Exploratory study of the D_s spectrum in 2+1 domain wall QCD with heavy overlap

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We present preliminary results for the D_s meson spectroscopy study on the 2+1 flavour domain wall fermion lattice configurations, generated with the Iwasaki gauge action at $\beta = 2.13$ by the RBC-UKQCD collaboration. The simulations are on $16^3 \times 32$ lattice with $L_s = 16$. We consider the charm quark propagating as an overlap fermion at fixed lattice spacing. The dispersion relation and mass splittings are evaluated.

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1. Introduction

The discoveries of new resonances D_{sJ} by the B factory experiments [1] and CLEO [2] have provoked much interest in heavy-light systems in general and in the D_s mesons in particular. The mass splittings can be understood in terms of heavy quark and chiral symmetry [3, 4].

In the double limit of heavy quark and chiral symmetry, the two heavy-light multiplets, $\{0^-, 1^-\}$ and $\{0^+, 1^+\}$, are degenerate. Then chiral symmetry breaking causes splitting between parity partners, such that the $1^+ - 1^-$ and $0^+ - 0^-$ are equal. Experimentally, the splittings, shown in Table 1, are remarkably close. The hyperfine splitting can also be understood in terms of heavy quark symmetry breaking effects.

Many previous lattice calculations [6, 7, 8, 9, 10, 11] tried to reproduce the features of these heavy mesons, most of them considering a static or non-relativistic heavy charm quark, with the exception of [9] which uses the Fermilab approach and [11] which describes the charm quark as a domain wall fermion. All these works are in the quenched approximation.

In this work the charm quark is described by an *overlap* [12] formalism, while the light strange quark is a domain wall fermion, DWF [13].

$0^+ - 0^-$	$1^+ - 1^-$	$1^- - 0^-$
349.1(4)	346.9(1.0)	143.8(4)

Table 1: Experimental values in MeV for the different mass splittings from [5].

2. Numerical details

The gauge ensembles used for our calculations are the 2+1 flavour dynamical DWF ensembles from RBC-UKQCD collaboration [14]. They were generated with the renormalized group improved Iwasaki gauge action at $\beta = 2.13$. The lattice volume is $16^3 \times 32$, with the fifth dimension $L_s = 16$ and the domain wall height $aM_5 = 1.8$. Three sea quark masses are considered, $am_{sea} = 0.01, 0.02, 0.03$, and the strange quark mass is fixed at $am_s = 0.04$ [15]. The correlators were measured with sources on multiple time planes, in order to improve our statistics. Details of the three ensembles used are listed in Table 2.

For the overlap charm [16] quark mass, two values are chosen, $am_c \sim 0.72, 0.9$. Correspondingly, we have two heavy-light mesons, indicated as H1, the lighter, and H2, the heavier, for each sea quark mass. Recall the expression of the massive overlap operator:

$$aD_{ov} = \rho(1+\mu) + \rho(1-\mu)\gamma_5 sgn(\gamma_5(aD_W - \rho)), \tag{2.1}$$

where $\mu = \frac{am_q}{2\rho}$ and ρ is any mass parameter that can be added to D_W without affecting the continuum limit: here it was chosen equal to 1.3 looking at the heavy-heavy pseudoscalar. The overlap operator was used to invert on hyp-smeared DWF gauge configurations for mass parameter $\mu \sim 0.277, 0.346$, corresponding to the two charm mass values above.

am_{sea}	N_{traj}(sep)	No. of Origins	p^2_{max}	N_{cfgs}
0.01	500-4000(50)	4	4	282
0.02	1000-4025(50)	2	4	122
0.03	1000-4000(50)	2	2	121

Table 2: Datasets used.

3. Analysis

In Figure 1 we show typical effective masses for the low-lying J^P states of the four channels we are interested in. The left plot is for the heavy-light meson containing the lighter charm quark, the right plot for the heavier one. For the pseudoscalar and vector channels, similarly reasonable plateau are found for higher momenta. Once computed the meson masses at different lattice momenta,

$$p_L a = \frac{2\pi\sqrt{n}}{La}, \quad n \in \mathbb{Z}^+ \tag{3.1}$$

we fit them to the dispersion relation.

The dispersion relation is defined such that the $\mathcal{O}(m^2 a^2)$ error is reflected in the deviation of c, the effective speed of light, from unity. We fit the energies to a quadratic expression as in eq. (3.2), (3.4)[1], as explained below.

The plot in Fig. 2 shows the dispersion relation for the pseudoscalar lightest meson, i.e. H1, in the $am_{sea} = 0.01$ case with 5 momenta. The value of the speed of light obtained from the fit to eq. (3.4), $c = 0.897(12)$, is higher than one might expect from [16]. One of the methods trying to overcome the problems with heavy quarks (i.e. $am_Q \sim 1$) is the Fermilab or Relativistic Heavy Quark approach [17]. It gives us an alternative interpretation of the dispersion relation: the basic idea is considering the expansion of the energy-momentum relation in powers of (lattice) momentum pa,

$$(Ea)^2 = (M_1 a)^2 + \frac{M_1}{M_2}(pa)^2 + K(pa)^4 + ... \tag{3.2}$$

where M_1 is the rest mass, $M_1 = E(0)$, and M_2 is the so-called kinetic mass, $M_2^{-1} = (\frac{\partial^2 E}{\partial p_i^2})_{\mathbf{p}=0}$. The relativistic mass shell will have $m_Q = M_1 = M_2$, and the expression above becomes

$$(Ea)^2 = (M_1 a)^2 + (c^2 = 1)(pa)^2. \tag{3.3}$$

In practice, at our non-relativistic mass, we can not truncate the expansion at p^2, but we have to consider higher order terms, i.e. including δE_{lat},

$$(Ea)^2 = (m_Q a)^2 + c^2 (pa)^2 + \delta E_{lat}. \tag{3.4}$$

It has been observed that the rest mass of non-relativistic particles decouples from the interesting dynamics.

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[1]Only three momenta are available for the $am_{sea} = 0.03$ ensemble, i.e. there are only three points in the plot of energies versus momenta, so we found the linear fit with $pa = 2\sin(\pi\sqrt{n}/La)$ (following [16]) be the best one in the 0.03 case.

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The suggestion from the Fermilab approach [17] is then considering M_2 instead of M_1 and tuning the couplings in the lagrangian so that M_2 takes the physical value. In this preliminary analysis we consider both M_1 and M_2 and look at the dependence of the mass splittings on them.
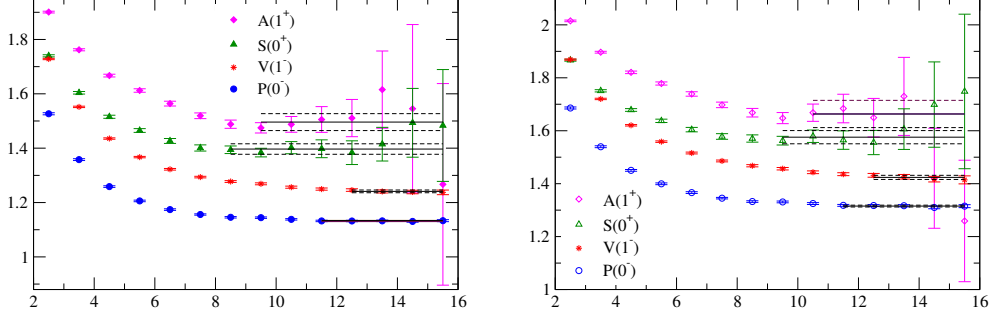
Figure 1: Effective mass for the low-lying heavy-light mesons with charm mass $am_c \sim 0.72$ (left) and $am_c \sim 0.9$ (right), for the $am_{sea} = 0.01$ ensemble.
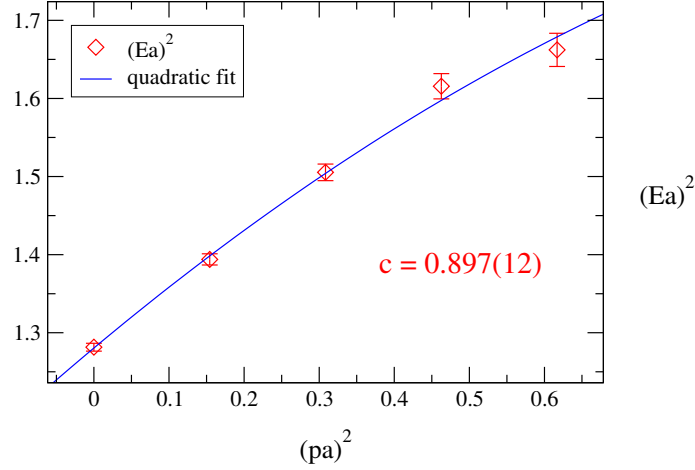
Figure 2: Dispersion relation plot for the pseudoscalar channel of the lighest heavy-light meson considered, in the $m = 0.01$ case.
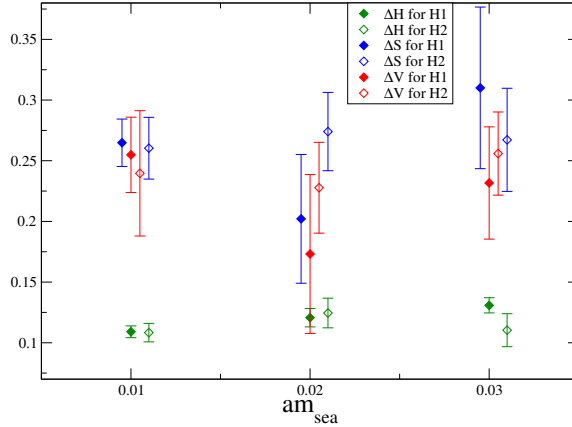
4. Results

First of all let's clarify the notation used for the mass splittings considered: $\Delta H = 1^- - 0^-$ is the hyperfine splitting, $\Delta S = 0^+ - 0^-$ and $\Delta V = 1^+ - 1^-$ are the scalar and vector parity splitting respectively. The values of these splittings obtained with all three ensembles for both our heavy-light mesons, H1 and H2, are summarized in Table 3. The same splittings values are plotted versus am_{sea} in Figure 3. We can notice a very small dependence on the sea quark masses.

The plots in Figure 4 summarize our main results. The two plots on the left show our splitting values versus $1/M_{PS}$, where M_{PS} is equal to M_1 in the upper left panel and to M_2 in the lower left one. The two plots on the right show the ratio of vector parity over scalar parity, $1^+ - 1^-/0^+ - 0^-$

m_{sea}	Meson	$a\Delta H$	$a\Delta S$	$a\Delta V$
0.01	H1	.109(5)	.265(20)	.255(31)
	H2	.108(8)	.260(25)	.240(52)
0.02	H1	.121(8)	.202(53)	.173(65)
	H2	.125(12)	.274(32)	.228(37)
0.03	H1	.131(6)	.310(67)	.232(46)
	H2	.110(14)	.267(43)	.256(34)

Table 3: Mass splittings value obtained for each ensemble and for both our heavy-light mesons, H1 and H2.


Figure 3: Mass splitting values listed in Table 3 versus the *am_{sea}*.

versus $1/M_{PS}$, as before. The horizontal line represents the experimental value. Results from all three m_{sea} values are shown. In all plots, the vertical line represents our estimate of the physical D_s meson, using $a^{-1} = 1.60(3)$ GeV [15]. We can see that the effect of using M_2 instead of M_1 is a shift in the *x* axis, as we expect from eq. (3.2) and (3.4): the difference between the two masses is entirely a lattice artefact (eq. 3.4).

What we can see from these four plots is in any case no heavy quark mass dependence of the mass splittings.

5. Conclusions

In the first stage of our study of D_s meson on 2+1 DWF QCD, with the charm quark as an overlap fermion, we found clear signals for all the four channels we were interested in. A very little dependence of the splittings on the sea quark mass is observed. For the dispersion relation analysis, both M_1 and M_2 were considered: as expected, no heavy quark mass dependance in the splittings is observed. The ratio of two parity splittings obtained is close to the experimental value within statistical errors, as shown in Figure 5. The $am_{sea} = 0.03$ ensemble data don't always follow the trend of the other two: investigations are in progress. In order to reduce our large error bars, we need more statistics. A quenched calculation is also in progress.
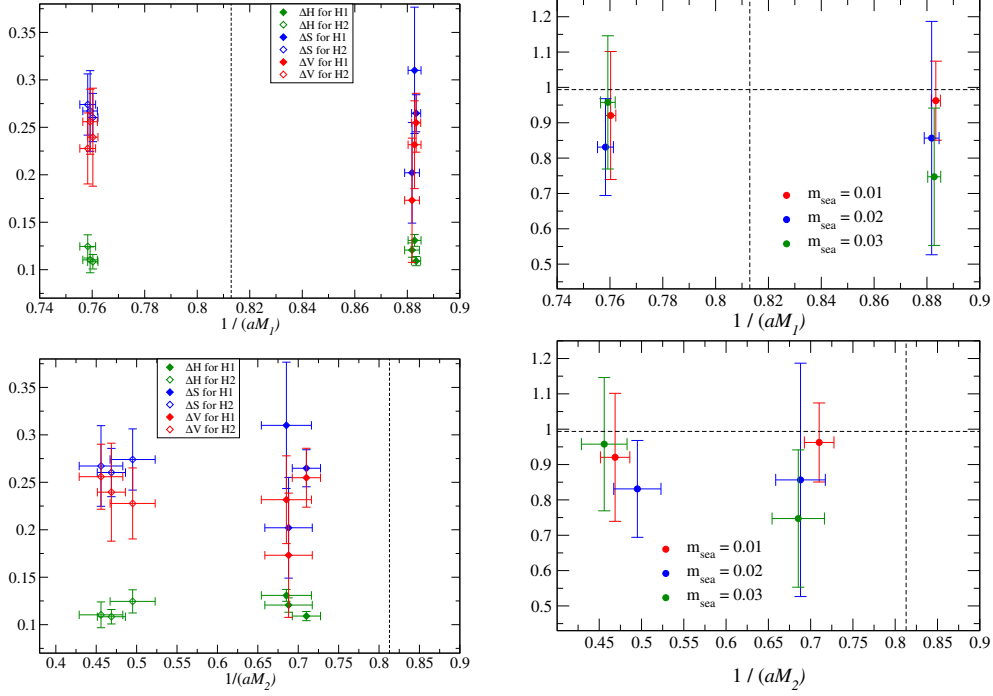
Figure 4: On the left the splitting values versus $1/M_{PS}$ are plotted, with M_{PS} equal to M_1 in the upper left panel and to M_2 in the lower left one. On the right the ratio $1^+ - 1^-/0^+ - 0^-$ obtained is plotted versus $1/M_{PS}$, with M_{PS} as before. The horizontal line corresponds to the experimental value. In all plots, the vertical line represents our estimate of the physical D_s meson, using $a^{-1} = 1.60(3)$ GeV [15].
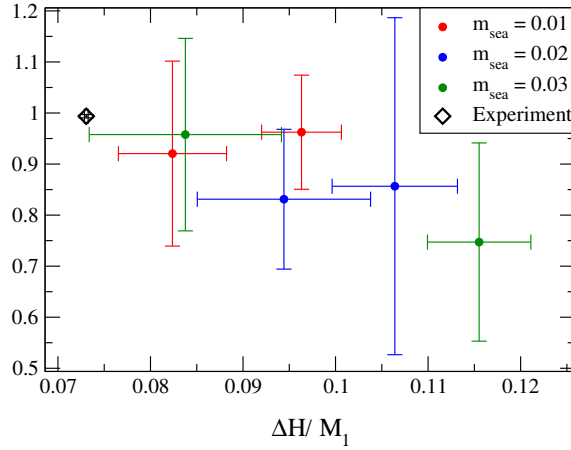
Figure 5: Plot of the $1^+ - 1^-/0^+ - 0^-$ ratio obtained for the three ensembles versus the hyperfine splitting over M_1. The experimental value is also plotted.

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