

Breakdown of QCD coherence ?

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We reconsider the calculation of a non-global QCD observable and find the possible breakdown of QCD coherence. This breakdown arises as a result of wide angle soft gluon emission developing a sensitivity to emission at small angles and it leads to the appearance of super-leading logarithms. We use the ‘gaps between jets’ cross-section as a concrete example and illustrate that the new logarithms are intimately connected with the presence of Coulomb gluon contributions. Numerical estimates of their potential phenomenological significance are presented.

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1. Introduction

The resummation of large logarithms associated with wide angle soft gluon emissions has been investigated for the last 20 years. For certain observables the contributions from non-global logarithms [1] have to be taken into account. One of the simplest of these non-global observables is the ‘gaps-between-jets’ cross-section. This is the cross-section for producing a pair of high transverse momentum jets (Q) with a restriction on the transverse momentum of any additional jets radiated in between the two jets, i.e. $k_T < Q_0$ for emissions in the gap. This observable has been studied [2, 3] and has been measured at HERA and the Tevatron [4].

In the original calculations [2] of the gaps-between-jets cross section, all terms proportional to $\alpha_s^n \ln^n(Q^2/Q_0^2)$ that can be obtained by dressing the primary $2 \rightarrow 2$ scattering in all possible ways with soft virtual gluons were summed. The restriction to soft gluons implies the use of the eikonal approximation. Let us focus on quark-quark scattering from now on. The corresponding resummed cross-section can be written

$$\sigma = \mathbf{M}^\dagger \mathbf{S}_V \mathbf{M} \quad \text{with} \quad \mathbf{M} = \exp \left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^Q \frac{dk_T}{k_T} \Gamma \right) \mathbf{M}_0. \quad (1.1)$$

Here, \mathbf{M} is the all-orders $qq \rightarrow qq$ amplitude (a 2-component vector in colour space), \mathbf{M}_0 is the hard scattering amplitude and \mathbf{S}_V represents the cut. The anomalous dimension matrix Γ [5] incorporates the effect of dressing a $qq \rightarrow qq$ amplitude with a virtual gluon in all possible ways. It receives contributions from two distinct regions of the loop-integral: the first corresponds to an on-shell gluon (to which one can assign a rapidity) and is identical, but with opposite sign, to the contribution from a real gluon. The second contribution, sometimes referred to as the ‘Coulomb gluon contribution’ [6] is purely imaginary ($i\pi$ terms) and stems from the region where the emitting parton is on-shell. Eq. (1.1) therefore corresponds to the independent emission of soft gluons, i.e. the iterative dressing of the $2 \rightarrow 2$ process with a softer gluon: due to perfect real/virtual cancellation outside the gap (the first line of Fig. 1 shows two contributions) one only has to consider virtual gluons in the gap and the Coulomb terms.

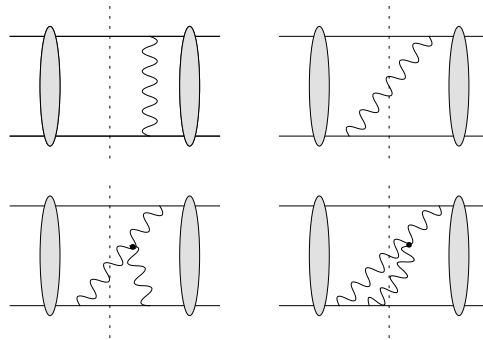


Figure 1: Illustrating the cancellation (and miscancellation) of soft gluon corrections.

However, there is another source of leading logarithms. Let us consider the two diagrams in the second line of Fig. 1. A real gluon (which is outside the gap by the definition of our observable)

emits a softer real or virtual gluon. The real-virtual cancellation is guaranteed only for the softest gluon. Since real gluons above Q_0 are forbidden in the gap, the two diagrams do not completely cancel; the left diagram with the virtual gluon being in the gap and its k_T being larger than Q_0 survives. The non-global nature of our observable has prevented the soft gluon cancellation which is necessary in order that Eq. (1.1) should be the complete result.

It is therefore necessary to include the emission of any number of soft gluons outside the gap region (real and virtual) dressed with any number of virtual gluons within the gap region. Clearly it is a formidable challenge to sum all leading logarithms, mainly because of the complicated colour structure. Progress has been made, working in the large N approximation [3]. Here, we keep the exact colour structure but instead we only compute the cross-section for one gluon outside the gap region. This can be viewed as the first term in an expansion in the number of out-of-gap-gluons.

2. Super-leading logarithms

In order to extract the leading logarithms we consider soft gluons strongly ordered in transverse momentum. The cross-section for one gluon outside and any number of gluons inside the gap is split into two parts corresponding to a virtual or real out-of-gap gluon:

$$\sigma_1 = -\bar{\alpha} \int_{Q_0}^Q \frac{dk_T}{k_T} \int_{\text{out}} \frac{dy d\phi}{2\pi} (\Omega_V + \Omega_R), \quad \bar{\alpha} \equiv \frac{2\alpha_s}{\pi} \quad (2.1)$$

$$\begin{aligned} \Omega_R = & \mathbf{M}_0^\dagger \exp\left(-\bar{\alpha} \int_{k_T}^Q \frac{dk'_T}{k'_T} \Gamma^\dagger\right) \mathbf{D}_\mu^\dagger \exp\left(-\bar{\alpha} \int_{Q_0}^{k_T} \frac{dk'_T}{k'_T} \Lambda^\dagger\right) \mathbf{S}_R \\ & \exp\left(-\bar{\alpha} \int_{Q_0}^{k_T} \frac{dk'_T}{k'_T} \Lambda\right) \mathbf{D}^\mu \exp\left(-\bar{\alpha} \int_{k_T}^Q \frac{dk'_T}{k'_T} \Gamma\right) \mathbf{M}_0, \end{aligned} \quad (2.2)$$

$$\Omega_V = \mathbf{M}_0^\dagger \exp\left(-\bar{\alpha} \int_{Q_0}^Q \frac{dk'_T}{k'_T} \Gamma^\dagger\right) \mathbf{S}_V \exp\left(-\bar{\alpha} \int_{Q_0}^{k_T} \frac{dk'_T}{k'_T} \Gamma\right) \gamma \exp\left(-\bar{\alpha} \int_{k_T}^Q \frac{dk'_T}{k'_T} \Gamma\right) \mathbf{M}_0 + \text{c.c.} \quad (2.3)$$

\mathbf{D}^μ and γ are the matrices that represent the emission of a real and a virtual gluon (k_T, y, ϕ) outside the gap, respectively. The major new ingredient is the matrix Λ [7] which incorporates the dressing of the $qq \rightarrow qqg$ process with a virtual gluon. The emission of the out-of-gap gluon is sandwiched between two exponentials: this accounts for all possible positions of the out-of-gap gluon within a chain of any number of k_T -ordered gluons within the gap.

The phase space of the out-of-gap gluon in Eq. (2.1) includes the configurations where it is collinear to either of the external quarks. One might suppose that the corresponding divergences cancel among Ω_R and Ω_V . This is true in case of the final state collinear limit. However, in the limit of the out-of-gap gluon becoming collinear to one of the initial state quarks, which corresponds to $|y| \rightarrow \infty, k_T > Q_0$, there is no cancellation:

$$[\Omega_V + \Omega_R]_{|y| \rightarrow \infty} \neq 0. \quad (2.4)$$

In particular, $(\Omega_V + \Omega_R)$ becomes independent of y in that limit. This has severe consequences. As the out-of-gap region stretches to infinity in rapidity, the integral Eq. (2.1) is divergent as it stands. This divergence however indicates that one needs to go beyond the soft approximation when considering the out-of-gap gluon. Strictly speaking we ought to work in the collinear (but not soft) approximation which means that the integral over rapidity ought to be replaced by

$$\int d^2k_T \int_{\text{out}} dy \left. \frac{d\sigma}{dyd^2k_T} \right|_{\text{soft}} \rightarrow \int d^2k_T \left[\int_{y_{\text{max}}}^{y_{\text{max}}} dy \left. \frac{d\sigma}{dyd^2k_T} \right|_{\text{soft}} + \int_{y_{\text{max}}}^{\infty} dy \left. \frac{d\sigma}{dyd^2k_T} \right|_{\text{collinear}} \right]. \quad (2.5)$$

In this equation y_{max} is a matching point between the regions in which the soft and collinear approximations are used. If y_{max} is in the region in which both approximations are valid the dependence on it should cancel in the sum of the two terms. Now we know that

$$\int_{y_{\text{max}}}^{\infty} dy \left. \frac{d\sigma}{dyd^2k_T} \right|_{\text{collinear}} = \int_{y_{\text{max}}}^{\infty} dy \left(\left. \frac{d\sigma_R}{dyd^2k_T} \right|_{\text{collinear}} + \left. \frac{d\sigma_V}{dyd^2k_T} \right|_{\text{collinear}} \right) \quad (2.6)$$

where the contribution due to real gluon emission can be written as

$$\int_{y_{\text{max}}}^{\infty} dy \left. \frac{d\sigma_R}{dyd^2k_T} \right|_{\text{collinear}} = \int_0^{1-\delta} dz \frac{1}{2} \left(\frac{1+z^2}{1-z} \right) \left(\frac{q(x/z, \mu^2)}{q(x, \mu^2)} - 1 \right) A_R + \int_0^{1-\delta} dz \frac{1}{2} \frac{1+z^2}{1-z} A_R \quad (2.7)$$

and the contribution due to virtual gluon emission is

$$\int_{y_{\text{max}}}^{\infty} dy \left. \frac{d\sigma_V}{dyd^2k_T} \right|_{\text{collinear}} = \int_0^{1-\delta} dz \frac{1}{2} \left(\frac{1+z^2}{1-z} \right) A_V. \quad (2.8)$$

In Eq. (2.7), $q(x, \mu^2)$ is the parton distribution function for a quark in a hadron at scale μ^2 and momentum fraction x . The factors A_R and A_V contain the z independent factors which describe the soft gluon evolution. Since we require $y > y_{\text{max}}$ ¹ the upper limit on the z integral is fixed: $\delta \approx k_T/Q \cdot \exp(y_{\text{max}} - \Delta y/2)$. We have already established that $A_R + A_V \neq 0$ due to Coulomb gluon contributions to the evolution. If it were the case that $A_R + A_V = 0$ then the virtual emission contribution would cancel identically with the corresponding term in the real emission contribution leaving behind a term regularised by the ‘plus prescription’ (since we can safely take $\delta \rightarrow 0$ in the first term of Eq. (2.7)). This term could then be absorbed into the evolution of the incoming quark parton distribution function by choosing the factorisation scale to equal the jet scale Q .

The miscancellation therefore induces an additional contribution of the form

$$\int_0^{1-\delta} dz \frac{1}{2} \left(\frac{1+z^2}{1-z} \right) (A_R + A_V) = \ln \left(\frac{1}{\delta} \right) (A_R + A_V) + \text{subleading} \quad (2.9)$$

$$\approx \left(-y_{\text{max}} + \frac{\Delta y}{2} + \ln \left(\frac{Q}{k_T} \right) \right) (A_R + A_V). \quad (2.10)$$

¹The approximation arises since we assume for simplicity that Δy is large and δ is small. This approximation does not affect the leading behaviour and can easily be made exact if necessary.

Provided we stay within the soft-collinear region in which both the soft and collinear approximations are valid, the y_{\max} dependence will cancel with that coming from the soft contribution in Eq. (2.5) leaving only the logarithm. The leading effect of treating properly the collinear region is therefore simply to introduce an effective upper limit $\Delta y/2 + \ln(Q/k_T)$ to the integration over rapidity in Eq. (2.5).

$$\frac{2\alpha_s}{\pi} \int_{Q_0}^Q \frac{dk_T}{k_T} \int_{Y/2}^{\ln(Q/k_T) + \Delta y/2} \frac{dy d\phi}{2\pi} = \frac{2\alpha_s}{\pi} \frac{1}{2} \ln^2(Q/Q_0) + \text{subleading}. \quad (2.11)$$

This is the super-leading logarithm: the failure of the ‘plus prescription’ has resulted in the generation of an extra collinear logarithm. The implications for the gaps-between-jets cross-section are clear: collinear logarithms can be summed into the parton density functions only up to scale Q_0 and the logarithms in Q/Q_0 from further collinear evolution must be handled separately.

The miscancellation Eq. (2.4) and hence the super-leading logarithm is intimately connected with the Coulomb phase terms. If one artificially switches off the $i\pi$ terms in the evolution matrices, then there is full cancellation in Eq. (2.4). Moreover, the super-leading logarithm makes its appearance at the lowest possible order in α_s , i.e. at $O(\alpha_s^4)$ relative to the Born cross-section. This is due to the fact that at lower orders any $i\pi$ term is cancelled by a corresponding term from the complex conjugate contribution. The first $i\pi$ terms and the first super-leading logarithm appear in case of four soft gluons:

$$\sigma_1 \sim \sigma_{\text{Born}} \left(\frac{2\alpha_s}{\pi} \right)^4 \ln^5 \left(\frac{Q}{Q_0} \right) \pi^2 Y. \quad (2.12)$$

At higher orders in α_s more gluons can be outside the gap. However, to resum the double logarithms to all orders a deeper understanding of the colour evolution of multi-parton systems seems necessary.

Indeed we appear to have uncovered a breakdown of QCD coherence: radiation at large angles does appear to be sensitive to radiation at low angles. However this striking conclusion was arrived at under the assumption that it is correct to order successive emissions in transverse momentum. Coherence indicates that one does not need to take too much care over the ordering variable, e.g. k_T , E and k_T^2/E are all equally good ordering variables but the super-leading logarithms arise counter to the expectations of coherence and in particular as a result of radiation which is both soft and collinear. It is therefore required to prove the validity of k_T ordering before we can claim without doubt the emergence of super-leading logarithms or confirm their size.

3. Numerical results

We numerically compute the out-of-gap cross section which is the sum of Eq. (2.2) and Eq. (2.3) and which we generically denote σ_1 . The purely superleading logarithmic part of σ_1 is obtained by considering the initial state collinear limit and performing the integral over rapidity over an interval of size $\ln(Q/k_T)$. The result is multiplied by 2 to account for the possibility that the out-of-gap gluon can be on either side of the gap. We refer to the cross section thus computed as ‘SLL’ in the figures. For comparison, we also compute the sum of Eq. (2.2) and Eq. (2.3) without making the collinear approximation. In this case the integral over y is over the region

$Y/2 < |y| < \Delta y/2 + \ln(Q/k_T)$ where $\Delta y = Y + 2$. This cross-section is labelled ‘all’ and necessarily includes a partial summation of the single logarithmic terms as well as the super-leading terms. The strong coupling is fixed at $\alpha_s = 0.15$. Fig. 2 shows σ_1 as a function of $L = \ln(Q^2/Q_0^2)$, normalized to the fully resummed cross-section σ_0 corresponding to zero gluons outside of the gap region, i.e. as determined by Eq. (1.1). The super-leading series is generally small relative to the ‘all’ result for $L \lesssim 4$, which indicates that the single logarithms are phenomenologically much more important than the formally super-leading logs at these values of L . Of course one should remember that our calculations are for the emission of one gluon outside the gap region and the full super-leading series requires the computation of any number of such gluons.

From a more theoretical perspective it is interesting to take a look at the cross-sections out to larger values of Y , see Fig. 3. Note that this time we have normalized the cross-section by the square of the in-gap cross-section. The cross-section saturates at large enough Y , i.e. $\sigma_1 \sim -\sigma_0^2$. In [9] we calculated the conventional gap-between-jets cross-section in the high energy limit and showed that it is equivalent to the BFKL result in the region in which both are valid. Here, we find that in the high-energy (large Y) limit the cross section for one emission outside the gap is proportional to the square of the conventional gap cross-section, offering a tantalizing clue to the structure of higher orders. A deeper understanding of this connection would almost certainly open new avenues to understanding non-global observables.

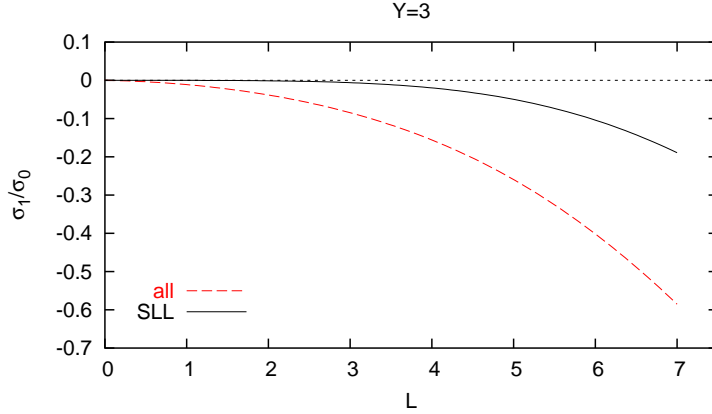


Figure 2: L dependence of the out-of-gap cross-section (normalized to the in-gap cross-section) at $Y=3$

4. Outlook

We appear to have found the breakdown of the intuitive picture of QCD coherence: superleading logarithms appear in the gaps-between-jets observable as the consequence of the sensitivity of soft wide angle gluon emission to collinear emission. The full confirmation of this finding though requires the proof of the validity of k_T -ordering. The new super-leading contributions are not restricted to the gaps-between-jets observable. We expect them to arise generally in non-global observables and potentially as additional leading logarithms also in global observables where non-global contributions are subleading. The new contributions will therefore possibly have an impact

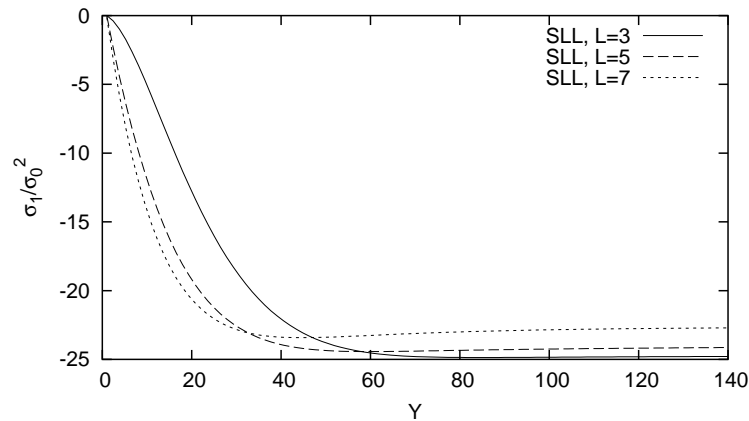


Figure 3: Y dependence of the out-of-gap cross-section normalized to the square of the in-gap cross-section at three different values of L and plotted out to very large Y .

on a wide spectrum of processes and observables, such as eventshape variables, k_T -distributions or particle production near threshold. The connection between the super-leading logarithms and high-energy QCD appears to offer intriguing clues for their resummation and the understanding of non-global observables.

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