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Mass loss from a viscous accretion disc in presence of cooling

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We compute the mass loss from a viscous advective disc, which undergoes centrifugal pressure mediated shock. We show that for same outer boundary condition of the disc, the mass outflow rate decreases with increasing viscosity, since viscosity weakens the centrifugal barrier that generates the shock. We also show that in presence of cooling the mass outflow rate decreases marginally, which shows that these outflows are basically centrifugally driven. We also show that the optical depth of the disc for external photons entering the post shock disc is decreased because of mass loss, and hence should soften the spectrum.

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1. Introduction

Jets are ubiquitous astrophysical objects and are known to accompany YSOs, aged stars, X-ray binaries (binary system comprising either neutron star or stellar mass black hole candidates and an ordinary stellar companion), AGNs (supermassive black hole candidates). While jets or outflows around gravitating centres with hard boundaries are quite natural, it is altogether a different proposition to consider jets from around a black hole. As black holes do not have either hard boundary or intrinsic atmospheres, jets/outflows have to originate from the accreting matter onto black holes. Therefore, the physics of accretion disc is of paramount importance in order to understand the ejection mechanism from the accretion disc.

Matter accreting onto black holes has to satisfy two inner boundary conditions without regards of the type of outer boundary condition. Namely, (a) matter crossing the horizon has to be super sonic and, (b) angular momentum of matter within the marginally stable orbit ($r_{ms} = 3r_g$, where $r_g = 2GM_B/c^2$, G, M_B, c are Schwarzschild radius, gravitational constant, mass of the black hole and speed of light, respectively) has to be sub-Keplerian. At large distances from the black hole, matter is evidently subsonic, therefore condition (a) automatically suggests that accretion flows onto black holes are globally transonic. A transonic, sub-Keplerian flow may admit two X-type critical points depending on the flow parameters. Matter crossing the outer critical point (x_o) becomes supersonic, and supersonic matter at around few$\times 10r_g$ may undergo shock due to centrifugal pressure [1, 2].

Such a shocked accretion disc-model was used to compute the spectral states of black hole candidates [4, 5, 14, 8] where the post-shock torus — CENBOL (CENtrifugal Pressure supported BOundary Layer), produces the hard power-law tail by inverse-Comptonizing the softer photons from the outer disc.

Numerically, it was also shown that for a two dimensional accretion flow, the unbalanced thermal gradient force along the z direction in the post-shock disc drives a significant portion of the inflowing matter as bipolar outflows [15]. In the analytical front, mass outflow rates were also computed from advective discs, and generally it was shown that as high as 10%-15% of the accreting matter can be ejected as bipolar outflows. However, these investigations were performed in the inviscid limit.

Earlier investigation of shock induced outflows from inviscid disc showed that outflows generated from a post-shock disc depends on the strength of the centrifugal barrier as well as the thermal driving from post shock disc or CENBOL [6, 11, 12].

In presence of viscosity, as matter flows inward angular momentum decreases while specific energy increases. Will the mass outflow rate increase because of enhanced viscous heating, or will it decrease because of the weakened centrifugal barrier? Moreover, since post-shock flows are denser and hotter than pre-shock flows, cooling processes in the CENBOL should be more effective. While viscosity transports angular momentum and increases the energy of the flow, cooling processes only reduce the energy of the flow, but leave angular momentum distribution unaffected. Thus, for the same outer boundary condition, a higher cooling rate should make the shock front move closer to the black hole, and would reduce the specific energy of the flow. Should this reduce the mass outflow rate further? However, if there is mass loss from the post-shock flow, then the density should go down, which should reduce the cooling rate. The question is, which one dominates, whether cooling reduces mass loss or mass loss reduces cooling? We want to investigate how cooling and mass loss affect each other in presence of viscosity in the disc.

In the next section we present the model assumptions and equations of motion. In section 3, we present the results and in finally in section 4, we draw conclusion.

2. Assumptions and equations of motion

We assume a stationary, axisymmetric viscous accretion flow in presence of cooling effects. The central black hole is assumed to be non-rotating and general relativistic effects are approximated by a Paczyński-Wiita potential [16]. However, the outflows are less dense and differential rotation is less effective, therefore we assume the outflows to be inviscid and adiabatic.

The equations of motion for accretion are,

the radial momentum equation :

$$u\frac{du}{dx}+\frac{1}{\rho}\frac{dP}{dx}-\frac{\lambda^2(x)}{x^3}+\frac{1}{2(x-1)^2}=0, \tag{1a}$$

the baryon number conservation equation :

$$\dot{M}=2\pi\Sigma ux, \tag{1b}$$

the angular momentum conservation equation :

$$u\frac{d\lambda(x)}{dx}+\frac{1}{\Sigma x}\frac{d}{dx}\left(x^2W_{x\phi}\right)=0, \tag{1c}$$

and the entropy generation equation :

$$\Sigma uT\frac{ds}{dx}=Q^+-Q^-, \tag{1d}$$

where, flow variables u, ρ, P and $\lambda(x)$ in the above equations are the radial velocity, density, isotropic pressure and specific angular momentum of the flow respectively. Here Σ and $W_{x\phi}=-\alpha_\Pi\Pi$ denote the vertically integrated density and the viscous stress (α_Π and Π are the viscosity parameter and total pressure respectively), s is the specific entropy of the flow, T is the local temperature. $Q^+(=\rho hH)$ and $Q^-(=\rho hC)$ are the heat gained and lost by the flow (integrated in the vertical direction) respectively, where $h=\sqrt{\frac{2}{\gamma}}ax^{1/2}(x-1)$ is local disc height and $a=\sqrt{\gamma P/\rho}$ is local sound speed.

The heating and cooling terms are [3, 7, 13, 17],

$$H=Ax(ga^2+\gamma u^2)\frac{d\Omega}{dx}, \tag{2a}$$

where $A=-\alpha_\Pi I_n/\gamma$ and $g=I_{n+1}/I_n$, Ω is the angular velocity. In the above expressions $n=1/(\gamma-1)$ is the polytropic index, and $I_n=(2^n n!)^2/(2n+1)!$. And,

$$C=\frac{\beta Sa^5}{ux^{3/2}(x-1)}, \tag{2b}$$

where, β is the cooling factor, and

$$S = \frac{32\eta \dot{m}\mu^2 e^4 1.44\times 10^{17}}{3\sqrt{2}m_e^3\gamma^{5/2}}\frac{1}{2GM_{\odot}c^3}, \tag{2c}$$

where η is the ratio between magnetic pressure and the gas pressure and in this paper has been kept fixed ($\eta = 0.1$), e is the electron charge, m_e is the electron mass, $\dot{m}$ is the accretion rate in units of Eddington rate, $M_{\odot}$ is the solar mass, and for fully ionized plasma $\mu = 0.5$.

The equations of motion of jets are,

$$\mathscr{E}_j = \frac{1}{2}v_j^2 + na_j^2 + \frac{\lambda_j^2}{2x_j^2} - \frac{1}{2(r_j-1)}, \tag{3a}$$

where, $\mathscr{E}_j$ is the specific energy of the jet. The integrated continuity equation:

$$\dot{M}_{\rm out} = \rho_j v_j \mathscr{A}, \tag{3b}$$

and instead of the entropy generation equation we have the polytropic equation ($p_j = K_j\rho_j^{\gamma}$, p_j, ρ_j are the local pressure and density of the jet, and K_j is a constant proportional to the entropy) of state for the jet.

The jet geometry and the computation of jet streamline are discussed in detail by Chattopadhyay & Das (2007) [10], therefore we present it only very briefly. The jet is assumed to flow in between two geometric surfaces, namely funnel wall (FW) and centrifugal barrier (CB), which are given by,

$$\frac{\lambda^2}{x_{FW}^2} = \frac{1}{r_{FW}-1}, \tag{3c}$$

where, $r_{FW}^2 = x_{FW}^2 + y_{FW}^2 \equiv$ spherical radius, and x_{FW} is the cylindrical radius of FW. The centrifugal barrier (CB) surface is mathematically presented as,

$$\frac{\lambda^2}{x_{CB}^3} = \frac{x_{CB}}{2r_{CB}(r_{CB}-1)^2}, \tag{3d}$$

where, $r_{CB}^2 = x_{CB}^2 + y_{CB}^2 \equiv$ spherical radius of CB. And x_{CB}, y_{CB} are the cylindrical radius and axial coordinate (or height at r_{CB}) of CB. The jet cylindrical radius is given by,

$$x_j = \frac{x_{FW}+x_{CB}}{2}, \qquad y_j = y_{FW} = y_{CB}. \tag{3e}$$

And, the jet cross-sectional area is,

$$\mathscr{A} = 2\pi(x_{CB}^2 - x_{FW}^2). \tag{3f}$$

Furthermore, we define another variable, which we call mass-outflow rate which is the fractional mass-outflow w.r.t the accretion rate at the outer sonic point, and is mathematically represented as,

$$R_{\dot{m}} = \frac{\dot{M}_{\rm out}}{\dot{M}_-} = \frac{Rv_j(x_s)\mathscr{A}(x_s)}{4\pi x_s h_+ u_-} = \frac{Rv_j(x_s)\mathscr{A}(x_s)}{4\pi\sqrt{\frac{2}{\gamma}}x_s^{3/2}(x_s-1)a_+u_-}, \tag{3g}$$

where, x_s is the shock location in accretion and $R(=\frac{\rho_+h_+}{\rho_-h_-})$ is the compression ratio at the shock.

2.1 Self consistent accretion-ejection solution

We solve the accretion-ejection by a novel iterative technique. The Rankine-Hugoniot shock conditions are modified for mass loss, *i.e.,* (a) the energy flux is continuous across the shock —

$$\mathscr{E}_+ = \mathscr{E}_-, \tag{4a}$$

(b) the mass flux is continuous across the shock —

$$\dot{M}_+ = \dot{M}_- - \dot{M}_{\rm out} = \dot{M}_-(1 - R_{\dot{m}}), \tag{4b}$$

and finally, (c) the momentum balance condition —

$$W_+ + \Sigma_+ u^2{}_+ = W_- + \Sigma_- u^2_-, \tag{4c}$$

where subscripts "$-$" and "$+$" refer, respectively, to quantities before and after the shock. The methodology is as follows:
(a) We assume $\dot{M}_{\rm out} = 0$, and solve for shocks in the accretion flow *i.e.,* for eqs. (1a-1d).
(b) Once we have a shock we assign $\mathscr{E}_j = \mathscr{E}_+$, $\lambda_j = \lambda_+$ and the density at the jet base to be the post-shock density $\rho_j(x_s) = \rho_+$.
(c) We solve for jet *i.e.,* eqs. (3a-3f), and then with this solution compute $R_{\dot{m}}$ from eq. (3g)
(d) Use the computed value of $R_{\dot{m}}$ in shock conditions [eqs.(4a-4c)], to compute a new x_s'.
(e) Repeat above steps till x_s' converges, the converged value of x_s' is the actual x_s.

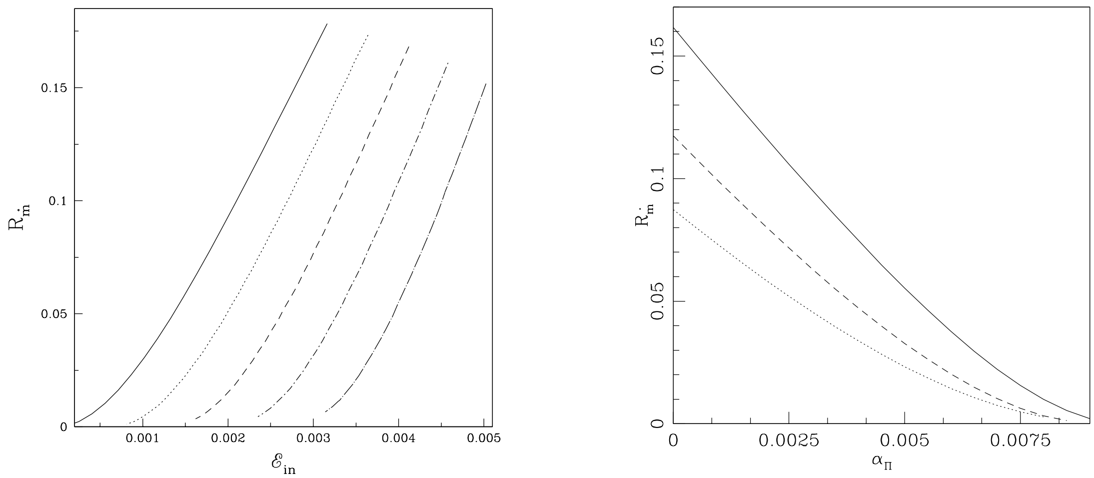
3. Results

As we have pointed out, viscosity transports angular momentum outwards but heats up the gas inwards, while cooling decreases energy along the accretion flow. To understand the proper role of each of the above mentioned processes on the accretion flow we first take up only viscous flow and then turn on cooling, *i.e.,* to say, we put $\beta = 0$ until otherwise stated.

Figure (1a), shows $R_{\dot{m}}$ as an increasing function of $\mathscr{E}_{\rm in}$ (energy at inner sonic point of the accretion flow). Each curve is parametrized by $\alpha_\Pi = 0$ (solid), 0.005 (dotted), 0.01 (dashed), and 0.015 (dashed-dotted), and 0.02 (long dashed) respectively. Specific angular momentum at inner sonic point (x_{ci}) is kept fixed at 1.75. We also see that $R_{\dot{m}}$ decreases with increasing values of α_Π (the viscosity parameter). In Fig. (1b), the dependence of $R_{\dot{m}}$ on α_Π is more explicitly shown. Though $R_{\dot{m}}$ decreases with increasing α_Π, but for increasing λ_i (specific angular momentum at x_{ci}), $R_{\dot{m}}$ increases.

As the accretion flow is shocked by centrifugal barrier, the post shock flow will be of higher thermal energy and the unbalanced thermal gradient forces along z direction will drive bipolar jets. Thus for higher post shock energy (represented by higher $\mathscr{E}_{\rm in}$) the thermal driving will be greater [*e.g.,* Fig. (1a)]. However, if λ is higher, the centrifugal barrier will be stronger and therefore stronger will be the shock, generating more outflow [*e.g.,* Fig. (1b)]. It is intriguing to know which of the above two phenomena is instrumental in creating outflows? Moreover, why does $R_{\dot{m}}$ decrease with α_Π?

Since shocks generate outflows, it will be intriguing to see how shocks are affected by viscosity.



(a) (b)

Figure 1: (a) $R_{\dot{m}}$ with $\mathscr{E}_{\rm in}$, for $\alpha_\Pi = 0$ (solid), 0.005 (dotted), 0.01 (dashed), and 0.015 (dashed-dotted), 0.02 (long-dashed) respectively, and $\lambda_i = 1.75$, $\beta = 0$. (b) $R_{\dot{m}}$ vs α_Π, for for $\lambda_i = 1.8$ (solid), 1.775 (dashed) and 1.75 (dotted), respectively. Inner sonic points are $x_{ci} = 2.313$, 2.375, and 2.445, respectively, where $\beta = 0$.

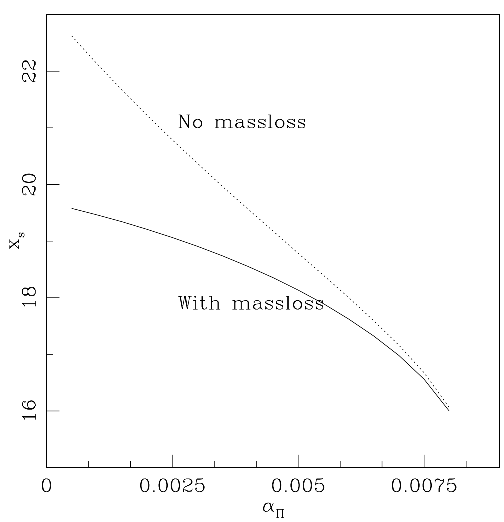
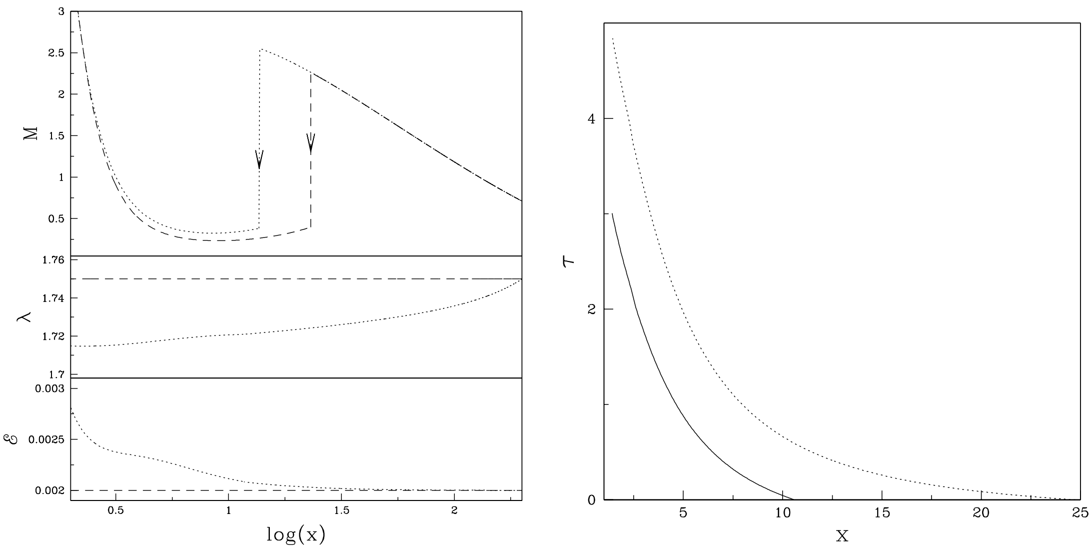


Figure 2: Variation of x_s with α_Π, $\lambda_i = 1.75$ $x_{ci} = 2.445$, $\beta = 0$. The dotted curve represents solution without mass loss, and solid represents x_s with mass loss.



(a) (b)

Figure 3: (a) Variation of M (upper panel), λ (middle panel), $\mathscr{E}$ (lower panel) with $log(x)$, for $\alpha_\Pi = 0$ (dashed) and $\alpha_\Pi = 0.003$ (dotted). The outer edge quantities are $x_{\rm inj} = 200$, $(\mathscr{E}_{\rm inj}, \lambda_{\rm inj}) = (0.002, 1.75)$. The shocks are $x_s = 20.3$ (dashed) and $x_s = 11.65$ (dotted). (b) Variation of τ in the post-shock disc with (x), for the same inflow parameters as Fig. (3a). Cooling parameter $\beta = 0$ for both the figures.

In Fig. 2, the shock location (x_s) is plotted with α_Π, for solutions without mass loss (dotted) and with mass loss (solid). The parameters are same as the dotted curve of Fig. 5a, *i.e.,* $(\lambda_i, x_{ci}) = (1.75, 2.445)$, which we choose to be a representative case. It is to be remembered that the dotted curve is a typical solution from Chakrabarti & Das (2004) [7]. We clearly show that mass loss from the post-shock region of the disc causes the shock to move in (the dotted curve is of higher value than the solid one). However, as α_Π is increased the two curves tend to converge. Since $R_{\dot m}$ decreases with α_Π, the difference in shock location also diminishes. From Fig. 2, it is quite clear that α_Π or viscosity also reduces x_s. As we mentioned before viscosity should increase the post shock energy of the flow. Is it not good enough to increase x_s?

In Fig. (3a), Mach number M (upper panel), λ (middle panel), and specific energy $\mathscr{E}$ (lower panel) are plotted with $log(x)$. Each curved is parametrized by $\alpha_\Pi = 0$ (dashed) and $\alpha_\Pi = 0.003$ (dotted), and are launched with same outer boundary condition $(\mathscr{E}_{\rm inj}, \lambda_{\rm inj}) = (0.002, 1.75)$, the outer edge being $x_{\rm inj} = 200$. Increasing α_Π decreases the shock location from $x_s = 24.8$ (dashed) to $x_s = 11.65$ (dotted), consequently the mass outflow rate decreases from $R_{\dot m} = 0.093$ (dashed) to $R_{\dot m} = 0.07$. It is clearly shown in this figure that as viscosity is increased $\mathscr{E}(x)$ increases and $\lambda(x)$ decreases, but increase in $\mathscr{E}(x)$ cannot compensate the decrease in $\lambda(x)$, and hence the shock moves inwards. As the shock moves inwards the jet area at the base decreases and the mass outflow rate decreases. So in this particular case when the flow variables at the outer boundary have been kept fixed the position of the shock is determined mainly by the centrifugal force of the flow. Therefore we may safely conclude that, *if the outer boundary is fixed, then by increasing viscosity, the shock*

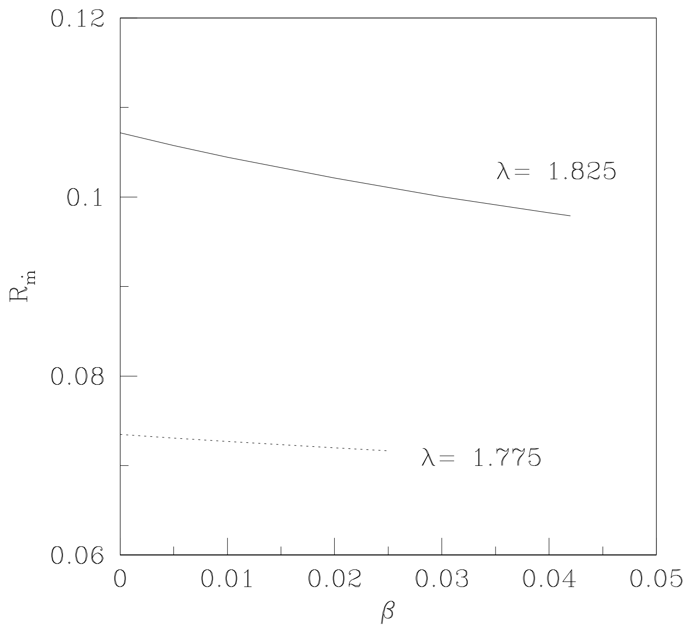


Figure 4: Variation of mass outflow rate $R_{\dot{m}}$ with the cooling parameter β. The curves are parametrized by $\lambda_i = 1.825$ (solid) and $\lambda_i = 1.775$ (dashed), and the viscosity parameter is kept fixed at $\alpha_\Pi = 0.005$.

location and consequently the mass outflow rate is decreased.

What would be the observational consequence of such accretion-ejection mechanism? Chakrabarti & Titarchuk (1995) did show that the post shock matter or the CENBOL is the source of high energy power-law photons which are generally observed in black hole candidates. What happens to the spectrum from such a disc when viscosity and mass loss is considered? Since the density in the CENBOL decreases due to mass loss and since the shock is located nearer to the black hole, so the CENBOL will be relatively less dense and smaller in size. Both these facts should reduce the optical depth. Which means more photons will be able to penetrate the CENBOL and for the photons inside it will be able to leave the CENBOL. The spectrum under such circumstances will be softer. In Fig. (3b), $\tau(x)$ is plotted only for the post-shock flow or the CENBOL, for the following viscosity parameter $\alpha_\Pi = 0$ without considering mass loss (dotted), and for $\alpha_\Pi = 0.003$ with (solid) mass loss. And indeed we see that the optical depth of CENBOL is reduced.

We have shown that the outflows generated by shocked accretion flow are basically centrifugally driven. The crucial point is, viscosity basically weakens the centrifugal barrier. Though viscosity increases energy of the flow (due to viscous heating), but this does not compensate the weakening of the centrifugal barrier, and consequently the size of the CENBOL decreases and with the decreased size of CENBOL the mass outflow rate decreases too.

We want to pursue this even further by including the cooling mechanism. We only incorporate the synchrotron cooling since bremsstrahlung cooling is too weak [9]. In Fig. (4) we have plotted $R_{\dot{m}}$ with the cooling parameter β, each curve being parametrized by $\lambda_i = 1.825$ (solid) and $\lambda_i = 1.775$ (dashed), with $\alpha_\Pi = 0.005$. Once again we see that $R_{\dot{m}}$ is higher for higher angular momentum. The post-shock CENBOL is denser and hotter than pre-shock disc. That means synchrotron cooling would be enhanced at CENBOL. Thus cooling should decrease $R_{\dot{m}}$. It does, but the effect is marginal. So we do see that the outflows generated by shocks are basically centrifugally driven.

4. Conclusion

In the present paper we have self consistently calculated mass outflow rates by solving accretion and jet equations simultaneously. We have also incorporated viscous effects as well as cooling effects.

We have shown that,
(i) If the accreting matter starts from the same outer boundary condition then the viscosity weakens the centrifugal barrier which makes the shocks to be located closer to the black hole.
(ii) As mass leaves the post shock disc, it reduces the post shock pressure, and that makes the shock to be located closer to the black hole.
(iii) As the centrifugal barrier is weakened by viscosity, the mass outflow rates are reduced.
(iv) If cooling is considered mass outflow rates are marginally reduced. Thus it shows that these outflows are basically centrifugally driven.
(v) The combined effect of mass loss and dissipation (viscosity+cooling) reduces the optical depth of the post-shock disc, which means that the spectrum from such a disc will be softer, in other words, “the disc will enter in its ‘hard’ state as the disc is shocked, however, because of mass loss the spectrum will become softer.” This means that the mass loss changes the spectral state from hard to intermediate hard states.

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