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An effectively chiral Yukawa model on a lattice

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A lattice action for naive fermions is presented which, near a weak-coupling phase transition, can have small massses for the 8 doublers of one chirality while rendering those of opposite chirality much heavier than $<\phi>_R$. The action is ultra-local and obeys all fundamental principles including exact gauge invariance when gauge fields are included. Analytical calculations have been done for models with both U(1) and SU(2) global symmetries. The same analysis is in reasonable agreement with old simulation results in the Smit-Swift model where the mass of the lightest unwanted doubler remains below $<\phi>_R$ near the weak-coupling transition. The possible relevance of my model toward a non-perturbative formulation of the Standard Model is also discussed.

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1. Introduction

After much effort, the most popular constructions for a chiral gauge model on a lattice are still not watertight. Here I present a simple yet less ambitious methodology illustrated below in terms of naive fermions. The idea is to make the doublers of unwanted chirality (e.g. with right-handed coupling to the gauge fields) heavy enough so that the effective model at low energy looks chiral. This idea was also pursued by Montvay[1], who constructed models in terms of mirror pairs of naive fermions which mixed through Wilson and Yukawa interactions. While Montvay's model and the model I'm presenting here are effectively chiral at low energy, they may not be useful toward a lattice formulation of the Standard Model without further developments (some possibilities discussed below). Heavy families, mirror or not, are difficult to reconcile with precision data. (See ref.[2] for a discussion plus references to earlier work, and see ref.[3] for a more recent assessment of the experimental situation.) The problem is that when fermions are made heavy through a Yukawa interaction, they also strongly couple to the scalar fields (radial and Goldstone modes). The net result is that loop diagrams involving these fermions are not suppressed, and have measurable consequences at low energy, for example, on the amount of $W^3 - B$ mixing.

The basic idea of my model is to lift the degeneracy between even and odd naive fermion modes through a modified kinetic term. This contrasts with older attempts to use Wilson-Yukawa terms as in the Smit-Swift model, which failed to do a very good job (see the review of ref. [4]). Both models have a weak-coupling phase transition where $< \phi >$ becomes non-zero. This paper is devoted to a study of the fermion spectrum near this transition in the broken phase. Both models also have a strong-coupling transition around which the fermions bind to the scalar field, producing an uninteresting left-right symmetric spectrum.

A truly chiral gauge model could be achieved in my model, or some variant of it, if there were phase transitions at intermediate coupling around which the even modes were weakly coupled to the scalar field, while the odd modes coupled strongly enough to bind. I believe such transitions may exist, but it has been difficult so far to come up with trustworthy analytical methods to rule one way or the other. Numerical simulations may be necessary.

2. Preliminaries

An "even" naive fermion mode is, of course, any one of 8 modes defined around a corner of the Brillouin zone having an even number of momentum components $= \pi$. (Here and below the lattice spacing is set to 1.) That is, even modes have p in the vicinity of

$$(0,0,0,0), (\pi,\pi,0,0), \ldots, (\pi,\pi,\pi,\pi)$$

while odd modes live around

$$(\pi,0,0,0), \ldots, (\pi,\pi,\pi,0), \ldots.$$

Consider the behavior of various vector and axial-vector objects under a transformation from an even to odd mode, e.g. under

$$\psi_n \to (-1)^{n_1} \gamma_1 \gamma_5 \psi_n.$$

The vector $\bar{\psi}_n\gamma_\mu\psi_{n+\mu}$ is even, (i.e. same for all modes) while the axial vector $\bar{\psi}_n\gamma_\mu\gamma_5\psi_{n+\mu}$ is odd (i.e. opposite sign for evens and odds) as is well known. It is less well known that there are objects with the opposite behavior: $\bar{\psi}_n\gamma_\mu\psi_{n+\mu+\sigma}$ is odd while $\bar{\psi}_n\gamma_\mu\gamma_5\psi_{n+\mu+\sigma}$ is even, where σ is any one of the 16 vectors $(\pm1,\pm1,\pm1,\pm1)$.

A kinetic term which is odd under the transformation is thus

$$K_{odd} = \bar{\psi}\Delta_{odd}\psi \equiv \frac{1}{32}\sum_{n,\mu,\sigma}\bar{\psi}_n\gamma_\mu(\psi_{n+\mu+\sigma} - \psi_{n-\mu-\sigma}),$$

which gives an inverse propagator

$$S^{-1}(p) = i\gamma_\mu \sin p_\mu \prod_\lambda \cos p_\lambda\,.$$

Now we are in a position to define a kinetic term which breaks the degeneracy between the even and odd modes:

$$K(\alpha) = \frac{1}{1+\alpha}(K_{even} + \alpha K_{odd}),$$

where α is a free parameter. To see how the breaking works, first consider free fermions with $S = K(\alpha) + m\bar{\psi}\psi$. Then the masses of the even and odd modes for $m \ll 1$ are

$$m_{even} \approx m\,, \quad m_{odd} \approx \frac{1+\alpha}{1-\alpha}m. \tag{2.1}$$

For $\alpha \to 1$, $m_{odd} \to 1$.

3. The Model

The main point of this paper is to study the effect of the kinetic term $K(\alpha)$ within an otherwise usual Yukawa model given by

$$S = K(\alpha) + y(\bar{\psi}_{R,n}\phi_n\psi_{L,n} + h.c.) + \kappa tr(\phi_n^\dagger\phi_{n+\mu} + h.c.) \tag{3.1}$$

where ϕ is an element of U(1) or SU(2). I will refer to this model as the Modified-Kinetic-Yukawa or MKY model. Adding a Wilson-Yukawa term does not make much difference in the results, and so is omitted. Standard mean-field methods reveal that the MKY model has phase transitions at weak and strong coupling as in the Smit-Swift model. As mentioned above, I have not established yet whether or not there are any transitions in the intermediate coupling regime, because neither mean-field nor Hartree-Fock methods are readily applicable there. Numerical simulations should be doable but slow because of the non-nearest-neighbor couplings in K_{odd}. In 4 dimensions, each fermion degree of freedom couples to 73 others (64 from K_{odd} plus 8 from K_{even} plus 1 from the Yukawa term), as compared to 9 in a model with only nearest-neighbor couplings.

4. Analysis

I'm going to present results only at $\kappa = 0$ in the broken phase, near the weak coupling transition. The critical coupling from mean-field methods for both U(1) and SU(2) is given by

$$2y_c^2\int_p \frac{1}{\sum_\mu s_\mu^2(\alpha)} = 1,$$

where

$$s_\mu(\alpha) = \frac{1}{1+\alpha}\sin p_\mu (1+\alpha\prod_\lambda \cos p_\lambda). \tag{4.1}$$

The results for 2 values of α are

$$y_c(.8) = .38,\; y_c(.9) = .29\ .$$

Since the $\kappa = 0$ model with these small values of y is essentially equivalent to a 4-fermion model with coupling y^2, the corrections to these values will be $O(y^4)$, which is small.

The masses of the even and odd modes are simply

$$m_{even} \approx y < \phi > ,\quad m_{odd} \approx \frac{1+\alpha}{1-\alpha} m_{even}$$

which I expect are reliable as long as m_{odd} is small, "small" estimated anecdotally from my numerics to be less than .17 or so, more or less depending on α, as determined by studying the poles of the propagator. When m_{odd} is not small, it is not so simply related to m_{even}.

To have any hope of effectively decoupling the odd modes or making their effects small at low energy, we need to see how m_{odd} compares to the mass of the gauge fields, or to $< \phi >_R$. Fortunately a mean-field estimate for this is easy to obtain. The recipe is as follows. First make the standard MF change of variables from compact ϕ to non-compact V. The saddle point value for V is the MF estimate for $< \phi >$. Expand around the saddle point by setting

$$V = (< \phi > + \rho)e^{i\pi}.$$

Then $< \phi >_R$ comes from the coefficient of the Goldstone part of the effective action:

$$S_{eff}(\rho, \pi) = < \phi >_R^2 p^2 \tilde{\pi}_p^2 + \dots\ .$$

This is equivalent to introducing gauge fields A with coupling g and defining $< \phi >_R \equiv M_A/g$. The result is

$$< \phi >_R^2 = 2m_{even}^2 \int \frac{\cos^2 p_1}{(\sum_\mu s_\mu(\alpha)^2 + m_{even}^2)^2}$$

where $s_\mu(\alpha)$ is given by eq. 4.1.

Here are a few sample numbers for SU(2). For $\alpha = .8$ and $y = .39 (= y_c(.8) + .01)$ I find $m_{even} = .018$, $m_{odd} = .162$, $< \phi >_R = .079$ and thus $m_{odd}/ < \phi >_R = 2.0$. Inspecting eq. 2.1 one might expect that making α closer to one would increase this ratio. This seems to be the case but the increase is small. For example for $\alpha = .9$ and $y = .30 (= y_c(.9) + .01)$ I find $m_{even} = .0056$, $m_{odd} = .114$ and $< \phi >_R = .0505$ which gives $m_{odd}/ < \phi >_R = 2.2$. For U(1) I also found ratios around 2.

Within the MF approximation I'm using here, I can't safely take α much closer to 1, because then the zero-order coefficient, $(1-\alpha)/(1+\alpha)$, of $\gamma_\mu p_\mu$ for an odd mode becomes comparable to its one-loop $O(y^4)$ correction.

To get some idea of the accuracy of the MF estimates versus simulation results, I have turned to the Smit-Swift model. For $y = .05$, $\kappa = 0$ and $w = .15$ (the coefficient of the Wilson-Yukawa term), the MF analysis gives $m_D = .16$ and $m_D/ < \phi >_R = .43$, where m_D is the mass of the lightest (unwanted) doubler. From simulations on $6^3 12$ and 8^4 lattices, Bock *et al* [5] find $m_D/ < \phi >_R = .30 \pm .05$, a bit lower than my estimate. Even if my MF estimates above for $m_{odd}/ < \phi >_R$ are 30% too high they are still significantly > 1.

5. Final Remarks

Getting the mass of the wrong-chirality doublers greater than $<\phi>_R$ near the weak-coupling transition is interesting, but, by itself, probably not going to lead to a phenomenologically acceptable formulation of the Standard Model, as discussed in the introduction. Some clever amendments to this model would be necessary. One would have to somehow offset the contributions of the heavy fermions to the so-called S-parameter(see ref.[2]). The quark sector gives the same sign to S and so cannot help. Perhaps scalar self-interactions can be tuned to do this. It may be more fruitful to look for phase transitions in the intermediate coupling regime where the even modes are light, while the odd modes bind to the scalar, and thus truly decouple.

References

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