PROCEEDINGS OF SCIENCE

Mass spectrum of 1^{-+} exotic mesons using the Maximum Entropy Method

M.S. Cook* **and H.R. Fiebig**

Physics Department, Florida International University, Miami, Florida 33199 USA
E-mail: mcook003@fiu.edu

Time correlation functions for an exotic hybrid meson, $J^{PC} = 1^{-+}$, are analyzed using the maximum entropy method (MEM). The MEM analysis provides both ground and excited state energies. Both the location and strength of each spectral mass density peak are determined when a simulated annealing process is employed with the MEM analysis. We extract ground and excited states of about 1.4, 1.6, and 2.2 GeV when extrapolating to the physical pion mass.

XXIVth International Symposium on Lattice Field Theory
23-28 July 2006
Tucson, Arizona, USA

*Speaker.

PoS(LAT2006)172

1. Introduction

Quantum Chromodynamics(QCD) allows both quarks and gluons to exist together in states having quantum numbers that are outside of the standard quark model. In the meson sector, these states are termed exotic hybrids, having quantum numbers such as $J^{PC} = 0^{+-}, 1^{-+}$ and 2^{+-}. According to [1], experimentally established 1^{-+} mesons are the $\pi_1(1400)$ and $\pi_1(1600)$. One expects the lattice formulation of QCD to eventually give properties, such as masses and decay widths, of these exotic hybrid states. Toward this end, we examine the mass spectrum of a 1^{-+} exotic meson using lattice QCD data generated previously for a decay width calculation [2]. The novel feature of this application is the use of Bayesian analysis, specifically the maximum entropy method (MEM).

The MEM offers several advantages in extracting hadron masses. First, the entire range of lattice time slices is used, thus eliminating the subjective choice of a plateau, as is typically done for an effective mass function. By using all time slices the MEM can deliver unbiased estimates of the extracted masses. Second, if multiple or exited states are present, these are naturally available with MEM. Third, one can also determine the probability of creating a certain state out of the QCD vacuum with a particular operator. We rely on these features to extract the 1^{-+} exotic hybrid meson mass spectrum, up to ≈ 2 GeV.

2. Lattice simulation

For this simulation we employ the same lattices and propagators used in [2]. There we expected systematic errors to dominate, and hence, kept the numerical effort modest by employing a simple Wilson gauge field action and Wilson fermions in the quenched approximation. Results will be presented from an anisotropic lattice of size $12^3 \times 24$ having bare aspect ratio $a_s/a_t = 2$ with a_s being the spatial and a_t the temporal lattice constants. A total of 200 gauge field configurations were used with a global gauge coupling $\beta = 6.15$.

For the exotic hybrid meson, referred to as h, we require an operator with quantum numbers $I = 1$ and $J^{PC} = 1^{-+}$. Following [3] we adopt an operator of the form,

$$O_{h^+;j}(t) = \frac{1}{\sqrt{V}} \sum_{\vec{x}} \sum_{i=1}^{3} \bar{d}_a(\vec{x}t)\gamma_i u_b(\vec{x}t)(F_{ij}^{ab}(\vec{x}t) - F_{ij}^{\dagger ab}(\vec{x}t)) . \tag{2.1}$$

where a, b are color indices, V is the spatial lattice volume, and F_{ij} is a product of SU(3) link matrices defined on paths forming a clover leaf in the i–j plane with center at $\vec{x}$, as described in [2]. Here the combination $F_{ij} - F_{ij}^{\dagger}$ is needed for positive charge conjugation, and the clover leafs are summed in the spatial planes only, thus representing magnetic type gluons.

Regarding charge conjugation symmetry, this can be made exact with the operator (2.1) by using equal amounts of $[U]$ and $[U^*]$ gauge fields. So with each $[U]$ in the 200 configurations of gauge fields we also compute propagators for $[U^*]$ because both $[U]$ and $[U^*]$ are equally probable. This doubles the number of propagators required, but besides making for exact charge conjugation, it also appears to reduce noise levels in the hybrid correlator signal.

A parity transformation $\mathscr{P}$ applied to (2.1) gives, as required, $\mathscr{P} O_h(t) \mathscr{P}^{-1} = -O_h(t)$. This relies on $\mathscr{P} U_i(\vec{x}, t) \mathscr{P}^{-1} = U_{-i}(-\vec{x}, t)$ for $i = 1, 2, 3$. Because the quenched gauge field action $S[U]$ satisfies $\mathscr{P} S[U] \mathscr{P}^{-1} = S[U]$ only in the limit of a large number of gauge field configurations, the

κ	$a_t m_\pi$	$m_\pi[GeV]$
.140	.50(1)	1.39(3)
.136	.60(1)	1.67(3)
.132	.71(1)	1.97(3)
.128	.82(1)	2.28(3)

Table 1: Hopping parameter values κ and resulting pion masses using units of the temporal lattice constant a_t, and in GeV, for a $12^3 \times 24$ lattice.

propagator for h only respects exact (negative) parity in the limit of a large number of gauge fields. Otherwise it's parity is only approximate. Thus, by using only 200 gauge fields, one can expect some contamination from the wrong (positive) parity state. This becomes evident in the discussion of results.

Wuppertal style smearing is done on the quark fields, using APE style gauge link fuzzing in the process. We use a common strength factor ($\alpha = 2.5$) and either $1,2$, and 3 smearing iterations. The same smearing is done at source and sink points. A total of nine combinations of these iterations are possible, and in this way, the correlation function expands to a 3×3 hermitian matrix $C_h(t,t_0)_{3\times 3}$ with elements

$$C_{h\{k,l\}}(t,t_0) = \langle O_{h\{k\}}(t) O^\dagger_{h\{l\}}(t_0)\rangle - \langle O_{h\{k\}}(t)\rangle\langle O^\dagger_{h\{l\}}(t_0)\rangle \,. \tag{2.2}$$

Here the separable terms are zero due to choice of flavor structure (2.1), and the number of smearing iterations is denoted with $k,l = 1,2,3$. Similar matrices are constructed for π, ρ and a_1 meson correlators. To keep elements of the correlation matrix at the same magnitude, a rescaling is done after each smearing step [4].

Four Wilson fermion hopping parameters, $\kappa = 0.140$, 0.136, 0.132, 0.128, were used with a multiple mass solver [5]. Pion masses for each κ value are shown in Table 1. Extrapolations to $m_\pi = 0$ are done for the hybrid, a_1, and ρ mesons using the three parameter model,

$$M = p + qx + r\ln(1+x) \quad \text{with } x = (a_t m_\pi)^2 \,, \tag{2.3}$$

where $M = a_t m$ is the spectral mass for each meson and p,q,r are parameters. This model matches most of the energy-versus-m_π^2 data produced in this work. Its choice is purely empirical. Setting the physical scale to the ρ meson mass results in a lattice constant of $a_t = 0.07(1)$ fm.

3. Analysis

Time evolution of the eigenvalues of $C_X(t,t_0)$ determines the mass spectrum of the meson X, where X is h, a_1, ρ, π. We choose to diagonalize $C_X(t,t_0)$ directly, using singular value decomposition (SVD). The SVD gives,

$$C_X(t,t_0) = U_X(t,t_0)\,\Sigma_X(t,t_0)\,V^\dagger_X(t,t_0)\,, \tag{3.1}$$

where $U_X(t,t_0)$ and $V_X(t,t_0)$ are unitary in our case, and $\Sigma_X(t,t_0) = \mathrm{diag}(\sigma_{X;1}(t,t_0)\ldots\sigma_{X;K}(t,t_0))$ contains the singular values satisfying $\sigma_{X;k}(t,t_0) \geq 0$. If $C_X(t,t_0)$ is non-degenerate and positive

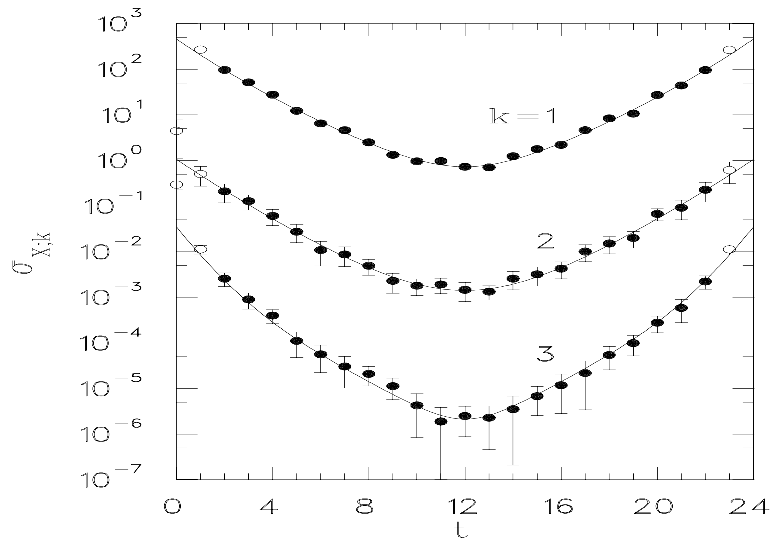

Figure 1: Eigenvalues $k = 1,2,3$ of the 3×3 correlation matrix $C_X(t,t_0)$ for the hybrid meson $X = h$, at the smallest pion mass. Error bars are statistical, derived from a jackknife procedure. The lines are MEM fits as explained in the text.

semi-definite then the set of eigenvalues $\{\lambda_{X;k}(t,t_0)|k = 1\ldots K\}$ and singular values $\{\sigma_{X;k}(t,t_0)|k = 1\ldots K\}$ are the same.

In Fig. 1 we show all eigenvalues, $\sigma_k = \lambda_k$, of $C_X(t,t_0)_{K\times K}$ for the hybrid meson operator, $X = h$, $K = 3$, computed at the smallest pion mass, i.e. the largest κ value. Diagonalizing is done independently on each time slice. Note that the eigenvalues are separated by almost three orders of magnitude.

The lattice provides correlation function data, say $\sigma(t)$, denoting any set $\sigma_{X;k}(t,t_0)$ on the chosen time slice range $t = 2\ldots 22$. Our data should be well described by the spectral model

$$F(\rho|t) = \int_0^\infty d\omega\,\rho(\omega)\cosh(\omega(t-t_c))\,, \tag{3.2}$$

where $t_c = 12$ and $\rho(\omega)$ is a spectral density function. To compute $F(\rho|t)$ given $\sigma(t)$ we use a conditional probability distribution function $P[\rho\leftarrow\sigma]$ with,

$$P[\rho\leftarrow\sigma] \propto P[\sigma\leftarrow\rho]P[\rho]\,. \tag{3.3}$$

Here $P[\sigma\leftarrow\rho]$ is known as the likelihood function, and $P[\rho]$ is the Bayesian prior [1, 6]. We construct $P[\sigma\leftarrow\rho]$ from the χ^2-distance between the data and the model

$$\chi^2 = \sum_{t_1,t_2} (\sigma(t_1) - F(\rho|t_1))\,\Gamma^{-1}(t_1,t_2)\,(\sigma(t_2) - F(\rho|t_2))\,, \tag{3.4}$$

with $\Gamma(t_1,t_2)$ being elements of the covariance matrix. Then $P[\sigma\leftarrow\rho] = \exp(-\chi^2/2)$ is the choice for the likelihood function. For the Bayesian prior we employ the Shannon-Jaynes entropy [7],

$$S = \int_0^\Omega d\omega\left(\rho(\omega) - m(\omega) - \rho(\omega)\ln\frac{\rho(\omega)}{m(\omega)}\right)\,. \tag{3.5}$$

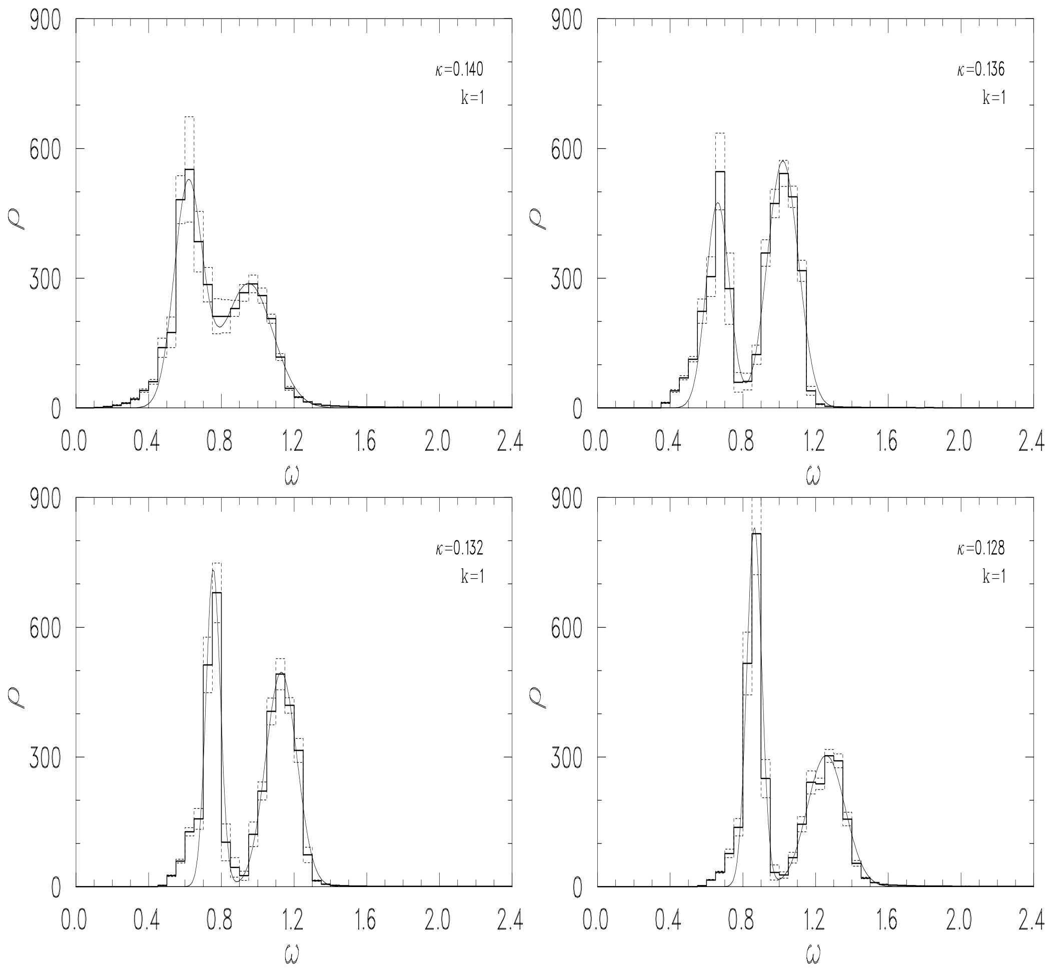

Figure 2: Spectral density functions (thick histogram lines) for the first (largest) eigenvalue set $\sigma_{h,k}$, $k = 1$, of the hybrid meson correlation matrix. The four panels correspond to increasing pion masses (decreasing hopping parameters κ) from left to right and top to bottom. The envelop histograms (dotted lines) indicate standard errors from 16 random starts. Gaussian fits are also shown (solid lines).

Here Ω is a cutoff energy, and the function $m(\omega)$ is called the default model. The idea then is to find a spectral density function ρ which maximizes $P[\rho \leftarrow \sigma]$, the posterior probability, at a fixed data set $\sigma(t)$. This problem is then solved by simulated annealing. Put in terms of a partition function Z_W,

$$Z_W = \int [d\rho] e^{-\beta_W W[\rho]} \quad \text{with} \quad W[\rho] = \chi^2/2 - \alpha S, \tag{3.6}$$

this involves generating equilibrium configurations $[\rho]$ while gradually increasing β_W from an initially small value, following some annealing schedule [6, 4].

Discretization of the ω integrals in (3.2) and (3.5) is required. We choose $a_t \Delta\omega = 0.05$ and $a_t \Omega = 2.4$ for the spectral mass cutoff in (3.5). The default model is a constant function $m(\omega) = 10^{-6} a_t^{-1}$ for all data sets $\sigma_{X;k}$, and the entropy strength α is slightly adjusted, in each case, to keep the ratio of αS to $\chi^2/2$ between 0.1 and 0.01, for the final $\rho(\omega)$. The parameter α, however, may be varied by several orders of magnitude without significantly effecting the results [6].

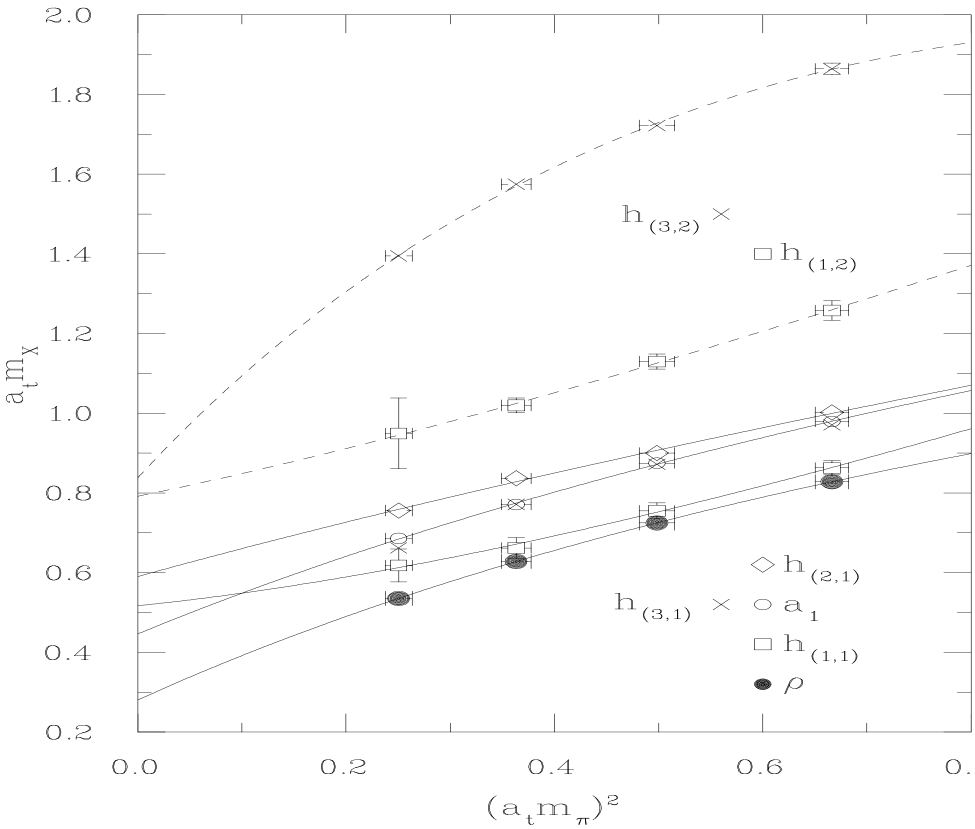

Figure 3: Plots of spectral meson masses versus the squared pion mass $x = (a_t m_\pi)^2$ and fits with the model (2.3). Solid lines refer to masses from primary ($n = 1$) spectral peaks, and dashed lines to secondary ($n = 2$) peaks, see Tab. 2. The extrapolation of the ρ meson mass to $x = 0$ is used to set the physical scale.

The spectral density functions for the first eigenvalue $\sigma_{h;1}$ of the hybrid meson correlation matrix are displayed in Fig. 2. Gaussian fits to $\rho(\omega)$ are used to resolve all of the spectral peaks in this work. Once the spectral peaks are known, they are then plotted versus $x = (a_t m_\pi)^2$, and extrapolations to $x = 0$ are attempted using the model (2.3). Results of these extrapolations are shown in Fig. 3 and in Tab. 2. A total of five spectral levels were uncovered for the hybrid meson h. Two eigenvalues of $C_h(t,t_0)_{3\times3}$ had two spectral peaks each, and the third eigenvalue had only one peak, for each pion mass used.

4. Discussion of results

We note, first, that the a_1 meson mass in Tab. 2 is very close to the experimental value 1230 MeV, which is the mass of the $a_1(1260)$ meson [1]. This leads us to believe the extrapolating model (2.3) is adequate, and therefore, the extrapolated results for the h mass spectrum, in Tab. 2, may also be accurate. Second, the levels $X_{(k,n)}$ for $h_{(1,1)}$ and $h_{(2,1)}$ in Tab. 2 are close to the experimental masses 1376 MeV and 1653 MeV of the 1^{-+} resonances known as $\pi_1(1400)$ and $\pi_1(1600)$ in Ref. [1]. Third, the level $h_{(3,1)}$ coincides with the one generated by the a_1 meson operator, especially at the three higher pion masses. This level comes from a correlator eigenvalue about four orders of magnitude less than the dominant one. It may, because of the limited number of gauge

X	k	n	$a_t E_X$	$a_t \Delta_X$	E_X[GeV]	Δ_X[GeV]
ρ	1	1	0.28(04)	0.08	0.7785	0.22
a_1	1	1	0.45(06)	0.17	1.23(0.17)	0.47
h	1	1	0.52(19)	0.37	1.43(0.53)	1.03
h	1	2	0.79(37)	0.65	2.19(1.03)	1.80
h	2	1	0.59(04)	0.27	1.63(0.12)	0.76
h	3	1	0.34(05)$\times$	0.20	0.94(0.13)$\times$	0.55
h	3	2	0.84(07)	0.11	2.32(0.18)	0.30

Table 2: Extrapolated spectral masses E_X and peak widths Δ_X, for mesons $X_{(k,n)}$. The eigenvalue label is k and the spectral peak number is n. The values for Δ_X and for the uncertainties (in parentheses) are obtained by randomization of the data points, as explained in [4].

fields used, single out the positive parity 1^{++} contamination of the lattice signal, which happens to be the quantum numbers of the a_1 meson [1]. Lastly, the largest levels $h_{(1,2)}$ and $h_{(3,2)}$ in Tab. 2 point at a mass somewhat above 2 GeV. We speculate that at least one of these levels coincides with the 1^{-+} resonance at 1.9 GeV uncovered in Ref. [2], where the space of operators was larger, including a πa_1 two-hadron field in addition to h, which could be the cause of a lowering of the energy level from ≈ 2 GeV to 1.9 GeV. This state could be a mixture of a hybrid meson and a two-meson state, thus supporting the outcome of Ref. [2].

5. Summary

Using the maximum entropy method, five distinct spectral levels have been uncovered for the $J^{PC} = 1^{-+}$ exotic meson. Two of the spectral levels correspond with the $\pi_1(1400)$ and $\pi_1(1600)$ from [1]. Two more levels possibly correspond with a resonance energy of 1.9 GeV previously determined by a decay width calculation [2]. The fifth spectral level, a consequence of inexact parity symmetry, tracks consistent with an operator representing the $a_1(1260)$ meson.

All of these spectral levels rely on extrapolations to $m_\pi = 0$ from relatively heavy pions. Although this may give rise to large systematic errors, the fact that the a_1 extrapolation came very close to its experimental value leads us to conclude at least two spectral levels for the 1^{-+} exotic meson will be below 2 GeV.

References

[1] S. Eidelman *et al.*, Physics Letters B, 592, 1+, 2004.

[2] M. S. Cook and H. R. Fiebig, Phys. Rev. D, 74, 034509, 2006.

[3] C. Bernard *et al.*, Phys. Rev. D, 56, 7040–7041, 1997.

[4] M. S. Cook and H. R. Fiebig, [hep-lat/0609010]

[5] U. Glässner *et al.*, Int. J. Mod. Phys. C, 7, 635, 1996, [hep-lat/9605008].

[6] H.R. Fiebig, Phys. Rev. D, 65, 094512, 2002.

[7] M. Jarrell and J.E. Gubernatis, Phys. Rep., 269, 133, 1996

PoS(LAT2006)172