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Static Extremal Black Holes

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The thermodynamic entropy and the attractor equations for extremal black hole solutions follow from a variational principle based on an entropy function. We review this variational principle for static extremal black holes in four space-time dimensions and we discuss a few examples in the context of $N = 2$ supergravity theories.

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1. Introduction

One of the important successes of string theory is that one can obtain a statistical derivation of the thermodynamic (macroscopic) Bekenstein-Hawking entropy of certain supersymmetric (BPS) black holes in terms of a microscopic state counting [1]. This state counting is reliably computed at weak coupling, whereas the computation of the thermodynamic entropy is performed at strong coupling. Even though microscopic and macroscopic entropies are computed in different regions in the coupling constant space, they can nevertheless be compared in a meaningful way since they only depend on the quantized charges of the black hole under consideration.

An important feature of these supersymmetric black holes is that they are supported by scalar fields (often called moduli fields). In the black hole background these scalar fields vary radially as one moves from spatial infinity to the horizon of the black hole, and they get attracted to specific values at the horizon which are determined by the black hole charges. These values are independent of the asymptotic values of the fields at spatial infinity. This is the so-called attractor mechanism, which was first noted in the context of supergravity [2 – 5] and then generalized to theories with higher-derivative terms in [6, 7]. As a result, the macroscopic entropy is entirely determined in terms of the black hole charges.

Much of the success in the matching of microscopic and macroscopic entropies is tied to supersymmetry. There are, however, examples of extremal non-BPS black holes for which the microscopic entropy based on state counting agrees with the thermodynamic entropy [8]. Thus, it appears worthwhile to study generic features of extremal, not necessarily supersymmetric black holes. One such feature is the attractor mechanism, which is not only a property of BPS black holes, but is also present for extremal non-BPS black holes [9 – 12]. The attractor behaviour is encoded in so-called attractor equations, which can be obtained by extremizing a so-called entropy function [11]. Moreover, the value of this function at the extremum yields the thermodynamic entropy of the extremal black hole.

In the following, we review the entropy function of [11] for static extremal black hole solutions in four space-time dimensions, using the approach of [13] based on electric/magnetic duality covariance. Then, following [14], we specialize to the case of $N = 2$ supergravity theories and we display the associated attractor equations at the two-derivative level. We also briefly discuss the entropy function for BPS black holes in the presence of a certain class of higher-curvature interactions. We display a few solutions to the attractor equations describing extremal black holes in heterotic string theory. We refer to [13] for a detailed discussion of these issues.

2. Entropy function and electric/magnetic duality covariance

Let us consider static extremal black hole solutions to the equations of motion of a general system of abelian vector gauge fields, scalar and matter fields coupled to gravity in four space-time dimensions. Following [11], we take the near-horizon geometry of such a black hole to be of the form $AdS_2 \times S^2$. Thus, we consider near-horizon solutions with spherical symmetry, which may be written as

$$\mathrm{d}s^2{}_{(4)} = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = v_1\Big(-r^2\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2}\Big) + v_2\Big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\Big)\,,$$

$$F_{rt}{}^I = e^I \,, \qquad F_{\theta\varphi}{}^I = \frac{p^I}{4\pi} \sin\theta \,. \tag{2.1}$$

Here the $F_{\mu\nu}{}^I$ denote the field strengths associated with a number of abelian gauge fields. The θ-dependence of $F_{\theta\varphi}{}^I$ is fixed by rotational invariance and the p^I denote the magnetic charges. The fields e^I are dual to the electric charges. In addition to the constant fields e^I, v_1 and v_2 there may be a number of other fields which for the moment we denote collectively by u_α.

As is well known theories based on abelian vector fields are subject to electric/magnetic duality, because their equations of motion expressed in terms of the dual field strengths,[1]

$$G_{\mu\nu I} = \sqrt{|g|}\, \varepsilon_{\mu\nu\rho\sigma} \frac{\partial \mathscr{L}}{\partial F_{\rho\sigma}{}^I} \,, \tag{2.2}$$

take the same form as the Bianchi identities for the field strengths $F_{\mu\nu}{}^I$. Adopting the conventions where $x^\mu = (t, r, \theta, \varphi)$ and $\varepsilon_{tr\theta\varphi} = 1$, and the signature of the space-time metric equals $(-,+,+,+)$, it follows that, in the background (2.1),

$$\begin{aligned} G_{\theta\varphi I} &= -v_1 v_2 \sin\theta \frac{\partial \mathscr{L}}{\partial F_{rt}{}^I} = -v_1 v_2 \sin\theta \frac{\partial \mathscr{L}}{\partial e^I} \,, \\ G_{rt\, I} &= -v_1 v_2 \sin\theta \frac{\partial \mathscr{L}}{\partial F_{\theta\varphi}{}^I} = -4\pi\, v_1 v_2 \frac{\partial \mathscr{L}}{\partial p^I} \,. \end{aligned} \tag{2.3}$$

These two tensors can be written as $q_I \sin\theta/(4\pi)$ and f_I. The quantities q_I and f_I are conjugate to p^I and e^I, respectively, and can be written as

$$\begin{aligned} q_I(e,p,v,u) &= -4\pi\, v_1 v_2 \frac{\partial \mathscr{L}}{\partial e^I} \,, \\ f_I(e,p,v,u) &= -4\pi\, v_1 v_2 \frac{\partial \mathscr{L}}{\partial p^I} \,. \end{aligned} \tag{2.4}$$

The q_I are constant by virtue of the equations of motion and correspond to the electric charges.

Electric/magnetic duality transformations are induced by rotating the tensors $F_{\mu\nu}{}^I$ and $G_{\mu\nu I}$ by a constant transformation, so that the new linear combinations are all subject to Bianchi identities. Half of them are then selected as the new field strengths defined in terms of new gauge fields, while the Bianchi identities on the remaining linear combinations are regarded as field equations belonging to a new Lagrangian defined in terms of the new field strengths. In order that this dualization can be effected the rotation betweeen the tensors must belong to $\mathrm{Sp}(2n+2;\mathbb{R})$, where $n+1$ denotes the number of independent gauge fields. Hence this leads to new quantities $(\tilde{p}^I, \tilde{q}_I)$ and $(\tilde{e}^I, \tilde{f}_I)$, where

$$\begin{aligned} \tilde{p}^I &= U^I{}_J\, p^J + Z^{IJ} q_J \,, \\ \tilde{q}_I &= V_I{}^J q_J + W_{IJ}\, p^J \,, \end{aligned} \tag{2.5}$$

and likewise for (e^I, f_I). Here $U^I{}_J$, $V_I{}^J$, W_{IJ} and Z^{IJ} are constant real $(n+1)\times(n+1)$ submatrices subject to

$$\begin{aligned} &U^{\mathrm{T}}V - W^{\mathrm{T}}Z = V^{\mathrm{T}}U - Z^{\mathrm{T}}W = \mathbb{1} \,, \\ &U^{\mathrm{T}}W = W^{\mathrm{T}}U \,, \qquad Z^{\mathrm{T}}V = V^{\mathrm{T}}Z \,, \end{aligned} \tag{2.6}$$

[1] Here and henceforth we assume that the Lagrangian depends on the abelian field strengths but not on their space-time derivatives. We also assume that the gauge fields appear exclusively through their field strengths.

so that the full matrix belongs to $\mathrm{Sp}(2n+2;\mathbb{R})$ [15]. Since the charges are not continuous but will take values in an integer-valued lattice, this group should eventually be restricted to an appropriate arithmetic subgroup.

Next, we define the reduced Lagrangian by the integral of the full Lagrangian over the horizon two-sphere S^2,

$$\mathscr{F}(e,p,v,u) = \int \mathrm{d}\theta\,\mathrm{d}\varphi\,\sqrt{|g|}\,\mathscr{L}\,. \tag{2.7}$$

We note that the definition of the conjugate quantities q_I and f_I takes the form,

$$q_I = -\frac{\partial\mathscr{F}}{\partial e^I}\,, \qquad f_I = -\frac{\partial\mathscr{F}}{\partial p^I}\,. \tag{2.8}$$

It is known that a Lagrangian does not transform as a function under electric/magnetic dualities. Instead we have [16],

$$\tilde{\mathscr{F}}(\tilde{e},\tilde{p},v,u) + \frac{1}{2}[\tilde{e}^I\tilde{q}_I + \tilde{f}_I\tilde{p}^I] = \mathscr{F}(e,p,v,u) + \frac{1}{2}[e^I q_I + f_I p^I]\,. \tag{2.9}$$

so that the linear combination $\mathscr{F}(e,p,v,u)+\frac{1}{2}[e^I q_I + f_I p^I]$ transforms as a function. It is easy to see that the combination $e^I q_I - f_I p^I$ transforms as a function as well, so that we may construct a modification of (2.7) that no longer involves the f_I and that transforms as a function under electric/magnetic duality,

$$\mathscr{E}(q,p,v,u) = -\mathscr{F}(e,p,v,u) - e^I q_I\,, \tag{2.10}$$

which takes the form of a Legendre transform in view of the first equation (2.8). In this way we obtain a function of electric and magnetic charges. Therefore it transforms under electric/magnetic duality according to $\tilde{\mathscr{E}}(\tilde{q},\tilde{p},v,u) = \mathscr{E}(q,p,v,u)$. Furthermore the field equations imply that the q_I are constant and that the action, $\int \mathrm{d}t\mathrm{d}r\,\mathscr{E}$, is stationary under variations of the fields v and u, while keeping the p^I and q_I fixed. This is to be expected as $\mathscr{E}$ is in fact the analogue of the Hamiltonian density associated with the reduced Lagrangian density (2.7), at least as far as the vector fields are concerned. The constant values of the fields $v_{1,2}$ and u_α are thus determined by demanding $\mathscr{E}$ to be stationary under variations of v and u,

$$\frac{\partial\mathscr{E}}{\partial v} = \frac{\partial\mathscr{E}}{\partial u} = 0\,. \tag{2.11}$$

The function $2\pi\mathscr{E}(q,p,v,u)$ coincides with the entropy function proposed by Sen [11]. The equations (2.11) are the so-called attractor equations and the thermodynamic entropy is directly proportional to the value of $\mathscr{E}$ at the stationary point,

$$\mathscr{S}_{\text{macro}}(p,q) \propto \mathscr{E}\Big|_{\text{attractor}}\,. \tag{2.12}$$

The precise proportionality factor is a priori undefined and depends on various normalization conventions used for the Lagrangian and the charges. The above derivation of the entropy function applies to any gauge and general coordinate invariant Lagrangian, and, in particular, also to Lagrangians containing higher-derivative interactions. The entropy computed by (2.12) is Wald's entropy [17] which, in the absence of higher-derivative interactions, reduces to the area law of Bekenstein and Hawking.

In the absence of higher-derivative terms, the reduced Lagrangian $\mathscr{F}$ is at most quadratic in e^I and p^I and the Legendre transform (2.10) can easily be carried out. For instance, consider the following Lagrangian in four space-time dimensions (we only concentrate on terms quadratic in the field strengths),

$$\mathscr{L}_0 = -\tfrac{1}{4}\,\mathrm{i}\sqrt{|g|}\Big\{\mathscr{N}_{IJ}\,F^{+\,I}_{\mu\nu}\,F^{+\mu\nu J} - \bar{\mathscr{N}}_{IJ}\,F^{-\,I}_{\mu\nu}\,F^{-\mu\nu J}\Big\}\,, \tag{2.13}$$

where $F^{\pm\,I}_{\mu\nu}$ denote the (anti)-selfdual field strengths. In the context of this paper the tensors $F^{\pm I}_{\underline{rt}} = \pm\mathrm{i}F^{\pm\,I}_{\underline{\theta\varphi}} = \frac{1}{2}(F_{\underline{rt}}{}^I \pm \mathrm{i}F_{\underline{\theta\varphi}}{}^I)$ are relevant, where underlined indices refer to the tangent space. From (2.13) and (2.1), we straightforwardly derive the associated reduced Lagrangian (2.7),

$$\mathscr{F} = \frac{1}{4}\left\{\frac{\mathrm{i}v_1\,p^I(\bar{\mathscr{N}}-\mathscr{N})_{IJ}\,p^J}{4\pi\,v_2} - \frac{4\mathrm{i}\pi\,v_2\,e^I(\bar{\mathscr{N}}-\mathscr{N})_{IJ}\,e^J}{v_1}\right\} - \frac{1}{2}e^I(\mathscr{N}+\bar{\mathscr{N}})_{IJ}\,p^J\,. \tag{2.14}$$

It is straightforward to evaluate the entropy function (2.10) in this case,

$$\mathscr{E} = -\frac{v_1}{8\pi\,v_2}\,(q_I - \mathscr{N}_{IK}\,p^K)\,[(\mathrm{Im}\,\mathscr{N})^{-1}]^{IJ}\,(q_J - \bar{\mathscr{N}}_{JL}\,p^L)\,, \tag{2.15}$$

which is indeed compatible with electric/magnetic duality. Upon decomposing into real matrices, $\mathrm{i}\mathscr{N}_{IJ} = \mu_{IJ} - \mathrm{i}\nu_{IJ}$, this result coincides with the corresponding terms in the so-called black hole potential

$$V_{\mathrm{BH}} = \frac{1}{2}(p,q)^T\,\mathscr{M}\begin{pmatrix} p\\ q\end{pmatrix}\quad,\quad \mathscr{M} = \begin{pmatrix} \mu+\nu\,\mu^{-1}\,\nu & \nu\,\mu^{-1}\\ \mu^{-1}\,\nu & \mu^{-1}\end{pmatrix}\,, \tag{2.16}$$

discussed in [9], and more recently in [12]. Namely, setting $v_1 = v_2$ (which enforces the vanishing of the curvature scalar) we obtain $\mathscr{E} = (4\pi)^{-1}\,V_{\mathrm{BH}}$.

3. Application to $N = 2$ supergravity

We now give the entropy function for $N = 2$ supergravity coupled to n abelian $N = 2$ vector multiplets at the two-derivative level. Here we follow the conventions of [7], where the charges and the Lagrangian have different normalizations than in the previous section.

The $N = 2$ vector multiplets contain complex physical scalar fields which we denote by $X^I, I = 0,\dots,n$. At the two-derivative-level, the action for the vector multiplets is encoded in a holomorphic function $F(X)$ [18]. The coupling to supergravity requires this function to be homogeneous of second degree, i.e. $F(\lambda X) = \lambda^2 F(X)$. The gauge coupling functions $\mathscr{N}_{IJ}$ in (2.13) are given in terms of derivatives of $F(X)$,

$$\mathscr{N}_{IJ} = \bar{F}_{IJ} + 2\mathrm{i}\frac{\mathrm{Im}F_{IK}\,\mathrm{Im}F_{JL}\,X^K\,X^L}{\mathrm{Im}F_{MN}\,X^M\,X^N}\,, \tag{3.1}$$

where $F_I = \partial F(X)/\partial X^I$ and $F_{IJ} = \partial^2 F(X)/\partial X^I\partial X^J$.

Imposing the vanishing of the Ricci scalar, i.e. setting $v_1 = v_2$, the resulting entropy function (2.15) can be brought into the equivalent form [19, 13],

$$\mathscr{E} = \tfrac{1}{2}\Sigma(Y,\bar{Y},p,q) + \tfrac{1}{2}N^{IJ}(\mathscr{Q}_I - F_{IK}\mathscr{P}^K)\,(\mathscr{Q}_J - \bar{F}_{JL}\mathscr{P}^L)\,, \tag{3.2}$$

where

$$\Sigma(Y,\bar{Y},p,q) = -\mathrm{i}\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right) - q_I(Y^I + \bar{Y}^I) + p^I(F_I + \bar{F}_I) \;, \tag{3.3}$$

and

$$\begin{aligned} \mathscr{P}^I &\equiv p^I + \mathrm{i}(Y^I - \bar{Y}^I)\,, \\ \mathscr{Q}_I &\equiv q_I + \mathrm{i}(F_I - \bar{F}_I)\,. \end{aligned} \tag{3.4}$$

Here the Y^I are related to the X^I by a uniform rescaling and F_I denotes the derivative of $F(Y)$ with respect to Y^I. Also $N_{IJ} = \mathrm{i}(\bar{F}_{IJ} - F_{IJ})$, where $F_{IJ} = \partial^2 F(Y)/\partial Y^I \partial Y^J$.

Varying the entropy function (3.2) with respect to the scalar fields Y^I yields the attractor equations

$$\left(\mathscr{Q}_I - F_{IJ}\,\mathscr{P}^J\right) - \frac{\mathrm{i}}{2}\left(\mathscr{Q}_K - \bar{F}_{KM}\,\mathscr{P}^M\right) N^{KP} F_{PIQ} N^{QL}\left(\mathscr{Q}_L - \bar{F}_{LN}\,\mathscr{P}^N\right) = 0 \;, \tag{3.5}$$

where $F_{PIQ} = \partial^3 F(Y)/\partial Y^P \partial Y^I \partial Y^Q$. The attractor equations determine the horizon value of the Y^I in terms of the black hole charges (p^I, q_I). Because the function $F(Y)$ is homogeneous of second degree, we have $F_{IJK} Y^K = 0$. Using this relation one deduces from (3.5) that $\left(\mathscr{Q}_J - F_{JK}\,\mathscr{P}^K\right) Y^J = 0$, which is equivalent to

$$\mathrm{i}(\bar{Y}^I F_I - Y^I \bar{F}_I) = p^I F_I - q_I Y^I \;. \tag{3.6}$$

Therefore, at the attractor point, we have

$$\Sigma = \mathrm{i}(\bar{Y}^I F_I - Y^I \bar{F}_I) \;. \tag{3.7}$$

With the normalizations used in this section, the entropy (2.12) reads

$$\mathscr{S}_{\text{macro}}(p,q) = 2\pi\,\mathscr{E}\Big|_{\text{attractor}} \;. \tag{3.8}$$

Supersymmetric (BPS) black holes are the subset of extremal black holes satisfying [4, 20]

$$\mathscr{Q}_I = \mathscr{P}^J = 0 \;, \tag{3.9}$$

which manifestly solves (3.5). Their entropy reads

$$\mathscr{S}_{\text{macro}} = \pi\,\Sigma|_{\text{attractor}} \;. \tag{3.10}$$

As an example, consider an $N = 2$ supergravity Lagrangian based on the holomorphic function

$$F(Y) = -\frac{Y^1 Y^2 Y^3}{Y^0} \;, \tag{3.11}$$

which arises in heterotic string compactifications on $K3 \times T^2$. Consider first a BPS black hole carrying the non-vanishing charges

$$q_0 = -Q\;, \qquad p^1 = Q\;, \qquad p^2 = p^3 = P\;, \tag{3.12}$$

with PQ positive. Solving the BPS equations (3.9) yields the attractor values

$$Y^0 = \frac{1}{2}P \;\;,\;\; Y^1 = \frac{\mathrm{i}}{2}Q \;\;,\;\; Y^2 = Y^3 = \frac{\mathrm{i}}{2}P\,, \tag{3.13}$$

and computing the entropy (3.10) gives

$$\mathscr{S}_{\rm macro} = 2\pi P Q\,. \tag{3.14}$$

Next, consider a non-supersymmetric black hole with non-vanishing charges [21]

$$q_0 = p^1 = Q\,, \qquad p^2 = p^3 = P\,, \tag{3.15}$$

with PQ positive. Solving the attractor equations (3.5) yields the following attractor values,

$$Y^0 = \frac{1}{4} P \quad , \quad Y^1 = \frac{\rm i}{4} Q \quad , \quad Y^2 = Y^3 = \frac{\rm i}{4} P\,, \tag{3.16}$$

and the entropy (3.8) is computed to be

$$\mathscr{S}_{\rm macro} = 2\pi P Q\,. \tag{3.17}$$

4. Inclusion of R^2-interactions

It is possible to incorporate higher-order derivative interactions involving the square of the Weyl tensor C_{abcd} into the discussion given above. This class of higher-order interactions can be dealt with by including the $N=2$ Weyl multiplet into the function F and preserving its homogeneity according to

$$F(\lambda Y, \lambda^2 \Upsilon) = \lambda^2 F(Y, \Upsilon)\,. \tag{4.1}$$

Here Υ denotes the square of the rescaled anti-selfdual field T^-_{ab} which belongs to the Weyl multiplet. In the presence of these higher-curvature interactions, there are thus additional terms in the Lagrangian proportional to the square of the Weyl tensor with gravitational coupling functions encoded in $F_\Upsilon = \partial F(Y,\Upsilon)/\partial \Upsilon$. These higher-curvature terms lead to a modification [14, 13] of the entropy function (3.2).

Let us consider BPS black holes in the following. Then, it can be shown that the field Υ takes the value $\Upsilon = -64$ at the horizon, and that the Ricci scalar continues to vanish, i.e. $v_1 = v_2$ [6]. The resulting entropy function takes the form [13]

$$\mathscr{E} = \tfrac{1}{2}\Sigma(Y,\bar{Y},p,q) + \tfrac{1}{2} N^{IJ} \left(\mathscr{Q}_I - F_{IK}\, \mathscr{P}^K\right) \left(\mathscr{Q}_J - \bar{F}_{JL}\, \mathscr{P}^L\right)\;, \tag{4.2}$$

where now

$$\Sigma(Y,\bar{Y},p,q) = -{\rm i}\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right) - 2{\rm i}\left(\Upsilon F_\Upsilon - \bar{\Upsilon}\bar{F}_\Upsilon\right) - q_I (Y^I + \bar{Y}^I) + p^I (F_I + \bar{F}_I)\;, \tag{4.3}$$

and where $F = F(Y,\Upsilon)$. The BPS attractor equations for the scalar fields continue to have the form $\mathscr{Q}_I = \mathscr{P}^J = 0$, and the entropy is given by

$$\mathscr{S}_{\rm macro}(p,q) = 2\pi\, \mathscr{E}\Big|_{\rm attractor} = \pi\, \Sigma\Big|_{\rm attractor}\,. \tag{4.4}$$

As an example, consider the holomorphic function

$$F(Y,\Upsilon) = -\frac{Y^1 Y^2 Y^3}{Y^0} - C\,\frac{Y^1}{Y^0}\Upsilon\,, \tag{4.5}$$

which arises in heterotic string compactifications of $K3 \times T^2$. Consider, in particular, a black hole with non-vanishing charges q_0 and p^1. In the absence of R^2-interactions, i.e. when $C = 0$, its entropy vanishes. In the presence of R^2-interactions, however, its entropy (4.4) is non-vanishing and proportional to $\sqrt{|q_0 p^1|}$. This black hole corresponds to a perturbative heterotic string state whose state degeneracy is in precise agreement with $\exp \mathscr{S}_{\rm macro}$ for large charges [22]. This two-charge black hole thus constitutes an example of a black hole for which higher-curvature interaction terms are crucial, and for which there exists a string theoretic microscopic description which precisely reproduces its macroscopic entropy to leading order.

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