

## Excited mesons from $N_f = 2$ dynamical Clover Wilson lattices

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We study mesons on the lattice with a special focus on excited states. For that purpose we construct several quark sources with different spatial smearings, including p-waves. These quark sources are then combined with the appropriate Dirac structures to form meson interpolators of definite spin. We use these operators to construct a cross correlation matrix from which we extract ground and excited meson states using the variational method. For the calculations we use gauge configurations with  $N_f = 2$  dynamical Clover Wilson fermions provided by the CP-PACS collaboration. We show preliminary results for pseudoscalar, scalar, vector and pseudovector mesons.

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## 1. Introduction

Ground state spectroscopy in lattice calculations appears to be well understood. Excited state spectroscopy, however, is still a challenging task. There are two major problems: First, one has to improve the overlap of the interpolating fields with excited states. Excited hadron states include both radial and orbital excitations and thus allowing for such excitations should be implemented in a lattice calculation to obtain more realistic results. The second problem is finding reliable means to disentangle ground and excited states in the hadron spectrum. Our method of choice is a variational approach [1, 2]. It not only allows us to extract excited states but also separates physical states from ghost contributions in quenched and partially quenched calculations.

## 2. Simulation details

We perform lattice calculations with dynamical gauge configurations with  $N_f = 2$  Clover Wilson Fermions. The configurations were generated by the CP-PACS collaboration, which made them publicly available. At the moment there are three lattices available: a  $12^3 \times 24$ , a  $16^3 \times 32$ , and a  $24^3 \times 48$ . All these lattices have approximately the same spatial volume of about 2.5 fm, but differ in their lattice spacing. This enables us to perform a continuum extrapolation. For the preliminary results we are presenting here we have used 100 configurations per sea quark mass of the  $16^3 \times 32$  lattice which has a lattice spacing of 0.1555(17) fm. For further information about the lattices and how they were generated, see Ref. [3, 4].

For our simulations, we make use of the Chroma software package [5] from the USQCD. This package has the advantage that most of the applications needed in lattice QCD are already implemented and it can be easily installed on various platforms, including QCDOC.

## 3. The Method

To obtain reliable results for excited states, we follow a procedure which has already proven to be very successful in quenched calculations [6, 7]. First, we generate several spatially different quark sources. In previous studies, we considered a narrow and a wide Gaussian source (in the following called  $n, w$ , respectively), obtained from gauge covariant smearing, to allow for a node in the radial wave function of the quark. Here we also include p-wave sources ( $p_x, p_y, p_z$ ) which we generate by acting with a covariant derivative on the wide smeared source. This enables us to not only explore the possibility of radial excitations, but also orbital ones. In addition to these sources we add a local source ( $L$ ) so that we can examine physical matrix elements.

We then combine these sources with the appropriate Dirac structures to obtain interpolating fields of definite spin. By doing so, we end up with a large number of interpolators for each meson channel (e.g., 15 different interpolators in the case of the pseudoscalar meson). From these interpolators we construct a matrix of correlators and then apply the variational method which has been proposed by Michael [1] and later refined by Lüscher and Wolff [2]. Here, one has to solve a generalized eigenvalue problem. This method has some advantages. First, the system has full freedom to choose relative contributions of the different interpolators in the diagonalization step. Second, this approach can separate ghost contributions in quenched and partially quenched results,

as has been shown in Ref. [8]. In the case where one uses more than one local operator one can also extract ratios of couplings to excited states, see Ref. [9].

The masses of the excited states are obtained by looking at the eigenvalues, which to leading order behave as

$$\lambda^{(k)}(t) \propto e^{-tM_k} . \quad (3.1)$$

From these eigenvalues we then construct effective mass plots by using

$$am_k^{eff} \left( t + \frac{1}{2} \right) = \ln \left( \frac{\lambda^{(k)}(t)}{\lambda^{(k)}(t+1)} \right) . \quad (3.2)$$

When we find a plateau in these plots and if the corresponding eigenvectors are steady we conclude that the signal of the considered state is disentangled from higher excitations. We then fit the eigenvalue to a single exponential according to Eq. (3.1). In the following we present the results of these fits for different mesons.

#### 4. Discussion of the Results

In Fig. 1, we show a collection of effective mass plots for the different meson channels pseudoscalar(PS), scalar(SC), vector(V), and pseudovector(PV). The effective masses are shown only for the completely degenerate case

$$\kappa_{val}^{(1)} = \kappa_{val}^{(2)} = \kappa_{sea} = \kappa . \quad (4.1)$$

With our limited statistics, we find good plateaus for most of the ground states. But the results for the excited states do not look encouraging. In fact, we are able to obtain a first excited state only in the PS and V channels.

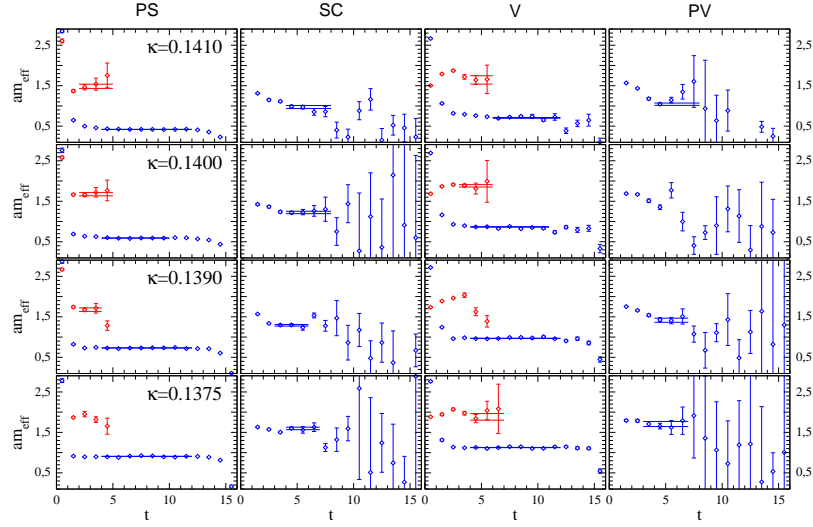
In Fig. 2-5, we plot the results of our fits versus

$$\frac{1}{\kappa} = \frac{1}{2} \left( \frac{1}{\kappa_{val}^{(1)}} + \frac{1}{\kappa_{val}^{(2)}} \right) . \quad (4.2)$$

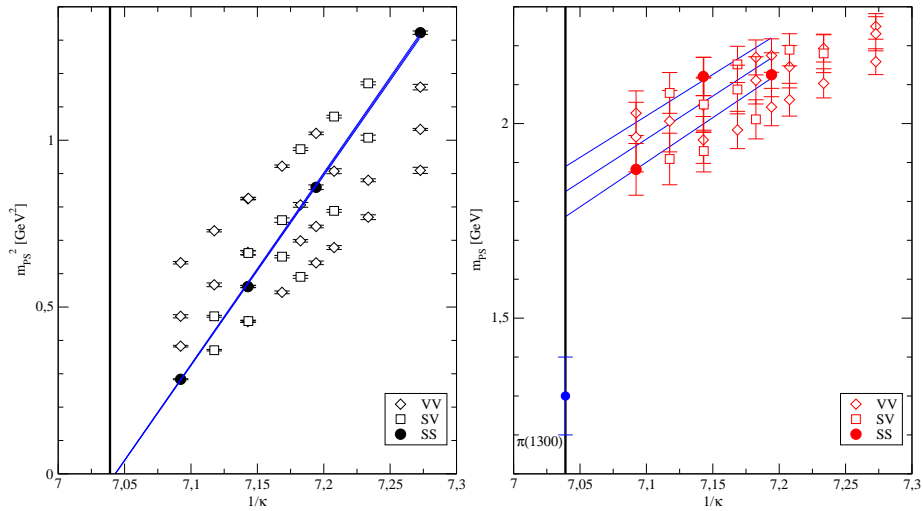
We distinguish three different cases: The case where both valence quark masses are equal to the sea quark mass, denoted as SS, the case where only one valence quark has the same mass as the sea quark, denoted as SV, and the VV case where both valence quarks have a mass which differs from the sea quark mass. In all the figures the vertical line represents  $1/\kappa_{crit}$  obtained by the CP-PACS collaboration using all available configurations. Since our results are still very preliminary we perform only linear fits for the chiral extrapolation.

In the left hand plot of Fig. 2, one can see that we obtain with our limited statistics a result for  $\kappa_{crit}$  which is comparable to the CP-PACS result for this quantity.

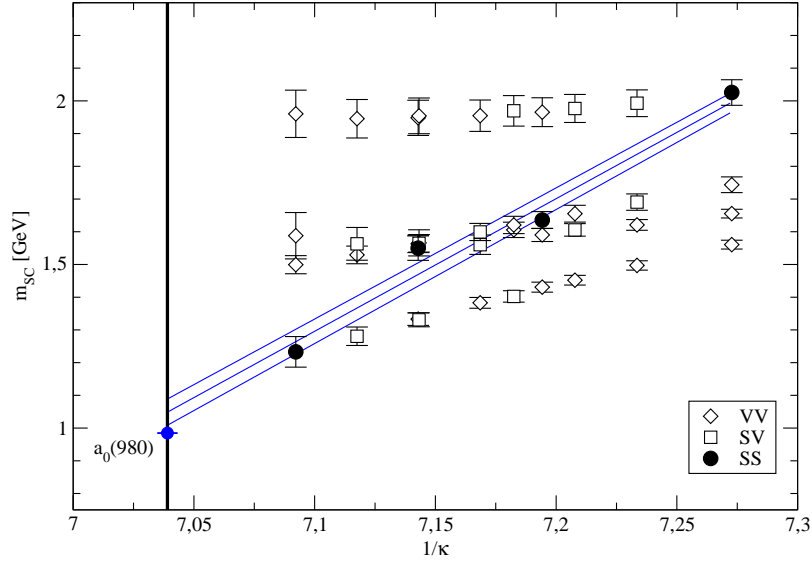
In all the cases we cannot exclude systematic effects which are due to the limited statistics we have so far. It might, for example, be possible, that with larger statistics some of the plateaus in Fig. 1 change and a fit interval starting one timeslice later might be more appropriate. Especially our excited states with their short plateaus can be strongly affected by such systematic shifts. This



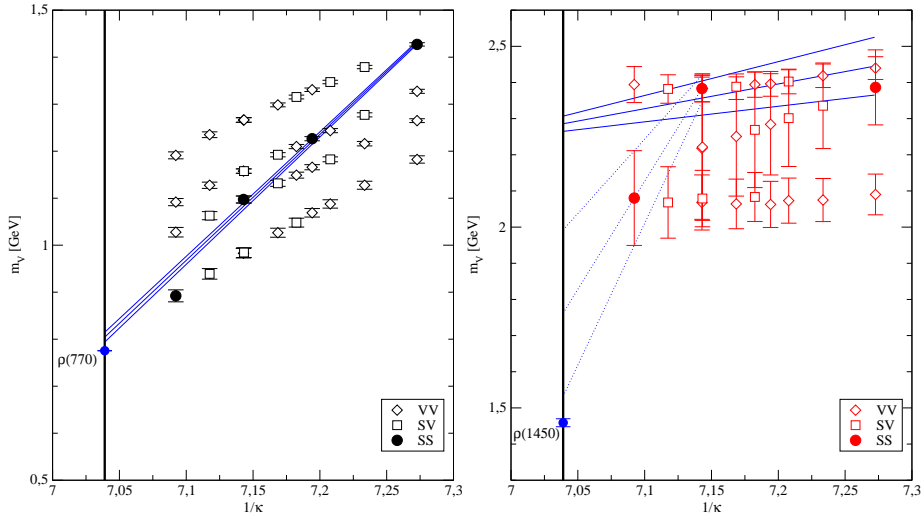
**Figure 1:** Effective masses for pseudoscalar(PS), scalar(SC), vector(V), and pseudovector(PV), shown for completely degenerate quark masses. The horizontal lines denote the time intervals of our fits and represent the fit results  $m \pm \sigma_m$ , where  $\sigma_m$  is the statistical error obtained from single elimination jackknife.



**Figure 2:** Results for the pseudoscalar meson ground (left plot) and excited state (right plot) versus  $1/\kappa$ . Filled circles denote the SS case, open squares the SV case, and open diamonds the VV case. The vertical line is  $1/\kappa_{crit}$  obtained by the CP-PACS collaboration. We also show a naive chiral extrapolation of the SS results. Also the experimental value is included.



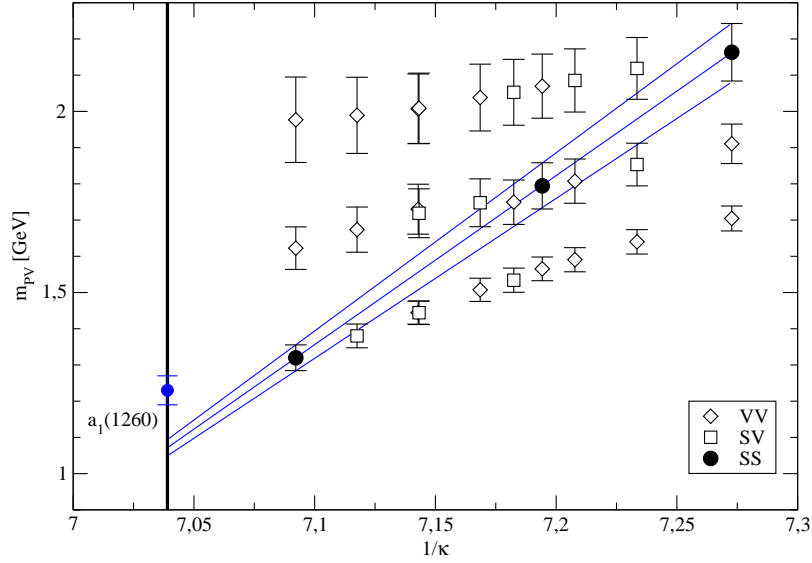
**Figure 3:** Same as in Fig. 2, but for the scalar meson ground state.



**Figure 4:** Same as in Fig. 2, but for the vector meson ground and excited state. For the excited state we tried two different chiral extrapolations.

might also explain why our results for the excited pseudoscalar and vector meson are extrapolating to values which are much higher than the experimental ones.

Apart from this, we also find that the widths of our Gaussian sources depend on the sea quark mass. This makes it rather difficult to tune the smearing parameters to obtain widths in physical units which are approximately the same. What we also find is that a cross correlation matrix of interpolators built from  $L$  and  $n$  sources gives much better results than using interpolators built from  $n$  and  $w$  sources. This suggests, that our narrow and wide smeared sources are too similar to allow to disentangle ground and excited states.



**Figure 5:** Same as in Fig. 2, but for the pseudovector meson ground state.

## 5. Summary and outlook

We have presented preliminary results of our calculation of ground and excited meson states from dynamical configurations with  $N_f = 2$  Clover Wilson fermions. For all considered channels we find results for the ground states and in the pseudoscalar and vector channels we are even able to extract a first excited state.

We encounter some difficulties to optimize the smearing parameters for our calculation since we find that the width of our Gaussian source depends on the sea quark mass. In addition to this, we observe that interpolators built from these Gaussian sources are giving worse signals for the excited states than interpolators built from one Gaussian and a local source. Possibly, this means that the Gaussians used are too much alike.

To circumvent this problem we are currently running simulations with a different set of sources. We use a narrow source with much smaller width and replace our old wide source by the narrow source upon which we apply a Laplacian (The Laplacian being a scalar operator does not change the quantum numbers of the hadron).

Our future plans also include performing the same calculations on the other lattices of the CP-PACS collaboration to be able to do a continuum extrapolation of our results. Especially the couplings to excited states are in that respect of great interest.

## 6. Acknowledgements

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