PROCEEDINGS
OF SCIENCE

B_s and B_c mesons in lattice QCD with exact chiral symmetry

TWQCD Collaboration: Ting-Wai Chiu[a], Tung-Han Hsieh[*b]

[a] *Department of Physics and Center for Theoretical Sciences, National Taiwan University,*
Taipei 10617, Taiwan
E-mail: twchiu@phys.ntu.edu.tw
[b] *Research Center for Applied Sciences, Academia Sinica, Taipei 115, Taiwan*
E-mail: thhsieh@twcp1.phys.ntu.edu.tw

We determine the masses and decay constants of the pseudoscalar mesons B_s and B_c, and also the masses of the vector mesons B_s^* and B_c^*, in quenched lattice QCD with exact chiral symmetry. For 100 gauge configurations generated with single-plaquette action at $\beta = 7.2$ on the $32^3 \times 60$ lattice, we compute point-to-point quark propagators for 33 quark masses in the range $0.01 \leq m_q a \leq 0.85$, and measure the time-correlation functions of pseudoscalar and vector mesons. The inverse lattice spacing and the charm quark bare mass are determined using the mass and decay constant of $\eta_c(2980)$. The bare masses of **s** and **b** quarks are chosen such that the masses of the corresponding vector mesons are in good agreement with $\phi(1020)$, and $\Upsilon(9460)$ respectively. Our preliminary results are: $m_{B_s} = 5385(27)$ MeV, $f_{B_s} = 253(8)$ MeV, $m_{B_c} = 6278(6)$ MeV, $f_{B_c} = 489(4)$ MeV, $m_{B_s^*} = 5409(29)$ MeV, and $m_{B_c^*} = 6315(6)$ MeV.

---

[*]Speaker.

1. Introduction

In view of recent experimental results from B factories, it is interesting to understand the B physics from the first principles, in the framework of lattice QCD with exact chiral symmetry. In order to be consistent with the Standard Model, here we treat *all* quark flavors (heavy and light) as Dirac fermions, without using any heavy quark and/or non-relativistic approximations. Thus our approach is theoretically appealing, fundamentally different from other lattice QCD calculations with heavy quark and/or non-relativistic approximations, in which the systematic errors are difficult to control. In spite of the large separation of mass scales for heavy-light systems, we can accommodate strange, charm and bottom quarks on a $32^3 \times 60$ lattice, with inverse lattice spacing $a^{-1} = 7.68$ GeV. Then we compute point-to-point quark propagators, and extract physical quantities from the time-correlation function of hadron interpolators containing **s**, **c**, and **b** quarks. In this paper, we determine the masses and decay constants of the pseudoscalar mesons B_s and B_c, and also the masses of the vector mesons B_s^* and B_c^*. Note that B_c^* has not been observed in high energy experiments. Thus our result serves as the first prediction of B_c^* from lattice QCD.

To implement exact chiral symmetry on the lattice [1, 2, 3, 4], we consider the optimal domain-wall fermion proposed by Chiu [5]. From the generating functional for n-point Green's function of the quark fields, the valence quark propagator in background gauge field can be derived as [5]

$$\langle q(x)\bar{q}(y)\rangle = (D_c + m_q)^{-1}_{x,y} = (1 - rm_q)^{-1}[D^{-1}_{x,y}(m_q) - r\delta_{x,y}], \quad r = (2m_0)^{-1} \tag{1.1}$$

$$D(m_q) = m_q + (m_0 - m_q/2)[1 + \gamma_5 S(H_w)] \tag{1.2}$$

where m_q is the bare quark mass, m_0 is a parameter in the range $(0,2)$, $S(H_w)$ is the Zolotarev approximation of the sign function of H_w ($H_w = \gamma_5 D_w$, and D_w is the standard Wilson Dirac operator minus m_0), and $D_c = 2m_0(\mathbf{I} + \gamma_5 S)(\mathbf{I} - \gamma_5 S)^{-1}$, which becomes exactly chirally symmetric (i.e. $D_c\gamma_5 + \gamma_5 D_c = 0$) in the limit $N_s \to \infty$ (where $N_s + 2$ is the number of sites in the 5th dimension.) Note that in this framework, the bare quark mass m_q in the valence quark propagator $(D_c + m_q)^{-1}$ is well-defined for any gauge field configuration.

In practice, there are two ways to evaluate the valence quark propagator (1.1): (i) To solve the linear system of the 5D optimal domain-wall fermion operator; (ii) To solve $D(m_q)Z = \mathbf{I}$ by nested conjugate gradient. Here we employ the scheme (ii), since we can attain the maximum efficiency if the inner conjugate gradient loop is iterated with Neuberger's 2-pass algorithm [6].

We generate 100 gauge configurations with single plaquette gauge action at $\beta = 7.2$ on the $32^3 \times 60$ lattice. For $m_0 = 1.3$ and $N_s = 128$, we fix the Zolotarev coefficient with $\lambda_{min} = 0.1$ and $\lambda_{max} = 6.4$, where $\lambda_{min} \leq \lambda(|H_w|) \leq \lambda_{max}$ for all gauge configurations. For each configuration, point-to-point quark propagators are computed for 33 bare quark masses in the range $0.01 \leq m_q a \leq 0.85$, with stopping criteria 10^{-11} and 2×10^{-12} for the outer and inner conjugate gradient loops respectively. Then the norm of the residual vector is $||(D_c + m_q)Y - \mathbf{I}|| < 2 \times 10^{-11}$, and the chiral symmetry breaking due to finite N_s is $|\langle S^2\rangle - 1| < 10^{-14}$ for every iteration of the nested conjugate gradient.

In this paper, we measure the time-correlation functions for pseudoscalar (P) and vector (V) mesons,

$$C_P(t) = \langle \sum_{\vec{x}} \text{tr}\{\gamma_5(D_c + m_Q)^{-1}_{x,0}\gamma_5(D_c + m_q)^{-1}_{0,x}\}\rangle_U, \tag{1.3}$$
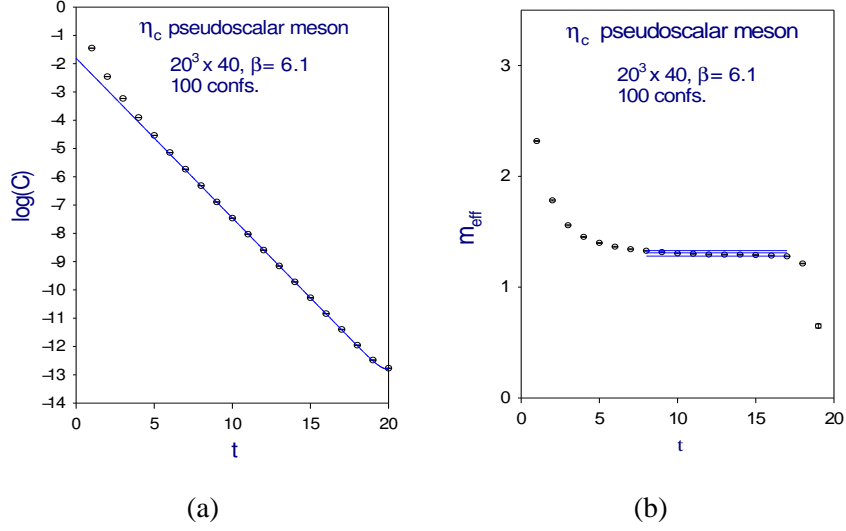
(a)   (b)

Figure 1: (a) The time-correlation function $C(t)$ of the η_c pseudoscalar meson operator $\bar{c}\gamma_5 c$ on the $20^3 \times 40$ lattice at $\beta = 6.1$. The solid line is the hyperbolic-cosine fit for $t \in [8,17]$. (b) The effective mass $M_{eff}(t) = \ln[C(t)/C(t+1)]$ of $C(t)$ in Fig. 1a.

$$C_V(t) = \langle \frac{1}{3} \sum_{\mu=1}^{3} \sum_{\vec{x}} \text{tr}\{\gamma_\mu (D_c + m_Q)^{-1}_{x,0} \gamma_\mu (D_c + m_q)^{-1}_{0,x}\}\rangle_U \tag{1.4}$$

where the subscript U denotes averaging over gauge configurations. Here $C_P(t)$ and $C_V(t)$ are measured for the following three categories: (i) Symmetric masses ($m_Q = m_q$) for 33 quark masses; (ii) $(m_Q, m_q) = (m_b, m_s)$; and (iii) $(m_Q, m_q) = (m_b, m_c)$.

2. Determination of a^{-1}, m_c, m_s, and m_b

In Ref. [7], we determine the inverse lattice spacing from the pion decay constant, with experimental input $f_\pi = 131$ MeV. However, in this paper, we do not use the same method since the smallest quark mass turns out to be rather heavy ($\simeq m_s$), thus chiral extrapolation to $m_q \simeq 0$ does not seem to be feasible. Nevertheless, we can use the mass and decay constant of the pseudoscalar meson $\eta_c(2980)$ to determine m_c and a^{-1} simultaneously. This can be seen as follows.

For symmetric masses $m_Q = m_q$, the pseudoscalar time-correlation function $C_P(t)$ (1.3) is measured, and fitted to the usual formula

$$\frac{z^2}{2m_Pa}[e^{-m_Pat} + e^{-m_Pa(T-t)}] \tag{2.1}$$

to extract the mass m_Pa and the decay constant

$$f_Pa = 2m_qa\frac{z}{m_P^2a^2} . \tag{2.2}$$
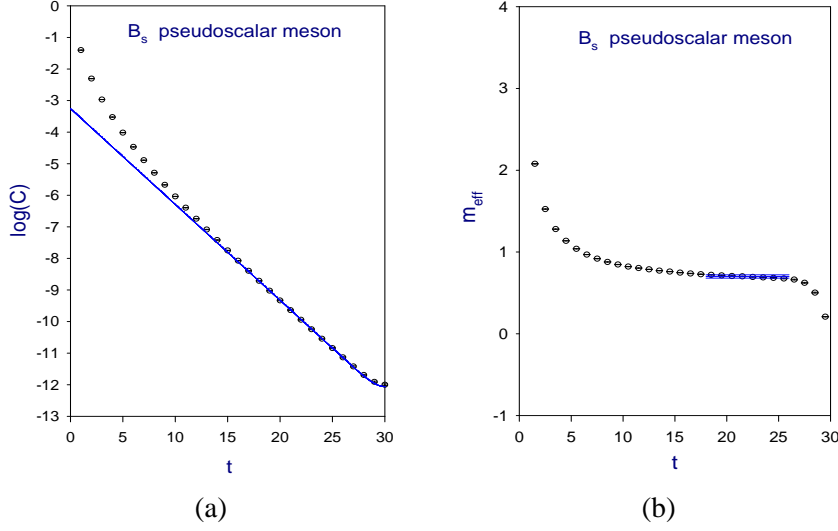
Then the ratio m_P/f_P can be obtained for each m_q.

3

Figure 2: (a) The time-correlation function $C(t)$ of the B_s pseudoscalar meson operator $\bar{\mathbf{s}}\gamma_5\mathbf{b}$ with $J^P = 0^-$, on the $32^3 \times 60$ lattice at $\beta = 7.2$. The solid line is the hyperbolic-cosine fit for $t \in [18, 27]$. (b) The effective mass $M_{eff}(t) = \ln[C(t)/C(t+1)]$ of $C(t)$ in Fig. 2a.

From our previous studies [7] of pseudoscalar mesons on the $20^3 \times 40$ lattice at $\beta = 6.1$, we obtain $m_{\eta_c} = 2944(4)$ MeV and $f_{\eta_c} = 453(4)$ MeV. (see Fig. 1). So we can use the ratio $2944/453 \simeq 6.5$ to discriminate which m_q can give the ratio m_P/f_P closest to 6.5. We find that at $m_q a = 0.16$, the ratio $m_P/f_P = 6.6(1)$, which is the closest to 6.5. Thus we fix $m_c a = 0.16$. Then we use the experimental mass of $\eta_c(2980)$ to determine a^{-1} through the relation

$$m_P a|_{m_c} = (2980 \text{ MeV}) \times a = 0.388(3)$$

and obtain $a^{-1} = 7680(59)$ MeV. To check the goodness of the values of m_c and a^{-1}, we compute the time-correlation function of $\bar{\mathbf{c}}\gamma_i\mathbf{c}$, and extract the mass of the vector meson to be $3087(20)$ MeV, in good agreement with $J/\Psi(3097)$. Note that the spatial size of our lattice ($L \simeq 0.8$ fm) seems to be small at first glance, however, even for the smallest quark mass $m_q a = 0.01$, its pseudoscalar mass satisfies $m_P L > 4$, thus the finite size effects are well under control.

The bare mass of strange quark is determined by extracting the mass of vector meson from the time-correlation function $C_V(t)$. At $m_q a = 0.02$, $m_V a = 0.1337(5)$, which gives $m_V = 1027(38)$ MeV, in good agreement with the mass of $\phi(1020)$. Thus we take the strange quark bare mass to be $m_s a = 0.02$. Similarly, at $m_q a = 0.68$, $m_V a = 1.2308(4)$, which gives $m_V = 9453(3)$ MeV, in good agreement with the mass of $\Upsilon(9460)$. Thus, we fix the bottom quark bare mass to be $m_b a = 0.68$.

3. The B_s and B_c Pseudoscalar Mesons

The pseudoscalar-meson decay constants play an important role in extracting the CKM matrix elements which are crucial for testing the flavor sector of the standard model via the unitarity of CKM matrix. Theoretically, lattice QCD with exact chiral symmetry provides a reliable framework
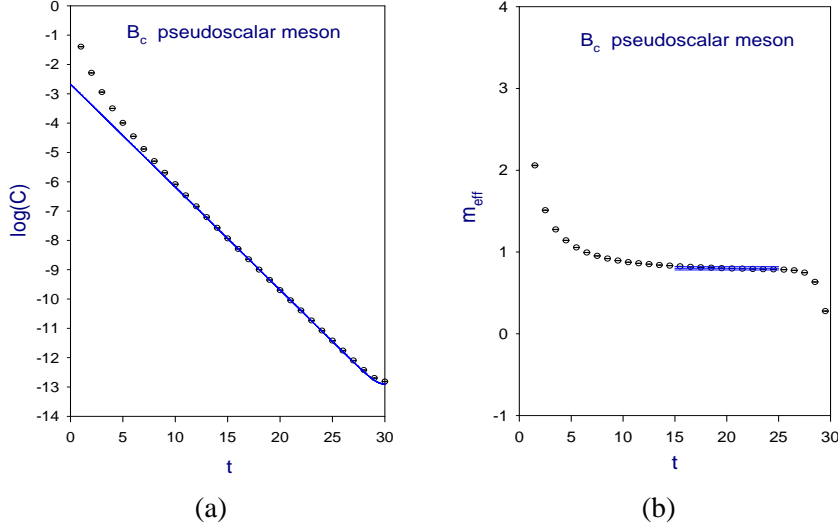
Figure 3: (a) The time-correlation function $C(t)$ of the B_c pseudoscalar meson operator $\bar{\mathbf{c}}\gamma_5\mathbf{b}$ with $J^P = 0^-$, on the $32^3 \times 60$ lattice at $\beta = 7.2$. The solid line is the hyperbolic-cosine fit for $t \in [15,25]$. (b) The effective mass $M_{eff}(t) = \ln[C(t)/C(t+1)]$ of $C(t)$ in Fig. 3a.

to compute the masses and decay constants of pseudoscalar mesons nonperturbatively from the first principles of QCD.

The decay constant f_P for a pseudoscalar meson P is defined by

$$\langle 0|A_\mu(0)|P(\vec{q})\rangle = f_P q_\mu$$

where $A_\mu = \bar{q}\gamma_\mu\gamma_5 Q$ is the axial-vector current. Using $\partial_\mu A_\mu = (m_q + m_Q)\bar{q}\gamma_5 Q$, one obtains

$$f_P = (m_q + m_Q)\frac{|\langle 0|\bar{q}\gamma_5 Q|P(\vec{0})\rangle|}{m_P^2} \tag{3.1}$$

where the pseudoscalar mass $m_P a$ and the decay amplitude $z \equiv |\langle 0|\bar{q}\gamma_5 Q|P(\vec{0})\rangle|$ can be obtained by fitting the pseudoscalar time-correlation function $C_P(t)$ to the usual formula (2.1).

In Fig. 2, the time-correlation function $C(t)$ of the B_s pseudoscalar meson operator $\bar{\mathbf{s}}\gamma_5\mathbf{b}$ is plotted versus time slices, together with the effective mass $\log[C(t)/C(t+1)]$, for 100 gauge configurations generated with single plaquette action on $32^3 \times 60$ lattice at $\beta = 7.2$. Here the "forward-propagator" $C(t)$ and the "backward-propagator" $C(T-t)$ are averaged to enhance the statistics. The same strategy is applied to all time-correlation functions in this paper. The solid line is the hyperbolic-cosine fit for $t \in [18,27]$. It gives $m_{B_s} = 5385(27)$ MeV and $f_{B_s} = 253(8)$ MeV, where m_{B_s} is in good agreement with the experimental value 5370 MeV. Since f_{B_s} has not been measured in high energy experiments, our result serves as the first prediction from lattice QCD with exact chiral symmetry.

In Fig. 3, the time-correlation function $C(t)$ of the B_c pseudoscalar meson operator $\bar{\mathbf{c}}\gamma_5\mathbf{b}$ is plotted versus time slices, together with the effective mass $\log[C(t)/C(t+1)]$. The solid line is the hyperbolic-cosine fit for $t \in [15,25]$. It gives $m_{B_c} = 6278(6)$ MeV, and $f_{B_c} = 489(4)$ MeV,
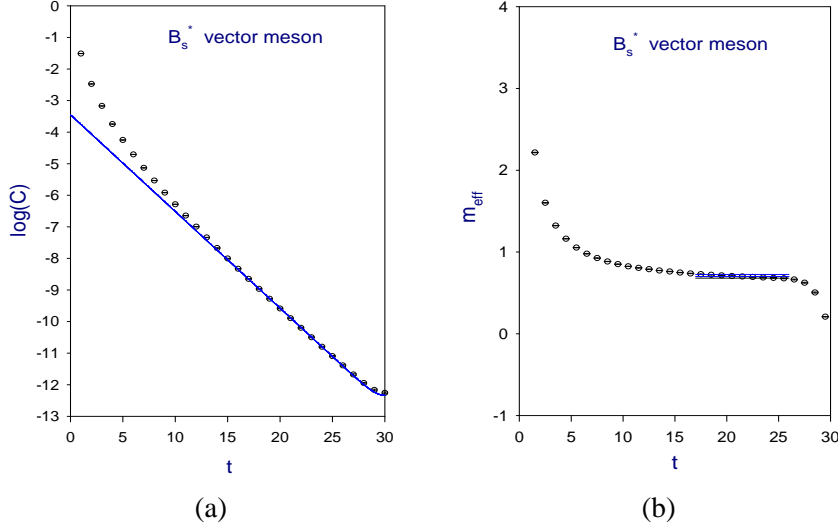
Figure 4: (a) The time-correlation function $C(t)$ of the B_s^* vector meson operator $\bar{\mathbf{s}}\gamma_i\mathbf{b}$ with $J^P = 1^-$, on the $32^3 \times 60$ lattice at $\beta = 7.2$. The solid line is the hyperbolic-cosine fit for $t \in [17,26]$. (b) The effective mass $M_{eff}(t) = \ln[C(t)/C(t+1)]$ of $C(t)$ in Fig. 4a.

where m_{B_c} is in good agreement with the experimental value 6287(5) MeV measured by CDF Collaboration. Since f_{B_c} has not been measured in high energy experiments, our result serves as the first prediction from lattice QCD.

4. The B_s^* and B_c^* Vector Mesons

In Fig. 4, the time-correlation function $C(t)$ (1.4) of the B_s^* vector meson operator $\bar{\mathbf{s}}\gamma_\mu\mathbf{b}$ is plotted versus time slices, together with the effective mass $\log[C(t)/C(t+1)]$. The solid line is the hyperbolic-cosine fit for $t \in [17,26]$. It gives $m_{B_s^*} = 5409(29)$ MeV, in good agreement with the experimental value 5417 MeV.

In Fig. 5, the time-correlation function $C(t)$ (1.4) of the B_c^* vector meson operator $\bar{\mathbf{c}}\gamma_\mu\mathbf{b}$ is plotted versus time slices, together with the effective mass $\log[C(t)/C(t+1)]$. The solid line is the hyperbolic-cosine fit for $t \in [15,25]$. It gives $m_{B_c^*} = 6315(6)$ MeV, which serves as the first prediction from lattice QCD.

5. Concluding Remark

In this paper, we have investigated heavy quark systems containing **b**, **c** and **s**, treating all quarks as Dirac fermions on the lattice, without using any heavy quark and/or non-relativistic approximations. Our results of the masses and decay constants of the pseudoscalar mesons B_s and B_c, and also the masses of the vector mesons B_s^* and B_c^* suggest that lattice QCD with exact chiral symmetry is a viable framework to study heavy quark physics from the first principles of QCD. For systems involving **u**/**d** quarks, one may use several quark masses in the range $m_{u/d} < m_q < m_s$ to perform the chiral extrapolation. To this end, one may choose a coarser lattice (e.g. $\beta = 7.0$), then it
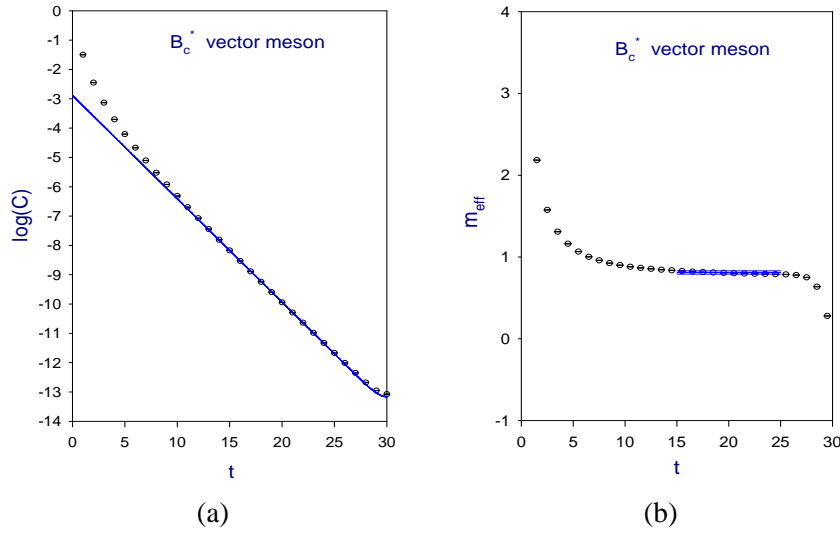
(a)   (b)

Figure 5: (a) The time-correlation function $C(t)$ of the B_c^* vector meson operator $\bar{\mathbf{c}}\gamma_i\mathbf{b}$ with $J^P = 1^-$, on the $32^3 \times 60$ lattice at $\beta = 7.2$. The solid line is the hyperbolic-cosine fit for $t \in [15, 25]$. (b) The effective mass $M_{eff}(t) = \ln[C(t)/C(t+1)]$ of $C(t)$ in Fig. 5a.

is possible to accommodate a wide range of quark masses $m_s/4 < m_q \leq m_b$ on the $48^3 \times 60$ lattice, without significant discretization and finite-size errors. Obviously, now, it has become feasible to treat all quarks as Dirac fermions on the lattice, in lattice QCD with exact chiral symmetry.

Acknowledgement

References

[1] D. B. Kaplan, Phys. Lett. B **288**, 342 (1992); Nucl. Phys. Proc. Suppl. **30**, 597 (1993).

[2] R. Narayanan and H. Neuberger, Nucl. Phys. B **443**, 305 (1995)

[3] H. Neuberger, Phys. Lett. B **417**, 141 (1998)

[4] P. H. Ginsparg and K. G. Wilson, Phys. Rev. D **25**, 2649 (1982)

[5] T. W. Chiu, Phys. Rev. Lett. **90**, 071601 (2003); Phys. Lett. B **552**, 97 (2003); hep-lat/0303008; Nucl. Phys. Proc. Suppl. **129**, 135 (2004).

[6] H. Neuberger, Int. J. Mod. Phys. C **10**, 1051 (1999); T. W. Chiu and T. H. Hsieh, Phys. Rev. E **68**, 066704 (2003)

[7] T. W. Chiu, T. H. Hsieh, J. Y. Lee, P. H. Liu and H. J. Chang, Phys. Lett. B **624**, 31 (2005)