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Hybrid charmonium from lattice QCD*

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We review our recent results on the $J^{PC} = 0^{--}$ exotic hybrid charmonium mass and $J^{PC} = 0^{-+}$, 1^{--} and 1^{++} nonexotic hybrid charmonium spectrum from anisotropic improved lattice QCD and discuss the relevance to the recent discovery of the $Y(4260)$ state and future experimental search for other states.

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1. Introduction

A hybrid meson $\bar{q}qg$ is a bound state of constituent quark q, anti-quark $\bar{q}$ and excited gluon g. The existence of hybrids is one of the most important predictions of quantum chromodynamics (QCD). There has been a lot of experimental activity[1, 2, 3, 4] in the search for hybrid mesons, for example: PEP-2(Babar), KEKB(Belle), 12 GeV Jefferson Lab upgraded, upgraded CLEO-c detector, and new BES3 detector.

For a conventional meson in the quark model, which is represented by the fermion bilinear $\bar{\psi}\Gamma\psi$, it can have the J^{PC} quantum numbers as $J = |L-S|$, $|L-S|+1$, ..., $L+S$, $P = (-1)^{L+1}$, and $C = (-1)^{L+S}$, with L the relative angular momentum of the quark and anti-quark, and S the intrinsic spin of the meson. For the gluon, the quantum numbers of the color electric field $\mathbf{E}$ and color magnetic field $\mathbf{B}$ are 1^{--} and 1^{+-} respectively. According to QCD, the operator of a hybrid meson is the gauge-invariant direct product of $\bar{\psi}\Gamma\psi$ and the color electric field $E_i^{c_1c_2} = F_{0i}^{c_1c_2}$ or color magnetic field $B_i^{c_1c_2} = \varepsilon_{ijk}F_{jk}^{c_1c_2}$. Therefore, the quantum numbers of a hybrid meson could be either exotic, with $J^{PC} = 1^{-+}$, 0^{+-}, 0^{--}, 2^{+-} ......, inaccessible to conventional mesons, or nonexotic, with $J^{PC} = 0^{++}$, 0^{-+}, 1^{--}, 1^{++}, 1^{+-}, 2^{++}, 2^{--}, 2^{-+}, ..., the same as conventional mesons.

Lattice gauge theory is the most reliable technique for computing hadron spectra. It involves discretization of the continuum theory on a space-time grid, and reduces to QCD when the lattice spacing a goes to zero. The implementation of the Symanzik program[5] with tadpole improvement[6] greatly reduces the discretization errors on very coarse and small lattices. Simulations on anisotropic lattices improve the signal in spectrum computations[7].

The 1^{-+}, 0^{+-}, and 2^{+-} exotic hybrid meson have been extensively studied on the lattice. Reviews can be found in Refs.[8, 9]. However, The 0^{--} hybrid meson spectrum has never been provided by lattice simulations due to the difficulties to extract high gluonic excitations from noise. Also, the first excited states of the nonexotic hybrid mesons are completely ignored in the literature[10].

In this paper, we review our investigation on the $J^{PC} = 0^{--}$ exotic hybrid charmonium and $J^{PC} = 0^{-+}$, 1^{--} and 1^{++} nonexotic hybrid charmonium, employing quenched lattice QCD with tadpole improved gluon[11] and quark[12] actions on the anisotropic lattice. We observe, for the first time, very good signals for $J^{PC} = 0^{--}$ exotic hybrid, and the very strong gluonic radial excitations in the first excited states of the nonexotic hybrids. We also predict their spectrum, useful for experimental search of these new particles.

2. Improved QCD on the anisotropic lattice

It has been argued in Refs. [12, 13] that such an Fermilab quark action[14] on the anisotropic works well in the charm quark regime and is valid even for heavier quarks $m_q a_s > 1$, with the lattice artifacts under well control. We modified the MILC code[15] for simulations on the anisotropic lattice. Our simulation parameters are listed in Tab. 1. We also did simulations on $8^3 \times 48$ and $12^3 \times 48$ at $\beta = 2.6$, $12^3 \times 36$ at $\beta = 2.8$, and $16^3 \times 48$ at $\beta = 3.0$, but there and throughout the paper we just list the results from the largest volume, i.e., $16^3 \times 48$ at $\beta = 2.6$ and $\beta = 2.8$ and $20^3 \times 60$ at $\beta = 3.0$. At each β, three hundred independent configurations were generated with the

$\beta = 6/g^2$	$L_s^3 \times L_t$	u_s	$a_t m_{q0}$	c_s	c_t	$a_s(1^3P_1 - 1S)$[fm]
2.6	$16^3 \times 48$	0.81921	0.229 0.260	1.8189	2.4414	0.1856(84)
2.8	$16^3 \times 48$	0.83099	0.150 0.220	1.7427	2.4068	0.1537(101)
3.0	$20^3 \times 60$	0.84098	0.020 0.100	1.6813	2.3782	0.1128(110)

Table 1: Simulation parameters at largest volume with $\xi = 3$ and $u_t = 1$. We employed the method in Ref. 18 to tune the parameters for the quark action. The last two columns are about the spatial lattice spacing and the lattice extent in physical units, determined from the charmonium mass splitting $m(\chi_{c1}(1^3P_1)) - m(1\overline{S})$, with the effective masses extracted by the method of Ref. 19.

improved gluonic action[11]. It is also important to check whether these lattice volumes are large enough. We found that when the spatial extent is greater than 2.2fm, the finite volume effects on the spectrum become very small. At $\beta = 2.6$, e.g., the effect on 0^{--} hybrid is less than 0.1% for the ground state. For the 1^{--} hybrid charmonium, the effect is less than 0.1% for the ground state, and 0.2% for the first excited state; For the 0^{-+} hybrid, the effect is less than 0.3%; For the heaviest one, i.e., the 1^{++} hybrid, the effect is about 0.9%, but still less than the errors.

We input the bare quark mass m_{q0} and then computed quark propagators using the improved quark action[12], the conventional quarkonium correlation function using the operators $0^{-+} = \bar{\psi}^c \gamma_5 \psi^c$, $1^{--} = \bar{\psi}^c \gamma_j \psi^c$, and $1^{++} = \bar{\psi}^c \gamma_5 \gamma_j \psi^c$, and the hybrid meson correlation function using the operators $0^{--} = \bar{\psi}^{c_1} \gamma_5 \gamma_j \psi^{c_2} F_{j0}^{c_1 c_2}$, $0^{-+} = \varepsilon_{ijk} \bar{\psi}^{c_1} \gamma_i \psi^{c_2} F_{jk}^{c_1 c_2}$, $1^{--} = \varepsilon_{ijk} \bar{\psi}^{c_1} \gamma_5 \psi^{c_2} F_{jk}^{c_1 c_2}$ and $1^{++} = \varepsilon_{ijk} \bar{\psi}^{c_1} \gamma_j \psi^{c_2} F_{0k}^{c_1 c_2}$ in Ref. [16].

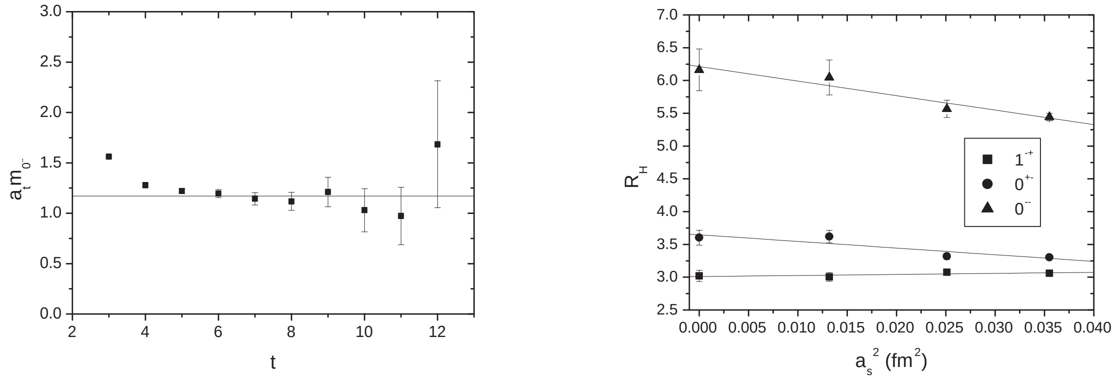
Our simulation parameters are listed in Tab. 1. At each $\beta = 6/g^2$, three hundred independent configurations were generated with the improved gluonic action[11]. Two hundred configurations are the minimum for obtaining stable results. We input two values of bare quark mass m_{q0} and then compute quark propagators using the improved quark action[12]. The data at two m_{q0} values were interpolated to the charm quark regime using $m(1\overline{S})_{exp} = [m(\eta_c)_{exp} + 3m(J/\psi)_{exp}]/4 = 3067.6$MeV.

3. Exotic hybrid charmonium

As mentioned in Sec. 1, it has been a long standing puzzle for the 0^{--} hybrid mesons[16]: no clear signal has ever been found. Thanks to the use of the tadpole improved gluon and quark actions on the anisotropic lattice. This is clearly shown in Fig. 1(1). From Fig. 1(2), we predict $m5.876(152)$MeV. Details could be found in Ref. [17].

4. Nonexotic hybrid charmonium

Figure 2 shows the correlation function $C(t)$ of the conventional 1^{--} and hybrid mesons. The effective masses of the ground and first excited states $a_t m_1$ and $a_t m_2$ are extracted by two different methods: (i) new correlation function method[18]; (ii) modified multi-exponential fit[19]. The multi-exponential fitting method has been widely used in the literature[10, 12, 13] for extracting the charmonium masses, and the results for the ground and first excited states are consistent with experiments; The MILC group[19] proposed an improved multi-exponential fitting method, which chooses the best fit according to some criteria. The recently proposed method (i) has been successfully applied to the investigation of the Roper resonance of the nucleon[18], where $a_t m_1$ is obtained

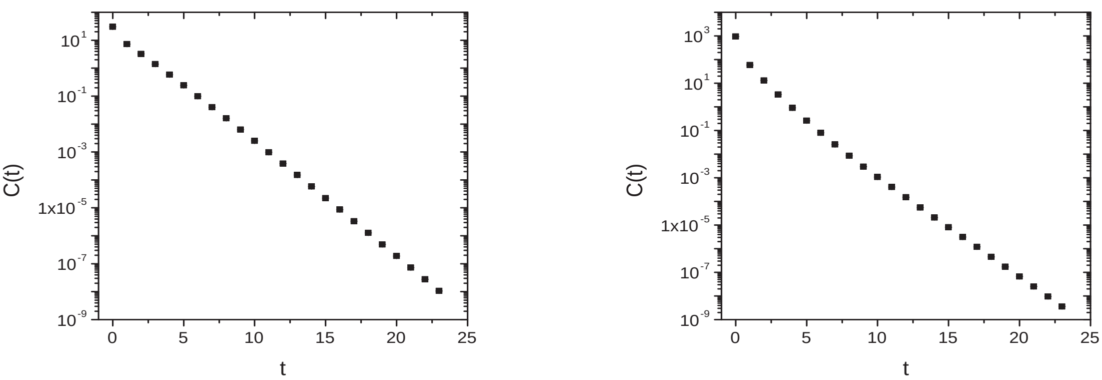


(1) (2)

Figure 1: (1) Effective mass of the 0^{--} hybrid meson for $\beta = 3.0$ and $a_t m_{q0} = 0.100$. The solid line is the fitted result, ranging from $t_i = 6$ to $t_f = 12$ with $\chi^2/d.o.f. = 0.4326$ and confidence level=0.7620. (2) Extrapolation of the 1^{-+}, 0^{+-}, and 0^{--} splitting ratio $R_H = \Delta M(1H - 1S)/\Delta M(1^1P_1 - 1S)$ to the continue limit. Here H stands for the exotic hybrids.

from $\ln(C(t)/C(t+1))$ in the large time interval $[t_i, t_f]$, and $a_t m_1 + a_t m_2$ from $\ln(C'(t)/C'(t+1))$ in the time interval $[t_i^*, t_f^*] < [t_i, t_f]$, with reasonable $\chi^2/d.o.f.$ and optimal confidence level. Here $C'(t) = C(t+1)C(t-1) - C(t)^2$. Two methods provide a cross-check of the results.

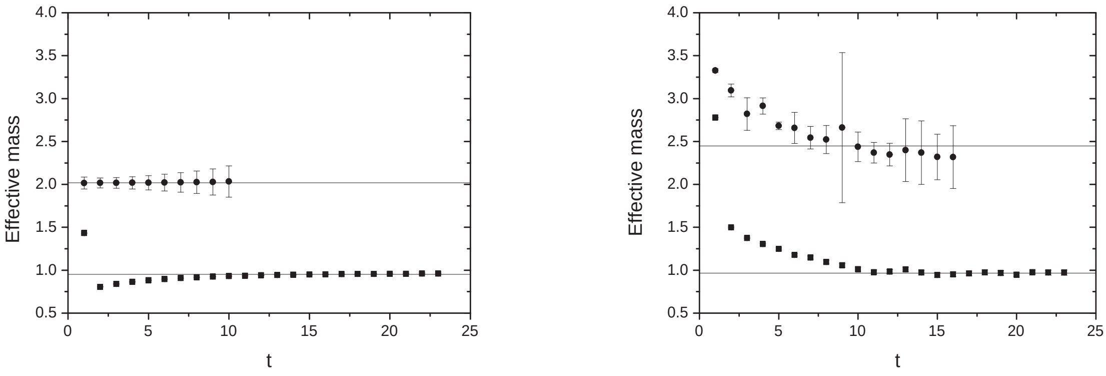
Figure 3(1) shows effective masses for the conventional 1^{--} quarkonium, where $a_t m_1$ and $a_t m_1 + a_t m_2$ are extracted respectively from the plateaux of the lower and upper curves, using the new method[18]. Figure 3(2) shows those for the 1^{--} hybrid meson.



(1) (2)

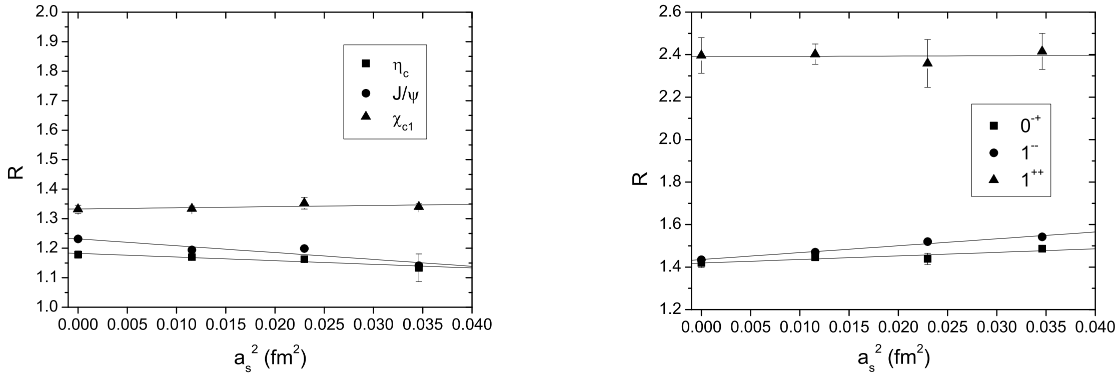
Figure 2: (1) Correlation function for the conventional 1^{--} quarkonium at $\beta = 2.6$ and $a_t m_{q0} = 0.229$; (2) Same as (1), but for the 1^{--} hybrid meson.

To extrapolate the quenched results to the continuum limit and determine the meson mass m in physical units, it is more convenient to consider the dimensionless ratio of effective masses $R = [a_t m]/[a_t m(1\bar{S})]$ or ratio of effective mass splittings $R' = [a_t m - a_t m(1\bar{S})]/[a_t m(\chi_{c1}(1^3P_1)) -$



(1) (2)

Figure 3: (1) Effective masses of the conventional 1^{--} quarkonium as a function of t for $\beta = 2.6$ and $a_t m_{q0} = 0.229$, using the new correlation function method. $a_t m_1 + a_t m_2$ and $a_t m_1$ are extracted respectively from the plateaux of the upper and lower curves, with $[t_i^*, t_f^*] = [1, 10]$ and $[t_i, t_f] = [11, 23]$; (2) The same as (1), but for the 1^{--} hybrid meson. $a_t m_1 + a_t m_2$ and $a_t m_1$ are extracted respectively from the plateaux of the upper and lower curves, with $[t_i^*, t_f^*] = [6, 16]$ and $[t_i, t_f] = [17, 23]$.



(1) (2)

Figure 4: (1) Extrapolation of $R = [a_t m]/[a_t m(1\overline{S})]$ to the continuum limit. Here $[a_t m]$ is the effective mass of the first excited state of a conventional charmonium, extracted by the method of Ref. [18]. (2) The same as (1), but for the hybrid charmonium.

$a_t m(1\overline{S})]$. For example, the ratio R for the first excited state of the conventional 0^{-+}, 1^{--} and 1^{++} charmonium mesons as a function of a_s^2 is plotted in Fig. 4(1), and those for the hybrids is plotted in Fig. 4(2). They indicate the linear dependence of R on a_s^2. The continuum results are obtained by linearly extrapolating the data to $a_s^2 \to 0$. After extrapolation, we determine m by inputting the experimental data $m(1\overline{S})_{exp}$ in R, or $m(\chi_{c1}(1^3P_1))_{exp} - m(1\overline{S})_{exp}$ and $m(1\overline{S})_{exp}$ in R'.

In the continuum limit, the masses of the 0^{-+}, 1^{--} and 1^{++} charmonium ground states are consistent with their experimental values 2.9804, 3.0969, and 3.5106 for $\eta_c(1S)$, J/ψ and $\chi_{c1}(1^3P_1)$. The results also show that the ground state for the nonexotic hybrid charmonium is degenerate with the conventional charmonium with the same quantum numbers. This might mislead people into giving up further study of the nonexotic hybrids.

The results for the conventional 0^{-+} and 1^{--} charmonium are in good agreement with the experimental data 3.638 and 3.686 for $\eta_c(2S)$ and $\psi(2S)$, which supports the reliability of the methods. Although there has not been experimental input for $\chi_{c1}(2^3P_1)$, our result is consistent with earlier lattice calculations[12, 13]. The minor differences between the data and experiments might be due to the quenched approximation used in the paper.

What new is that the first excited states of nonexotic charmonium hybrids are completely different from the conventional ones. The results show the masses of the 0^{-+} and 1^{--} hybrids to be about 0.7GeV heavier, and the 1^{++} about 3.2GeV heavier. These are very strong indications of gluonic excitations. This implies that radial excitations of the charmonium hybrids are completely different from the conventional non-hybrid ones, although their ground states overlap. This is clearly demonstrated in Figs. 2-5. This also teaches a very important lesson. One should carefully study not only the ground state, but also the excited states. Sometimes, the excited states show more fundamental properties of a hadron.

There is the issue as to whether the excited hybrid states extracted correspond to actual resonances or multi-particle scattering states. One important step is to show the volume dependence of each energy level. The spectral weights of the scattering states are very sensitive to the spatial volume. If they were scattering states, the spectral weights[1] would be proportional to $1/L_s^3$. Let L_s^{small} and L_s^{large} denote smaller and larger spatial lattice extent respectively. The averaged spectral weight ratio $W(L_s^{small})/W(L_s^{large})$ is respectively 1.05(40), 1.22(45) and 1.14(33) for the excited state of 0^{-+}, 1^{--} and 1^{++} hybrid. Example of $W(L_s^{small})/W(L_s^{large})$ for the 1^{--} hybrid is shown in Fig. 5. It confirms the nature of the resonance (bound) state. Details about other nonexotic hybrids could be found in Ref. [30].

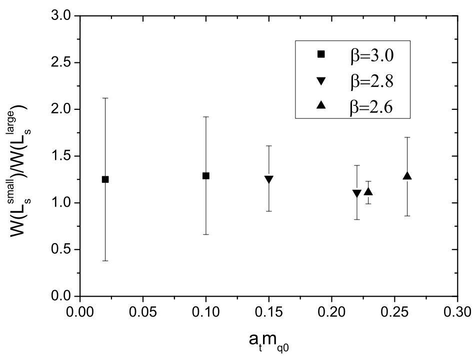


Figure 5: Ratio of spectral weights for the first excited 1^{--} hybrid state at different $a_t m_{q0}$ and β.

Finally, we discuss the new state $Y(4260)$, recently observed by the BaBar experiment[22] in the $J/\psi\pi^+\pi^-$ channel. It has the quantum numbers $J^{PC} = 1^{--}$. The discovery has attracted broad interest. There have been several phenomenological descriptions[23, 24, 25, 26, 27, 28] of this state: as tetra-quarks, a molecule of two mesons, $\psi(4S)$, or as a hybrid meson; However, most these assumptions were not based on QCD spectrum computations.

If $Y(4260)$ is a hybrid meson, our results indicate that it could certainly not be identified as the ground state of the 1^{--} hybrid meson. However, according to our lattice QCD spectrum

[1]For detailed discussions about the volume dependence of the scattering states, please refer to Refs. [20, 21].

calculations, it is most probably the first excited state of the 1^{--} hybrid charmonium. Further experimental study of the decay modes will clarify this issue.

After submission of Ref. [30], we noticed that the CLEO Collaboration announced their new experimental measurements[29] of $Y(4260)$, which strongly support the interpretation of a 1^{--} hybrid state.

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