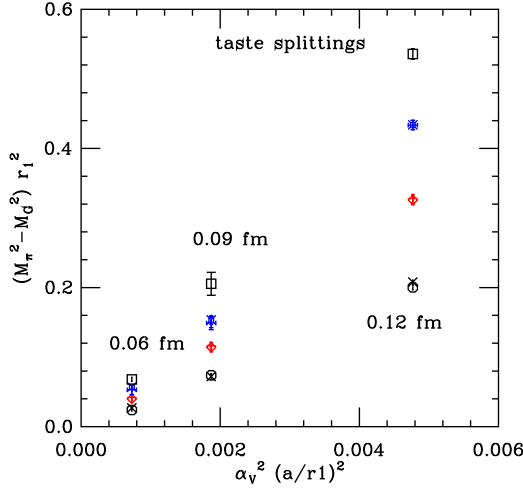
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Figure 1: Taste splitting among the pions for lattice spacings 0.12, 0.09 and 0.06 fm with $\hat{m}' = 0.4 m'_s$.
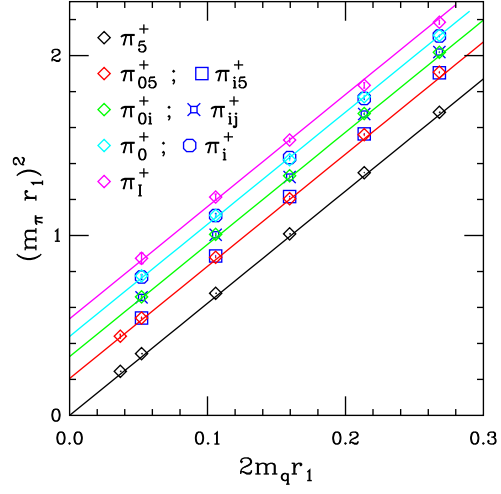

Figure 2: Squared pion masses as a function of the light quark mass on the coarse lattices.

renormalization constant, relative to that of the fine lattices, has been included so that data from all lattice spacings can be presented on the same plot. Lines through the data points come from the SX̄PT fit to the entire data set for decay constants and masses discussed above. The upward slope in these lines at small valence quark masses is caused by partially quenched chiral logs, which are of the form $(m^2_{sea,G} + \Delta_i a^2 \alpha^2_S) \log(m^2_{val,G} + \Delta_i a^2 \alpha^2_S)$, where $m_{val,G}$ and $m_{sea,G}$ are the valence and sea Goldstone pion masses, respectively, and Δ_i parametrizes the splitting of the taste-i pion. The effect is especially pronounced in the super-fine data because, not only are the taste-violating mass splittings rather small, but the light sea quark mass is rather large, $\hat{m}' = 0.4 m'_s$. The red line is the fit function in "full continuum QCD" (valence and sea quark masses set equal, and extrapolation of the parameters to the continuum limit). It is much smoother because it does not have partially quenched chiral logs.

4. Preliminary Results

Our previously published work [1, 2, 3] was based on analysis of the coarse and fine ensembles. Here we present preliminary results obtained by including the coarser and super-fine ensembles, and additional data from the $\hat{m}' = 0.1 m'_s$ run in the fine ensemble.

Figure 4 shows fit for the leptonic decay constant of the pion as a function of the sum of the valence quark masses, in units of the scale r_1. Lines through the data points come from the same SX̄PT fit as in Fig. 3. The red lines represent the fit function in "full continuum QCD" with the strange sea quark mass fixed to its physical value. The red plus shows the extrapolated value of f_π to the physical point, in agreement with experiment (black burst) to within systematic errors (blue bar).

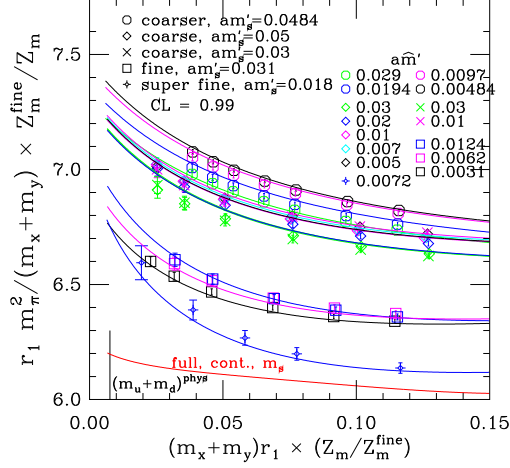Preliminary numerical results for f_π and f_K obtained from our most recent data are

Figure 3: The square of the pion mass divided by the sum of the valence quark masses as a function of the sum of the valence quark masses in units of r_1.
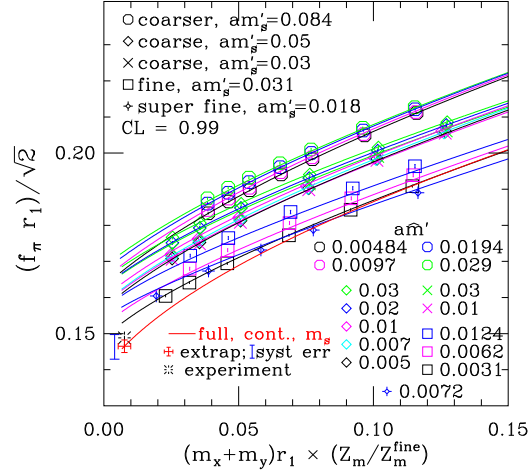
Figure 4: The pion decay constant as a function of the sum of the valence quark masses in units of r_1.

$$f_\pi = 128.6 \pm 0.4 \pm 3.0 \text{ MeV} \quad [129.5 \pm 0.9 \pm 3.5 \text{ MeV}] \tag{4.1}$$

$$f_K = 155.3 \pm 0.4 \pm 3.1 \text{ MeV} \quad [156.6 \pm 1.0 \pm 3.6 \text{ MeV}] \tag{4.2}$$

$$f_K/f_\pi = 1.208(2)(^{+}_{-14}) \quad [1.210(4)(13)]. \tag{4.3}$$

Here the numbers on the left are the new values, and those on the right in square brackets are from Refs. [1, 2, 3]. In each case the first error is statistical, and the second systematic. The result for f_π obtained from the experimental rate for $\pi \to \mu\nu$ coupled with the value of V_{ud} from super allowed nuclear beta decay is $f_\pi = 130.7 \pm 0.1 \pm 0.4$ MeV. The agreement of our result with the experimental one provides important evidence that we do understand and can control our errors. Marciano has pointed out [14] that lattice results for f_K/f_π can be combined with the experimentally determined rate for $K \to \mu\nu$ and V_{ud} to calculate V_{us}. We find

$$|V_{us}| = 0.2223(^{+26}_{-14}) \quad [0.2219(26)]. \tag{4.4}$$

Again, our latest result is on the left with the previous one in square brackets on the right. The Particle Data Group (2006) gives $V_{us} = 0.2257(21)$ [15] from the $K \to \pi\mu\nu$ experimental rate and non-lattice theory. Lattice errors continue to dominate experimental ones in our determination of V_{us}, and they will be reduced as additional super-fine lattices become available.

The up, down and strange quark masses can be determined from the masses of the π and K mesons using the SχPT fits. To do so we must distinguish between experimental masses, QCD masses in which electromagnetism has been turned off, and those in which both electromagnetism and isospin violations are turned off. The last of these, which we denote by $m_{\hat\pi}$ and $m_{\hat K}$, are used

to determine the physical values of the bare quark masses $\hat{m}$ and m_s from the SχPT fits. They are related to the QCD masses by

$$m_{\hat{\pi}}^2 \approx (m_{\pi^0}^{QCD})^2 \tag{4.5}$$

$$m_{\hat{K}}^2 \approx [(m_{K^0}^{QCD})^2 + (m_{K^+}^{QCD})^2]/2. \tag{4.6}$$

To relate the QCD masses to the experimental ones requires some continuum input [16]. Details can be found in Ref. [1]. Once $\hat{m}$ and m_s have been determined, m_u can be obtained from $m_{K^+}^{QCD}$. Preliminary results from our current data set are:

$$m_s^{\overline{MS}} = 90(0)(5)(4)(0) \text{ MeV} \quad [76(0)(3)(7)(0) \text{ MeV}] \tag{4.7}$$

$$\hat{m}^{\overline{MS}} = 3.3(0)(2)(2)(0) \text{ MeV} \quad [2.8(0)(1)(3)(0) \text{ MeV}] \tag{4.8}$$

$$m_s/\hat{m} = 27.2(0)(4)(0)(0) \quad [27.4(1)(4)(0)(1)] \tag{4.9}$$

$$m_u^{\overline{MS}} = 2.0(0)(1)(1)(1) \text{ MeV} \quad [1.7(0)(1)(2)(2) \text{ MeV}] \tag{4.10}$$

$$m_d^{\overline{MS}} = 4.6(0)(2)(2)(1) \text{ MeV} \quad [3.9(0)(1)(4)(2) \text{ MeV}] \tag{4.11}$$

$$m_u/m_d = 0.42(0)(1)(0)(4) \quad [0.43(0)(1)(0)(8)] \tag{4.12}$$

Again, new results are on the left, and early ones in this case from Refs. [1, 13, 17] are in square brackets on the right. Errors are from statistics, simulation systematics, perturbation theory and electromagnetic effects. The $\overline{MS}$ scale is $\mu = 2$ GeV. The main difference between the new and old results comes from the perturbation theory calculation of the renormalization constants needed to match the lattice masses to the $\overline{MS}$ masses. Here we use the new two-loop results of Mason, Trottier and Horgan [18], whereas the old results were based on a one-loop calculation. A non-perturbative mass renormalization calculation is in progress. Note that our result for m_u/m_d rules out a massless m_u at the 10σ level.

Finally, we have determined a number of the low energy constants L_i at the scale of m_η. Our preliminary results, in units of 10^{-3}, are:

$$2L_6 - L_4 = 0.5(1)(2) \quad [0.5(2)(4)] \tag{4.13}$$

$$2L_8 - L_5 = -0.1(1)(1) \quad [-0.2(1)(2)] \tag{4.14}$$

$$L_4 = 0.1(2)(2) \quad [0.2(3)(3)] \tag{4.15}$$

$$L_5 = 2.0(3)(2) \quad [1.9(3)(3)]. \tag{4.16}$$

Again, the new values are on the left, and the older ones from Ref. [1] are in square brackets on the right. These results are consistent with "conventional" ones summarized in Ref. [19]: $L_5 = 2.2(5)$, $L_6 = 0.0(3)$, and $L_4 = 0.0(5)$. Our value for $2L_8 - L_5$ is far from the range $-3.4 \leq 2L_8 - L_5 \leq -1.8$ that would allow $m_u = 0$, which is consistent with, but not independent of, our direct determination of m_u. Our old results for the low energy constants already play a significant role in setting constraints on the $\pi - \pi$ scattering lengths a_0^2 and a_0^0 [20]. It will be interesting to see the impact of the new ones.

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