Baryons in 2+1 flavour domain wall QCD

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Preliminary results for some of the light baryon masses and their excited states in 2+1 flavour domain wall QCD are presented. The spectrum was determined on two volumes, $16^3 \times 32$ and $24^3 \times 64$, both with $L_S = 16$. The degenerate light quark mass is less than a third of the strange quark mass. All data was produced jointly by the RBC and UKQCD collaborations on QCDOC machines.

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1. Introduction

Baryon physics is an important topic for lattice QCD. The lowest lying states in the spectrum are well determined, so a comparison of lattice results with experiment can be used to calibrate how good the calculation is. The baryon spectrum is a good place to test lattice QCD as these states are sufficiently complex to reveal physics hidden in the mesonic sector. Moreover, they are the simplest system in which the non-abelian character of QCD is evident: There are three quarks in the proton because there are three colours.[1]

The nature of the excited baryon spectrum is not completely resolved. Some of these states, for example the N(1440), the so called "Roper" resonance, cannot be explained in terms of the simple quark model. That is, they do not follow the pattern of alternating negative and positive parity states which is observed elsewhere in the spectrum. This has led to speculation that these states are not traditional three quark baryons, but somehow exotic. In principle, lattice QCD calculations can help resolve these questions, by determining the excited spectrum [2].

There is also diverse phenomenology in the matrix elements of baryons. Moments of structure functions are important for collider physics, where significant progress has been made [3]. The strangeness content of the nucleon is relevant for Cold Dark Matter models, the electric-dipole moment of the neutron is relevant for SUSY models, and proton decay is a prediction of GUTs and SUSY GUTs. The matrix elements needed for these quantities can all be determined in lattice QCD.

This rich phenomenology provides quite a challenge for lattice calculations. To calculate the properties of baryons, a fermion formulation which has the correct flavour structure is required. To go beyond a crude calculation, the light quark mass must be light enough to match onto chiral perturbation theory (χPT). A lattice formulation which has chiral symmetry is advantageous as a continuum-like (χPT) can be used. Baryons are large objects, and their excited states larger still so big volumes are needed. Finally, baryon correlators are noisy objects requiring large ensembles to reduce statistical uncertainties.

This work presents a preliminary study of the lowest lying, valence degenerate states of the baryon spectrum; $\{N, \Delta, \Omega, N^\star\}$ on two different volumes at a single lattice spacing with 2+1 flavours, and three different light sea quark masses. The Domain Wall Fermion (DWF) formulation is used as this fulfills the criteria set out above, *i.e.* flavour and approximate chiral symmetries at fixed lattice spacing.

2. Details of the calculation

The DWF action is given in [4] with the Pauli-Villars field in [5] for the dynamical simulation. The gauge fields were generated with renormalisation group (RG) improved action, as follows:

$$S_G = -\frac{\beta}{3}\left[(1-8)c_1 \sum_{x,\mu\nu} P(x)_{\mu\nu} + c_1 \sum_{x,\mu\neq\nu} R(x)_{\mu\nu}\right] \qquad (2.1)$$

where $c_1 = -0.331$ for the Iwasaki action [6] and the coupling was chosen to be $\beta = 2.13$. The ensembles were all generated on QCDOC machines, using the exact RHMC algorithm [7, 8, 9], for

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[1]paraphrasing N. Isgur in *why $N^\star$'s are important* [1]

m_f	volume	N_{traj}	N_{meas}	N_{ind}	physical length
0.03/0.04	$16^3 \times 32$	4000	700	70	
0.02/0.04	$16^3 \times 32$	4000	700	70	$L = 2\text{fm}$
0.01/0.04	$16^3 \times 32$	4000	1400	70	
0.03/0.04	$24^3 \times 64$	3000	100	50	
0.02/0.04	$24^3 \times 64$	3600	200	50	$L = 3\text{fm}$
0.01/0.04	$24^3 \times 64$	4000	300	77	

Table 1: Properties of the ensembles used in this study.

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two volumes of $16^3 \times 32$ and $24^3 \times 64$, and fifth dimension of length $L_S = 16$, the details are listed in Table 1.

The trajectory length in this study is $\tau = 1$. The integrated autocorrelation times are estimated to be $\mathscr{O}(50)$ for hadronic correlators. To maximise statistics, the correlation functions were over-sampled and then binned. The correlators were computed from up to four sources on different time-planes with two different smeared sources. The large volume production and measurement has yet to be completed. The details of the ensembles are given in [10]. The lattice spacing is $a^{-1} = 1.60(3)$ GeV as set by (m_ρ,r_0,Method-of-planes), and $m_{\text{res}} = 0.003$. Details of these determinations, and the mesonic spectrum are given in [11, 12].

3. Preliminary Results

The standard baryon interpolating operators are given by

$$\Omega(x) = \varepsilon_{ijk} \left[\psi_i(x) C \Gamma \psi_j(x) \right] \psi_k(x) \tag{3.1}$$

For the $I = \frac{1}{2}$ baryons, $\{N, N^\star\}$, two operators were used: $\Gamma = \{\gamma_5, \gamma_5\gamma_4\}$. For the $I = \frac{3}{2}$ baryons, $\{\Delta, \Omega\}$, six operators were used: $\Gamma = \{\gamma_k, \gamma_5\gamma_k\}$. For baryon correlators in a finite box with (anti)periodic boundary conditions in space (time), the backward propagating state is the negative parity partner, that is

$$C(t) = A_+ e^{-m_+ t} + A_- e^{-m_-(T-t)} \tag{3.2}$$

The masses of the positive, m_+, and negative parity, m_-, states were determined by a simultaneous fit to equation (3.2). The effective mass for these states are shown in Figure 1(Left). For the $I = \frac{3}{2}$ states there are two mass combinations, $m_{\text{val}} = m_{\text{light}}$, a Δ-like baryon and $m_{\text{val}} = m_{\text{strange}}$, an Ω-like baryon. Typically the signal for the negative parity partner of the $I = \frac{3}{2}$ baryons was to weak to satisfactorily resolve.

Shown in Figure 1(Right) is the Edinburgh plot, overlayed with earlier data obtained on ensembles with $L_S = 8$ [13]. Despite the obvious caveats regarding different volumes and lattice spacings, it is remarkable that the data appears to be crudely following a uniform curve. Moreover, there is good agreement between the lattice data and the quark model curve. In principle QCD should smoothly interpolate between the heavy quark limit and the physical data. It is therefore reassuring that this preliminary data appears to lie on such a curve, and supports the use of dynamical DWF.
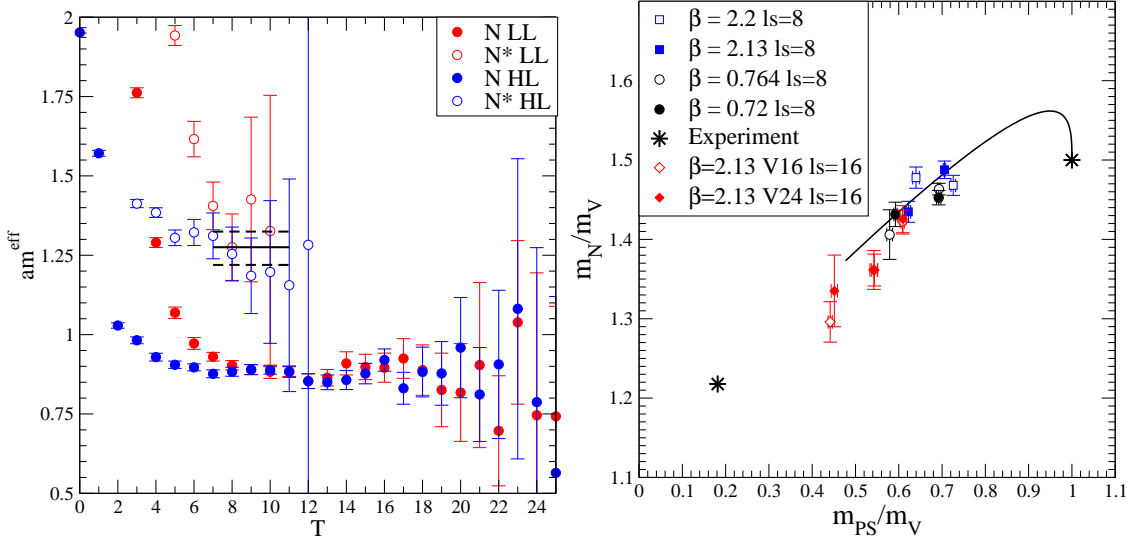
Figure 1: (left) The effective mass of $I = \frac{1}{2}$ baryon correlator on the $24^3 \times 64$,$am_l/am_s = 0.02/0.04$ ensemble. The red (blue) symbols show data obtained using a local (smeared) data. (Right) The Edinburgh plot with the $L_S = 8$ data [13] overlayed.

Shown in figure 2 is the chiral behaviour of the $\{N, N^\star\}$ (left) and $\{\Delta, \Omega\}$ (right). The nucleon data show no obvious finite size effects (FSE). Whilst $m_{PS}L$ is not the only metric for FSE, previous lattice studies have suggested that with an $m_{PS}L > 4$ FSE for the nucleon are less than a few percent. The lightest, small volume data has $m_{PS}L \sim 3.9$, so the absence of FSE is not surprising. Although we have only three data points, there is no sign of any deviation from linear behaviour. It will be interesting to see if this behaviour persists lighter quark mass.

The data for the negative parity partner is noisy, and resolving the plateau was problematic. This will improve with more data. The large volume data appears to be falling rather faster with quark mass than is naively expected. The solid blue line, is not a fit to the data, but lattice nucleon mass plus the experimental $N - N^\star$ splitting. The large volume data are below this line. One possible explanation for this is the dashed black line. This is the mass of the pseudo-scalar plus the nucleon. When the quark mass is light enough the decay channel $N^\star \to N + \pi$ is open. The large volume data appears to follow this line for the lighter data. Has the $N^\star$ decayed? and rather than measure the mass of the $N^\star$, are these operators measuring the mass of the $N + PS$? Whilst this interpretation of the data is tantalising, the data is not yet good enough to support it. This question can only be resolved by more data.

Shown in figure 2 are the decuplet baryon masses against the pseudo-scalar meson mass squared. For the deltas, the sea and valence quark mass varies, for the omegas, only the sea quark mass is varied. With statistical uncertainties of a few percent and three datum it would be difficult to interpret the apparent curvature of the delta data using χPT. Within these uncertainties, the data is linear. A two dimensional linear extrapolation in sea and valance quark masses to the decuplet baryons suggests that there is a FSE for the delta on the small volume data and possibly for the omega as well. This is not unreasonable as the delta is expected to be a larger state than the
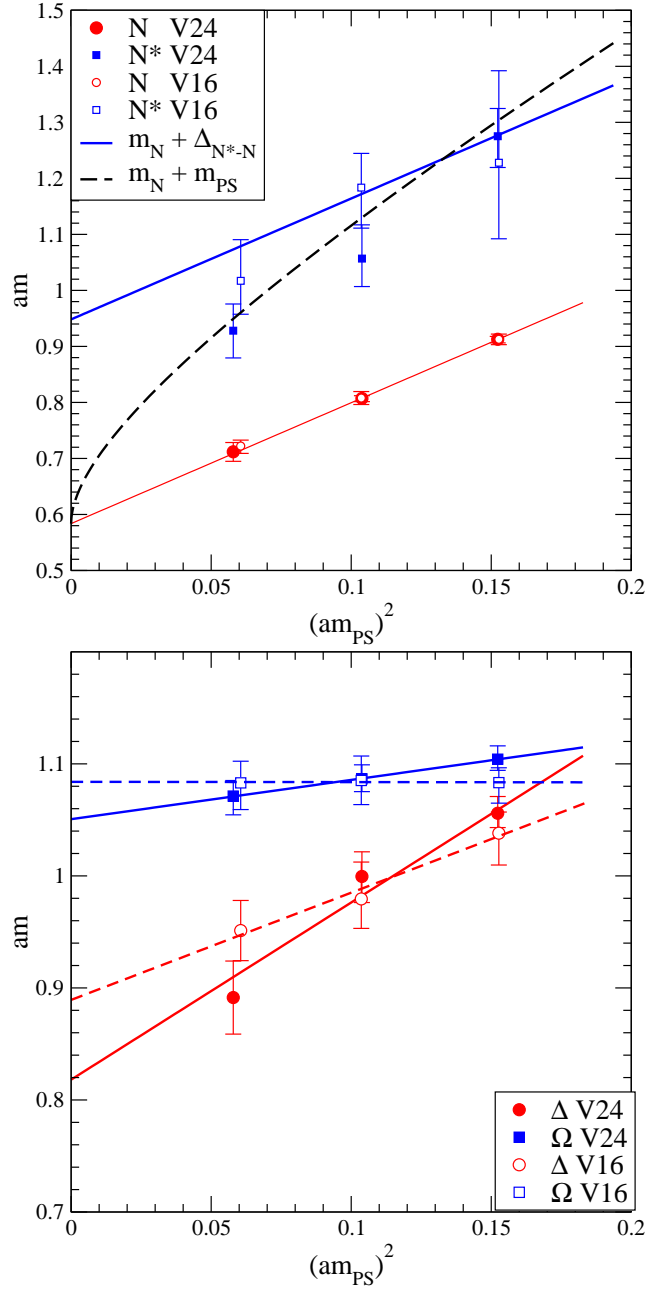
Figure 2: Chiral extrapolation of the baryon masses. Upper $I = \frac{1}{2}$, Lower $I = \frac{3}{2}$.
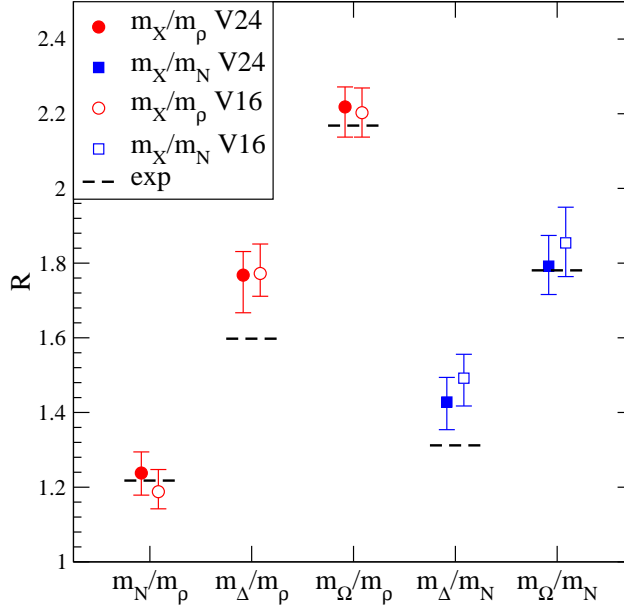
Figure 3: Ratios of baryon masses.

nucleon.

Rather than present results in physical units on preliminary results, shown in figure 3 are the ratios of the baryon masses over other lattice quantities, and compared to the experimental ratios. The spectrum of the lowest lying states crudely reproduces experiment. There is a probable finite size effect on the small volume data for decuplet baryons. Plotting the ratios against the nucleon, the decuplet baryons are closer to experiment, suggesting perhaps that the chiral extrapolations of the nucleon and the ρ are not quite consistent. On this preliminary data, both these channels were noisy, this is not to suggest the data is unreliable, but that with more data the extrapolations can be improved.

4. Conclusions

Preliminary results for the lowest lying, valence degenerate baryon spectrum, $\{N, \Delta, \Omega, N^{\star}\}$ on two volumes at one lattice spacing with three different sea quark masses $m_l/m_s \sim \{0.77, 0.53, 0.30\}$ have been presented. The data for the Edinburgh plot appears to lie on a smooth curve interpolating between the heavy quark limit and the physical datum, as predicted by the quark model. This suggests that the chiral behaviour of the data is continuum-like even at finite lattice spacing. This is corroborated by the comparison of the spectrum to experiment, albeit with FSE for the decuplet baryons. A possible interpretation of the $N^{\star}$ data, is that it has decayed at the lighter quark masses.

With better statistics the data will improve. A lighter quark mass of $m_l/m_s \sim 0.19$ is currently in production, and a finer lattice spacing is planned. These will contribute to controlling the systematic uncertainties of the chiral extrapolation and lattice artifacts. With these reductions in the uncertainties further calculations of the full spectrum and matrix elements as outlined in the introductions promises an exciting program of baryon physics.

Acknowledgements

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