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The isospin breaking effect on baryons with $N_f = 2$ domain wall fermions

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We study the isospin breaking effect on octet baryons. Using the two-flavor dynamical domain-wall QCD configurations combined with the quenched non-compact QED configurations, the electromagnetic mass splittings between isomultiplets $(p,n),(\Sigma^+,\Sigma^0,\Sigma^-),(\Xi^0,\Xi^-)$ are investigated. We evaluate the main source of statistical fluctuations in the two-point correlation function, and find that the elimination of $\mathcal{O}(e)$ fluctuation (e: the QED charge) is essential to extract the signal. Preliminary results for $m_p - m_n$ as well as other mass splittings are presented. Possible origin of systematic uncertainty is also discussed.

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1. Introduction

Recent development of the lattice QCD simulation enables us to access hadron properties with great accuracy. In fact, as a precise first principle calculation of QCD, the result of lattice QCD is now being used for examination of the fundamental theory, such as a unitarity triangle test of the standard model[1]. Yet, there are various quantities remained unsettled so far. Among them, we focus on the isospin breaking effect on baryons, such as proton-neutron ($p-n$) mass difference. Actually, this mass difference is one of the most fundamental quantities in nuclear physics. For instance, this quantity governs the β-decay of the neutron, i.e., the life time of the neutron. Note also that β-decay/electron capture are the basic ingredients in the understanding of the nucleosynthesis and the history of the universe: if the mass ordering between proton and neutron was opposite, the universe may not exist as it is now. In the laboratory-experiment, the isospin breaking effect on the baryon spectrum is observed not only for $p-n$ mass splitting but also for splitting between other isomultiplets in octet/decuplet members. The charge symmetry breaking in the $N-N$ interaction is also observed, which is caused by the isospin breaking of nucleons.

The isospin breaking of hadrons originates in two ingredients: one is the up and down quark mass difference in QCD, and the other is the electromagnetic (EM) effect based on QED. In this sense, the determination of u, d quark mass through the study of isospin breaking corresponds to fixing the Yukawa coupling constant between u, d quark and the Higgs particle, which are the fundamental (and unknown *a priori*) parameters in the standard model. Moreover, u, d quark masses are particularly interesting from the viewpoint of the so-called strong CP problem. In fact, it is proposed that the existence of massless quark(s) can absorb the phase θ of the vacuum, and thus resolve the strong CP problem. By studying the isospin breaking in hadron spectrum, we can examine whether or not such a scenario happens in our realistic world. On the other hand, the inclusion of QED in the QCD calculation is becoming the urgent task to develop the new physics search beyond the standard model. For instance, in the theoretical calculation of muon anomalous magnetic moment, it is know that hadronic light-by-light scattering contribution makes the large uncertainty, and it is proposed to resolve this problem using the QCD + QED simulation[2]. From this viewpoint, the study of isospin breaking on the spectrum is a suitable topic to build a firm foundation for the QCD + QED simulation.

In the literature of lattice QCD, the first work has been done by Duncan *et al.*[3], where they employed the quenched non-compact QED combined with quenched QCD with the Wilson fermion. In this pioneering work, however, both of the QCD quenching artifact and the artificial chiral symmetry breaking contamination are unavoidable. In this work, we eliminate these uncertainties by employing $N_f = 2$ dynamical domain-wall QCD configuration. Our study for the meson sector (such as $\pi^+ - \pi^0$, $K^+ - K^0$ mass splitting) has been already reported in Ref.[4], and we report the study of the EM effect on octet baryons in this proceeding. The effect of u, d quark mass difference will be reported elsewhere[5]. There is another work[6] on splitting between meson isomultiplets with improved action but still at the quenched level. Recently, the splitting of $p-n$ is also studied[7], while only the effect of u, d quark mass difference is considered there.

2. Formalism

We study the EM effect on baryons through the mass difference between isomultiplets, such as $m_p - m_n$, $m_{\Sigma^+} - m_{\Sigma^0}$, $m_{\Sigma^-} - m_{\Sigma^0}$, $m_{\Sigma^+} - m_{\Sigma^-}$, $m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0}$, $m_{\Xi^-} - m_{\Xi^0}$ using the QCD + QED simulation. In the QCD sector, we employ the $N_f = 2$ unquenched QCD gauge configuration $U_\mu^{\rm QCD}(x)$ generated by the RBC collaboration[8]. The domain-wall fermion action and the DBW2 gauge action is used with the parameters of $V = L^3 \times T = 16^3 \times 32$, $L_s = 12$, $M_5 = 1.8, \beta = 0.8$. The ensembles are generated for three different (u,d-) sea quark masses of $m_{\rm sea} = 0.02, 0.03, 0.04$. The lattice unit is determined to be $1.691(53)$ GeV so as to reproduce ρ meson mass $m_\rho = 770$MeV. The sea quark mass and the physical volume roughly correspond to $m_{\rm sea} \simeq \frac{1}{2} m_s - m_s (m_s : \text{strange quark mass})$ and $L^3 \simeq (1.9\text{fm})^3$, respectively. We pick up about 200 configurations from ~ 5000 trajectories available at each sea quark mass ensemble. The analysis is performed with bin size of 2, (i.e., bin size of 50 trajectory separation), in order to suppress the possible autocorrelation.

In the QED sector, we employ a non-compact formulation of $\mathscr{L}_{\rm QED} = \frac{1}{4} \sum_{x,\mu,\nu} \left(\partial_\mu A_\nu^{\rm QED}(x) - \partial_\nu A_\mu^{\rm QED}(x) \right)^2$ at the quenched level[3]. In the generation, we first generate $A_\mu^{\rm QED}$ in the momentum space under the Coulomb plus residual gauge fixing condition together with a boundary condition for the constant modes. The configuration in the coordinate space is obtained by Fourier transformation. The advantage of this formulation is that the generation of $A_\mu^{\rm QED}$ leads to just a Gaussian random number generation, and thus there is no autocorrelation between the configuration even for arbitrary small coupling. In addition to that, we do not have to worry about the renormalization of the QED coupling constant, because the quenched QED in non-compact formulation is a free theory. Given the QED configuration described above, we obtain a $U(1)$ link variable by $U_\mu^{\rm QED}(x) = \exp[-iA_\mu^{\rm QED}(x)]$, and then construct the QCD + QED configuration as $U_\mu^{\rm QCD}(x) \times (U_\mu^{\rm QED}(x))^Q$, where $Q = +2/3e, -1/3e$ for u and d quark, respectively. In order to study the QED charge dependence of the mass splitting, we use not only the physical QED charge, $\alpha_{\rm em} \equiv e^2/4\pi = \frac{1}{137} (\equiv \alpha_{\rm em}^{\rm phy})$, but also the charges of $\alpha_{\rm em} = (0), \frac{(0.6)^2}{4\pi}, \frac{(0.85)^2}{4\pi}, \frac{(1.0)^2}{4\pi}$ (except for $\alpha_{\rm em} = \frac{(0.85)^2}{4\pi}$ at $m_{\rm sea} = 0.02$).

The mass of baryon B is measured using the two-point correlation function $\Pi_{BB}(t) = \sum_{\vec{x}} \langle J_B(x) \bar{J}_B(0) \rangle$ with the use of positive parity projection. We use the operator J_B which has non-relativistic limit, for instance, $J_p = \varepsilon_{abc} (u_a^T C \gamma_5 d_b) u_c$ as the proton operator. The operators for other octets and singlet are obtained by the SU(3) rotation. Although these procedures are valid for $p, n, \Sigma^+, \Sigma^-, \Xi^-, \Xi^0$ baryons, additional treatment is necessary in the $Q = 0, S = -1$ channel, i.e., for $\Sigma^0, \Lambda_8, \Lambda_1$ baryons. In fact, the mixing between Λ_8 and Λ_1 occurs because of the SU(3) breaking, and the mixing between $\Sigma^0, \Lambda_8, \Lambda_1$ occurs because of the SU(2) breaking. Note here that $\Sigma^0(1193)$ is massive than $\Lambda(1116)$, experimentally. Therefore, in order to determine the mass splitting such as $m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0}$, we have to extract the Σ^0 state as a first excited state in $Q = 0, S = -1$ channel. For this purpose, we employ the so-called variational method[9]. In practice, we calculate not only the diagonal correlation functions as $\Pi_{B_f B_i}(t) = \sum_{\vec{x}} \langle J_{B_f}(x) \bar{J}_{B_i}(0) \rangle, B_i = B_f \in \{\Sigma^0, \Lambda_8, \Lambda_1\}$ but also the off-diagonal correlation functions as $\Pi_{B_f B_i}(t), B_i \neq B_f \in \{\Sigma^0, \Lambda_8, \Lambda_1\}$. By performing the diagonalization of the 3×3 correlation function matrix of $\Pi(t = t_0)^{-1} \cdot \Pi(t)$, we can extract not only the ground state Λ but also the first excited state Σ^0.

3. Numerical results and discussions

Using $N_f = 2$ unquenched domain-wall QCD configuration combined with the quenched QED configuration, we calculate the baryon two-point correlation functions for all octet and singlet baryons including the off-diagonal correlation functions in $Q = 0, S = -1$ channel. By solving the inverse of the Dirac operator under the QCD + QED configuration, the EM effect is automatically taken into account for the valence quark. In order to suppress the contamination from excited states, we employ the wall source and point sink correlation function under the Coulomb gauge fixing. In this study, we consider the isospin breaking effect up to the first order. Therefore, in the evaluation of the EM effect, we can use the quark masses without the EM effect. Namely, we perform the lattice simulation only at the unitarity point, $m_{u,\text{valence}} = m_{d,\text{valence}} = m_{\text{sea}} = 0.02, 0.03, 0.04$ for u, d quarks, and we use $m_s = 0.0446$ for s quark, which is determined from K-input[8].

In the $Q = 0, S = -1$ channel, we use the variational method to extract the first excited state Σ^0 as well as the ground state Λ. As described previously, we first calculate 3×3 correlation matrix $\Pi(t)$ using the flavor bases of $\Sigma^0, \Lambda_8, \Lambda_1$, and then diagonalize $\Pi(t = t_0)^{-1} \cdot \Pi(t)$, where the extracted eigenvalues correspond to the exponentially decaying correlation function for each ground/excited state. In the following analysis, we fix the arbitrary parameter t_0 as $t_0 = 1$. The dependence on t_0 is discussed later. In the study of the mass splitting between Σ triplets, we perform the similar procedure for Σ^+, Σ^- as well, i.e., we use $\Pi_{\Sigma\Sigma}(t)/\Pi_{\Sigma\Sigma}(t = t_0)$ instead of simple $\Pi_{\Sigma\Sigma}(t)$ where $\Sigma = \Sigma^+, \Sigma^-$. By employing this procedure, we can take the full advantage of the statistical correlation among Σ triplets and obtain the reasonable signal.

In order to study the QED charge dependence of the mass splitting, we evaluate the QED charges of $\alpha_{\text{em}} = (0), \alpha_{\text{em}}^{\text{phy}}, \frac{(0.6)^2}{4\pi}, \frac{(0.85)^2}{4\pi}, \frac{(1.0)^2}{4\pi}$ (except for $\alpha_{\text{em}} = \frac{(0.85)^2}{4\pi}$ at $m_{\text{sea}} = 0.02$). For each α_{em}, we calculate the correlation function for not only $e = +\sqrt{4\pi\alpha_{\text{em}}}$ but also $e = -\sqrt{4\pi\alpha_{\text{em}}}$ and take the average between them. This corresponds to the use of the QED configuration of $\{A_\mu^{\text{QED}}\}$ $\to \{A_\mu^{\text{QED}}, -A_\mu^{\text{QED}}\}$ with binning between A_μ^{QED} and $-A_\mu^{\text{QED}}$. In this procedure, $\mathcal{O}(e)$ contamination in the correlation function can be eliminated *a priori*. In fact, because the EM effect on physical observables appears only from $\mathcal{O}(e^2)$, such $\mathcal{O}(e)$ contamination is nothing but a statistical noise. Practically, we find that this procedure actually improve the S/N drastically, and is essential for the study of the baryon EM splitting.

In order to demonstrate the signal of the mass splitting, we analyze the ratio between the correlation functions of the isomultiplets. Considering the $p - n$ mass difference for example, we can express the correlation function for proton and neutron as $\Pi_{pp}(t) = \lambda_p \exp[-m_p t]$, $\Pi_{nn}(t) = \lambda_n \exp[-m_n t]$, respectively, where λ_p (λ_n) is the proton (neutron) overlap constant between the state and the operator. Noting that $(\lambda_p - \lambda_n)$ and $(m_p - m_n)$ are $\mathcal{O}(e^2)$, we can write the ratio of the proton and neutron correlation functions as

$$R_{p/n}(t) \equiv \Pi_{pp}(t)/\Pi_{nn}(t) = 1 + 2(\lambda_p - \lambda_n)/(\lambda_p + \lambda_n) - (m_p - m_n) \cdot t + \mathcal{O}(e^4). \tag{3.1}$$

Therefore, the slope of $R_{p/n}(t)$ in terms of the Euclidian time t directly corresponds to the $p - n$ mass splitting. In Fig. 1, we plot $R_{p/n}(t)$ in terms of t. For each QED charge, we find a clear negative linear slope, which indicates $m_p > m_n$ from the EM effect.

In the practical calculation of the mass splitting, we perform the exponential fit for each baryon correlation function and evaluate the mass difference, where the statistical error is estimated by the

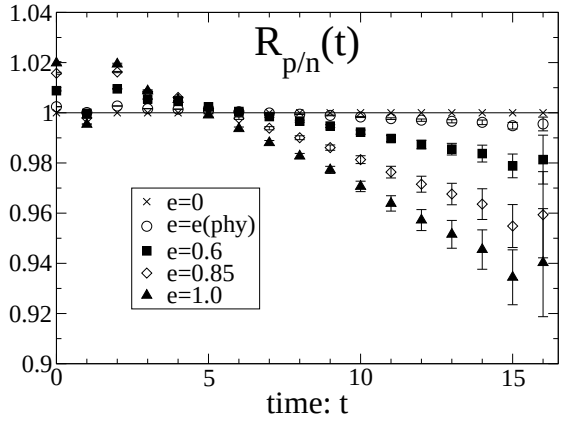

Figure 1: The ratio of the correlation function of proton and neutron $R_{p/n}(t)$ at $m_{\rm sea} = 0.03$ is plotted against t for each QED charge of $e = (0), e_{\rm phy}, 0.6, 0.85, 1.0$. Negative linear slope corresponds to $m_p > m_n$ from the EM effect.

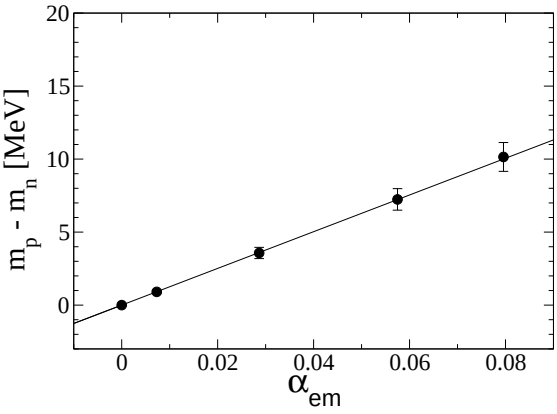

Figure 2: The proton-neutron mass difference $m_p - m_n$ from the EM effect at $m_{\rm sea} = 0.03$ is plotted against $\alpha_{\rm em} \equiv e^2/(4\pi)$. The result at $\alpha_{\rm em}^{\rm phy}$ is shown as the second point from the left. The solid line corresponds to the best linear fit in terms of $\alpha_{\rm em}$.

jackknife method. In Fig. 2, we plot the $p-n$ splitting determined at each QED charge $\alpha_{\rm em}$. We observe that the splitting behaves linearly in terms of $\alpha_{\rm em}$, and there is no indication of the appearance of higher order EM effect, $\mathcal{O}(\alpha_{\rm em}^2)$. Because we consider the EM effect up to $\mathcal{O}(\alpha_{\rm em})$ as a framework, this observation guarantees that our procedure is self-consistent.

Finally, we fit linearly the splitting in terms of $\alpha_{\rm em}$, and determine the splitting at $\alpha_{\rm em} = \alpha_{\rm em}^{\rm phy}$. We perform this procedure at each lattice simulation with quark mass of $m = m_{\rm valence} = m_{\rm sea} = 0.02, 0.03, 0.04$. Fig. 3 shows the result for $p-n$ mass splitting from the EM effect. We observe the non-trivial EM effect on $p-n$ for each m. Note that these are the first results obtained non-perturbatively using dynamical lattice simulation. It is interesting to see that the result at each m is roughly consistent with the model estimation using Cottingham formula, $m_p - m_n = 0.76(30)$MeV[10]. The splitting in the real world can be obtained by the chiral extrapolation of the lattice data. We, however, find that the result at $m = 0.02$ is afflicted with larger statistical fluctuation, and the reliable chiral extrapolation becomes difficult. In fact, the linear chiral extrapolation in terms of m leads to only zero-consistent result for $p-n$ mass difference. In order to extract better signal, the statistical improvement is in progress.

In Fig. 4, we also show the result of the EM splitting for $m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0}$ at each $m =$ 0.02, 0.03, 0.04. The splitting $m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0}$ is an interesting quantity from the viewpoint of discrimination of the two ingredients in isospin breaking, i.e., the EM effect and the u,d- quark mass difference effect. In fact, if we consider the SU(2) rotation of $\exp[-i\pi\sigma^2/2]$ (i.e., u,d exchange), the Σ triplets rotate as $\Sigma^+ \to \Sigma^-$, $\Sigma^- \to \Sigma^+$, $\Sigma^0 \to \Sigma^0$. Therefore, in the splitting of $m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0}$, there is no $\mathcal{O}(m_u - m_d)$ term and there exists only $\mathcal{O}(\alpha_{\rm em})$ EM effect up to the first order of the isospin breaking effect. Under this consideration, the result from the EM effect can be directly compared with the experimental value of 1.5MeV, in principle. Unfortunately, because of the statistical fluctuation, we obtain only the zero-consistent result for $m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0}$ after the linear chiral extrapolation. The upper bound of the lattice result for this quantity is found to be

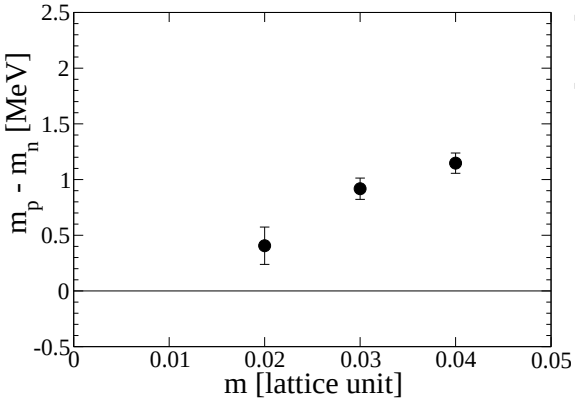

Figure 3: The proton-neutron mass difference from the EM effect at physical QED charge $\alpha_{\rm em}^{\rm phy} = 1/137$ plotted at each quark mass of $m = m_{\rm valence} = m_{\rm sea}$.

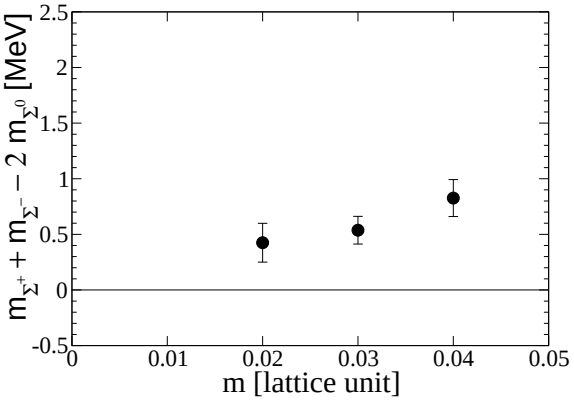

Figure 4: The mass difference $m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0}$ from the EM effect at physical QED charge $\alpha_{\rm em}^{\rm phy}$ plotted at each quark mass of $m = m_{\rm valence} = m_{\rm sea}$.

smaller than the experiental value, while this discrepancy may be attributed by the finite volume artifact[3].

Before closing this section, we comment on the systematic error in these results. In order to check the stability of the analysis, we examine the several alternative methods to extract the mass difference. For example, we perform the linear fit for the ratio of proton to neutron correlation functions (i.e., for the ratio shown in Fig.1) in terms of t, instead of exponential fit of each correlation function. We find that the result is consistent with each other and confirm that the results are reliable. In the variational method, we choose several t_0 instead of $t_0 = 1$ choice when diagonalizing $\Pi(t = t_0)^{-1} \cdot \Pi(t)$, and check the dependence on t_0. It is found that the results are insensitive to t_0, and we confirm that our variational procedure is stable. Yet, the current lattice results are afflicted by the statistical noise, particularly in light quark mass sector, and it is difficult to perform the definite chiral extrapolation. Further calculation is desirable to achieve the better statistics, which is actually our ongoing work. Although the uncertainty in the result extracted from the current lattice setup has been evaluated as describe above, the most troublesome artifact remained is the finite volume artifact, because the QED interaction is a long-range interaction. In fact, the model-based calculation[3] suggests that such artifact is sometimes comparable to the results of the lattice simulation. At this moment, we cannot evaluate the finite volume artifact without using model calculations, and the results given above would receive some modifications. The explicit calculation of the finite volume artifact is planned with the use of $N_f = 2+1$ configurations generated by the RBC-UKQCD collaboration[11], with which the configurations with different volumes are available.

4. Summary and outlook

We have investigated the electromagnetic (EM) effect on octet baryon spectroscopy. By employing $N_f = 2$ dynamical domain-wall QCD configuration combined with non-compact quenched QED configuration, the u, d sea quark effect in QCD has been incorporated. The mass split-

tings between isomultiplets $(p,n),(\Sigma^+,\Sigma^0,\Sigma^-),(\Xi^0,\Xi^-)$ have been studied by evaluating the two-point correlation function, where the variational method has been adopted in the $Q=0,S=-1$ channel in order to extract the Σ^0 state as a first excited state in this channel. In order to study the QED charge dependence of the mass splitting, we have chosen the QED charges of $\alpha_{\rm em} = (0),\alpha_{\rm em}^{\rm phy},\frac{(0.6)^2}{4\pi},\frac{(0.85)^2}{4\pi},\frac{(1.0)^2}{4\pi}$. We have found that it is essential to calculate both of $e=\pm\sqrt{4\pi\alpha_{\rm em}}$ for each $\alpha_{\rm em}$, in order to cancel the $\mathcal{O}(e)$ contamination and to achieve the reasonable S/N in the baryon EM mass splitting. By fitting the mass splitting linearly in terms of $\alpha_{\rm em}$, we have obtained the first result from lattice dynamical simulation for the baryon EM mass splitting at each $m=0.02,0.03,0.04$. The investigation of the effect of u, d quark mass difference is also in progress[5]. There remains uncertainty originates from the finite volume artifact. In order to investigate it explicitly, we are planning to perform the analysis using the $N_f=2+1$ dynamical domain-wall configuration generated by the RBC-UKQCD collaboration[11]. There, we can also eliminate the quenching artifact for the strange quark as well. In future, the inclusion of the dynamical QED effect is interesting to investigate[12], in which we expect the definite calculation of the isospin breaking effect is possible with all the uncertainties in the simulation under control.

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