PROCEEDINGS OF SCIENCE

Production of ρ_L^0-meson pair in $\gamma^*\gamma^*$ collisions

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We have shown that the lowest order QCD amplitude, i.e. the quark exchange contribution, to the forward production of a pair of longitudinally polarized ρ mesons in the scattering of two virtual photons $\gamma^*(Q_1)\gamma^*(Q_2) \to \rho_L^0\rho_L^0$ factorizes in two different ways: the part with transverse photons is described by the QCD factorization formula involving the generalized distribution amplitude of two final ρ mesons, whereas the part with longitudinally polarized photons takes the QCD factorized form with the $\gamma_L^* \to \rho_L^0$ transition distribution amplitude. Perturbative expressions for these, in general, non-perturbative functions are obtained in terms of the $\rho-$meson distribution amplitude.

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1. Introduction

The exclusive reaction

$$\gamma^*(q_1)\gamma^*(q_2) \to \rho_L^0(k_1)\rho_L^0(k_2) \tag{1.1}$$

is a beautiful laboratory to explore the application of perturbative QCD and in particular the factorization theorems which decouple large distance from short distance phenomena. We have calculated [1] the Born order of this process and demonstrated how the factorization properties of the amplitude emerge. At higher energies, the gluon exchange contribution dominates, both at the Born order [2] and in the resummed BFKL approach [3]. The Born order contribution with quark exchanges is described by the same set of diagrams which contributes to the scattering of real photons producing pions at large momentum transfer, studied long ago [4] in the framework of the factorized form of exclusive processes at fixed angle. In these processes, mesons are described by their light-cone distribution amplitudes (DAs), as illustrated in Fig. 1. Our present study can be seen as

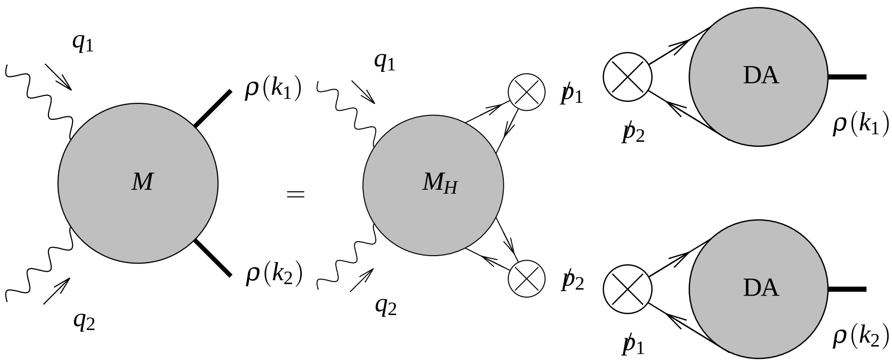

Figure 1: The amplitude of the process $\gamma^*(Q_1)\gamma^*(Q_2) \to \rho_L^0(k_1)\rho_L^0(k_2)$ in the collinear factorization, in which we show the M_H pertubative amplitude and the non-pertubative DA of ρ-mesons.

a complement of Ref. [4] for the case of the scattering with virtual photons, i.e. with transverse and longitudinal polarizations, in the forward kinematics (see also Ref. [5]). The virtualities $Q_i^2 = -q_i^2$, $i = 1,2$, supply the hard scale to the process (1.1) which justifies the use of the QCD collinear factorization methods and the description of ρ mesons by means of their distribution amplitudes.

First, we calculate, within this scheme, the scattering amplitude of (1.1) at Born level. Then, we turn to the study of two particular kinematical regions and show how the amplitude with transverse photons factorizes in a hard subprocess and a Generalized Distribution Amplitude (GDA) [6,7], while on the other hand the amplitude with longitudinal photons factorizes in a hard subprocess and a $\gamma^* \to \rho$ Transition Distribution Amplitude (TDA) [8].

2. The Born order amplitude

The kinematics is quite simple in the forward regime that we are investigating. We use a Sudakov decomposition with two lightlike vectors p_1 and p_2 with $2p_1.p_2 = s$ and write the photon and the meson momenta respectively as

$$q_1 = p_1 - \frac{Q_1^2}{s}p_2 \quad , \quad q_2 = p_2 - \frac{Q_2^2}{s}p_1 \quad \text{and} \quad k_1 = \left(1 - \frac{Q_2^2}{s}\right)p_1 \quad , \quad k_2 = \left(1 - \frac{Q_1^2}{s}\right)p_2 \,. \tag{2.1}$$

The energy's positivity of produced ρ's requires that $s \geq Q_i^2$. The squared invariant mass of the two $\rho-$mesons W^2 and the minimum squared momentum transfer are

$$W^2 = (q_1+q_2)^2 = s\left(1-\frac{Q_1^2}{s}\right)\left(1-\frac{Q_2^2}{s}\right) \quad \text{and} \quad t_{min} = (q_2-k_2)^2 = -\frac{Q_1^2Q_2^2}{s}\,. \tag{2.2}$$

Contrarily to the case studied in Ref. [4, 5], notice that t_{min} may not be large with respect to Λ^2_{QCD} , depending on the respective values of Q_1^2, Q_2^2 and W^2.

The scattering amplitude $\mathscr{A}$ of the process (1.1) can be written in the form $\mathscr{A} = T^{\mu\,\nu}\varepsilon_\mu(q_1)\varepsilon_\nu(q_2)$, where the tensor $T^{\mu\,\nu}$ has in the above kinematics a simple decomposition

$$T^{\mu\,\nu} = \frac{1}{2}g_T^{\mu\,\nu}\,(T^{\alpha\,\beta}g_{T\,\alpha\,\beta}) + (p_1^\mu + \frac{Q_1^2}{s}p_2^\mu)(p_2^\nu + \frac{Q_2^2}{s}p_1^\nu)\,\frac{4}{s^2}(T^{\alpha\,\beta}p_{2\,\alpha}\,p_{1\,\beta})\,, \tag{2.3}$$

and where $g_T^{\mu\,\nu} = g^{\mu\nu} - (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu)/(p_1.p_2)$. The longitudinally polarized ρ^0-meson DA $\phi(z)$ is defined by the standard non-local correlator of quark anti-quark fields on the light cone.

The Born order contribution to the amplitude is calculated in the Feynman gauge in a similar way as in the classical work of Brodsky-Lepage [4] but in very different kinematics. In our case the virtualities of photons supply the hard scale, not the transverse momentum transfer.

The scalar components of the scattering amplitude (2.3) read :

$$T^{\alpha\,\beta}g_{T\,\alpha\,\beta} \;=\; -\frac{e^2(Q_u^2+Q_d^2)\,g^2\,C_F\,f_\rho^2}{4N_c\,s}\int\limits_0^1 dz_1\,dz_2\,\phi(z_1)\,\phi(z_2) \tag{2.4}$$

$$\times\left\{2\left(1-\frac{Q_2^2}{s}\right)\left(1-\frac{Q_1^2}{s}\right)\left[\frac{1}{(z_2+\bar{z}_2\frac{Q_1^2}{s})^2(z_1+\bar{z}_1\frac{Q_2^2}{s})^2} + \frac{1}{(\bar{z}_2+z_2\frac{Q_1^2}{s})^2(\bar{z}_1+z_1\frac{Q_2^2}{s})^2}\right] + \right.$$

$$\left.\left(\frac{1}{\bar{z}_2\,z_1} - \frac{1}{\bar{z}_1\,z_2}\right)\left[\frac{1}{1-\frac{Q_2^2}{s}}\left(\frac{1}{\bar{z}_2+z_2\frac{Q_1^2}{s}} - \frac{1}{z_2+\bar{z}_2\frac{Q_1^2}{s}}\right) - \frac{1}{1-\frac{Q_1^2}{s}}\left(\frac{1}{\bar{z}_1+z_1\frac{Q_2^2}{s}} - \frac{1}{z_1+\bar{z}_1\frac{Q_2^2}{s}}\right)\right]\right\}$$

$$T^{\alpha\,\beta}p_{2\,\alpha}\,p_{1\,\beta} \;=\; -\frac{s^2 f_\rho^2 C_F e^2 g^2(Q_u^2+Q_d^2)}{8N_c Q_1^2 Q_2^2}\int\limits_0^1 dz_1\,dz_2\,\phi(z_1)\,\phi(z_2) \tag{2.5}$$

$$\times\left\{\frac{(1-\frac{Q_1^2}{s})(1-\frac{Q_2^2}{s})}{(z_1+\bar{z}_1\frac{Q_2^2}{s})(z_2+\bar{z}_2\frac{Q_1^2}{s})} + \frac{(1-\frac{Q_1^2}{s})(1-\frac{Q_2^2}{s})}{(\bar{z}_1+z_1\frac{Q_2^2}{s})(\bar{z}_2+z_2\frac{Q_1^2}{s})} + \frac{1}{z_2\bar{z}_1} + \frac{1}{z_1\bar{z}_2}\right\}\,,$$

where $Q_u = 2/3$ ($Q_d = -1/3$) denote the charge of the quark u (d), $C_F = (N_c^2-1)/(2N_c)$ and $N_c = 3$. All integrals (2.4)-(2.5) over quarks momentum fractions z_i are convergent.

3. $\gamma_T^*\gamma_T^* \to \rho_L^0\rho_L^0$ in the generalized Bjorken limit

In the region where the scattering energy W is small compared to the highest photon virtuality Q_1

$$\frac{W^2}{Q_1^2} = \frac{s}{Q_1^2}\left(1-\frac{Q_1^2}{s}\right)\left(1-\frac{Q_2^2}{s}\right) \approx 1-\frac{Q_1^2}{s} \ll 1\,, \tag{3.1}$$

which leads to kinematical conditions very close to the ones considered in Ref. [6,7] for the description of $\gamma^*\gamma \to \pi\pi$ near threshold. In Ref. [6,7] it was shown that, with initial transversally polarized photons, the scattering amplitude factorizes at leading twist as the convolution of a perturbatively calculable coefficient function and a generalized distribution amplitude (GDA). We recover a similar type of factorization with a GDA of the expression (2.4) also in the process of our case (1.1), as illustrated in Fig. 2.

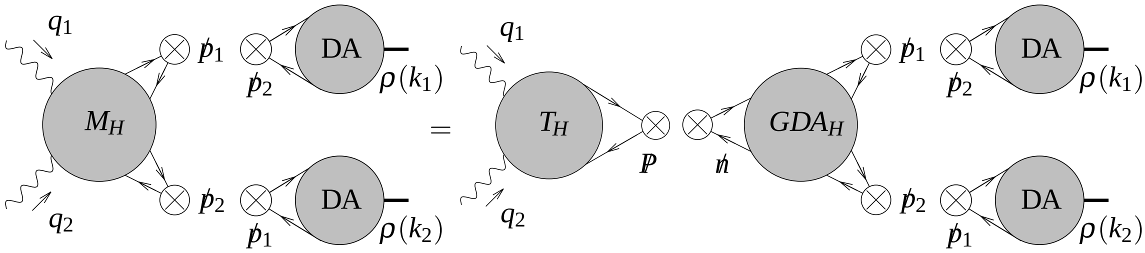

Figure 2: Factorization of the amplitude in terms of a GDA which is expressed in a pertubatively computed GDA_H convoluted with the DAs of the ρ-mesons.

We first assume that Q_1 and Q_2 are not parametrically close, i.e. $1-Q_1^2/s \ll 1-Q_2^2/s$.

The leading contribution is given by the very last term in (2.4),

$$T^{\alpha\beta}g_{T\,\alpha\beta} \approx \frac{e^2(Q_u^2+Q_d^2)\,g^2\,C_F\,f_\rho^2}{4N_c W^2}\int_0^1 dz_1 dz_2\,\phi(z_1)\phi(z_2)\left(\frac{1}{\bar z_2 z_1}-\frac{1}{\bar z_1 z_2}\right)\left(\frac{1}{\bar z_1+z_1\frac{Q_2^2}{s}}-\frac{1}{z_1+\bar z_1\frac{Q_2^2}{s}}\right). \quad (3.2)$$

To interpret this result in a factorized formula, let us first recall the definition of the leading twist GDA for a ρ_L^0 pair. We introduce the vector $P = k_1 + k_2 \approx p_1$, whereas the field coordinates are the ray-vectors along the light-cone direction $n^\mu = p_2^\mu/(p_1.p_2)$. In our kinematics the variable $\zeta = (k_1 n)/(Pn)$ characterizing the GDA equals $\zeta \approx 1$. Thus we define the GDA of the ρ_L^0 pair $\Phi_q(z,\zeta,W^2)$ for $q = u,d$ by the formula

$$\langle\rho_L^0(k_1)\,\rho_L^0(k_2)|\bar q(-\alpha n/2)\,\not n\,\exp\left[\int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} dy\,n_\nu A^\nu(y)\right] q(\alpha\,n/2)|0\rangle = \int_0^1 dz\,e^{-i(2z-1)\alpha(nP)/2}\Phi^{\rho_L\rho_L}(z,\zeta,W^2)\,.$$

We can then calculate the GDA $\Phi_q(z,\zeta,W^2)$ in the Born order of the perturbation theory. The gluonic Wilson line does not give any contribution in our particular kinematics. The remaining contributions to the correlator at order g^2 lead to the result

$$\Phi^{\rho_L\rho_L}(z,\zeta\approx 1,W^2) = -\frac{f_\rho^2\,g^2\,C_F}{2N_c W^2}\int_0^1 dz_2\,\phi(z)\,\phi(z_2)\left[\frac{1}{z\bar z_2}-\frac{1}{\bar z z_2}\right]. \quad (3.3)$$

In the case of a quark of a given flavour, the hard part T_H of the amplitude equals

$$T_H(z) = -4e^2\,N_c\,Q_q^2\left(\frac{1}{\bar z+z\frac{Q_2^2}{s}}-\frac{1}{z+\bar z\frac{Q_2^2}{s}}\right). \quad (3.4)$$

Eqs. (3.3, 3.4) taken together with the flavour structure of ρ^0 enables us to write (3.2) as

$$T^{\alpha\beta}g_{T\,\alpha\beta} = \frac{e^2}{2}\left(Q_u^2+Q_d^2\right)\int_0^1 dz\left(\frac{1}{\bar z+z\frac{Q_2^2}{s}}-\frac{1}{z+\bar z\frac{Q_2^2}{s}}\right)\Phi^{\rho_L\rho_L}(z,\zeta\approx 1,W^2)\,, \quad (3.5)$$

which shows the factorization of $T^{\alpha\beta}g_{T\,\alpha\beta}$ into the hard part and the GDA (up to corrections of order W^2/Q_1^2) . The Eq. (3.5) is the limiting case for $\zeta \to 1$ of the original equation derived by D. Müller et al. [6]. Let us finally note that in general GDAs are complex functions. In our case, i.e. in the Born approximation, the hard part (3.4) and the GDA (3.3) are real quantities. This is due to the use of the real DA of ρ-mesons and to the absence of the $s-$channel cut of hard diagrams in the Born approximation.

The peculiar case of parametrically close photon virtualities deserves particular attention. In this case, there are subtle problems for defining the light-cone vector P since the two outgoing mesons should be now treated in an almost symmetric way. In order to circumvent this difficulty, it is useful to start from the factorized formula (3.5) in the kinematical domain (3.1), assuming $Q_1 > Q_2$. Let us continue (3.5) in Q_2 up to $Q_2 = Q_1$. To control this continuation, one should restore the Q_1^2 and Q_2^2 dependence, encoded in ζ and W^2 through $W^2/\zeta = s(1-Q_1^2/s)$, as

$$\Phi^{\rho_L\rho_L}(z,\zeta,W^2) = -\frac{f_\rho^2 g^2 C_F\,\zeta}{2N_c W^2}\int_0^1 dz_2\,\phi(z)\,\phi(z_2)\left[\frac{1}{z\bar{z}_2}-\frac{1}{\bar{z}z_2}\right]. \tag{3.6}$$

The hard part in (3.5) is proportional to $1-Q_2^2/s$. This factor $1-Q_2^2/s$, now of the order of $1-Q_1^2/s$, starts to play the role of a suppression factor. The amplitude (3.5) which is proportional to $\frac{1}{s}\frac{1-Q_2^2/s}{1-Q_1^2/s}$, now behaves as $1/s \sim 1/Q^2$. In the leading twist approximation, this factorized result vanishes. This observation is confirmed by the result (2.4) of the direct calculation. Indeed, in the case $Q_1 = Q_2 = Q$, the magnifying factor $1/(1-Q^2/s)$ in the two terms of the last line of (2.4) is not present anymore. Thus, the resulting amplitude should be considered as a higher twist contribution.

4. $\gamma_L^*\gamma_L^* \to \rho_L^0\rho_L^0$ in the generalized Bjorken limit

In the regime $Q_1^2 \gg Q_2^2$, it has been advocated [8] that the amplitude with initial longitudinally polarized photons, should factorize as the convolution of a perturbatively calculable coefficient function and a $\gamma\to\rho$ transition distribution amplitude (TDA) defined from the non-local quark correlator

$$\int \frac{dz^-}{2\pi} e^{-ixP^+z^-}\langle\rho(p_2)|\bar{q}(-z^-/2)\gamma^+q(z^-/2))|\,\gamma(q_2)\rangle\,, \tag{4.1}$$

which shares many properties (including the QCD evolution equations) with the generalized parton distributions [6,9] succesfully introduced to describe deeply virtual Compton scattering.

The factorization properties of the scattering amplitude in this domain are conventionally described by using the now standard notations of GPDs. For that we rewrite the momenta of the particles involved in the process as

$$q_1 = \frac{1}{1+\xi}n_1 - 2\xi n_2\,,\quad k_1 = \frac{1-\frac{Q_2^2}{s}}{1+\xi}n_1,\quad q_2 = -\frac{Q_2^2}{(1+\xi)s}n_1 + (1+\xi)n_2\,,\quad k_2 = (1-\xi)\,n_2\,, \tag{4.2}$$

where ξ is the skewedness parameter which equals $\xi = Q_1^2/(2s-Q_1^2)$ and the new n_i Sudakov light-cone vectors are related to the p_i's as $p_1 = \frac{1}{1+\xi}n_1$ and $p_2 = (1+\xi)\,n_2$ with $p_1\cdot p_2 = n_1\cdot n_2 = s/2$. We also introduce the average "target" momentum $P = \frac{1}{2}(q_2+k_2)$ and the momentum transfer $\Delta = k_2 - q_2$. We still restrict our study to the strictly forward case with $t = t_{min} = -2\xi Q_2^2/(1+\xi)$.

Defining the new variable x through $z_2 = (x-\xi)/(1-\xi)$ with $x \in [\xi, 1]$, the expression in Eq. (2.5) can be put into the form:

$$\int_0^1 dz_1\, dz_2\, \phi(z_1)\, \phi(z_2)\, \{....\} = \int_{-1}^1 dx \int_0^1 dz_1\, \phi(z_1) \left(\frac{1}{\bar{z}_1(x-\xi)} + \frac{1}{z_1(x+\xi)} \right) \tag{4.3}$$

$$\times \left[\Theta(1 \geq x \geq \xi) \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1) \phi\left(\frac{1+x}{1-\xi}\right) \right] .$$

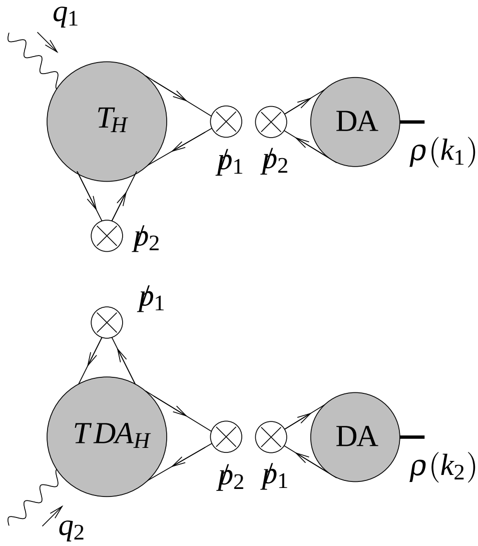

Figure 3: Factorization of the amplitude in terms of a TDA (lower part) which is itself the convolution of a hard term TDA_H and a DA of the ρ-meson.

To rewrite Eq. (4.3) in a form corresponding to the QCD factorization with a TDA, as illustrated in Fig. 3, we first calculate the hard part $T_H(z_1, x)$ of the scattering amplitude, at order g^2, for a meson built from a quark with a single flavour:

$$T_H(z_1,x) = -i f_\rho g^2 e Q_q \frac{C_F \phi(z_1)}{2N_c Q_1^2} \varepsilon^\mu(q_1) \left(2\xi n_{2\mu} + \frac{1}{1+\xi} n_{1\mu} \right) \left[\frac{1}{z_1(x+\xi-i\varepsilon)} + \frac{1}{\bar{z}_1(x-\xi+i\varepsilon)} \right] \tag{4.4}$$

and obviously coincides with the hard part of the $\rho-$meson electroproduction amplitude (4.3). The tensorial structure $2\xi\, n_2^\mu + \frac{1}{1+\xi} n_1^\mu = p_1^\mu + Q_1^2/s\, p_2^\mu$ coincides with the one present in Eq.(2.3).

Passing to the TDA, let us consider the definition of $\gamma_L^*(q_2) \to \rho_L^q(k_2)$ TDA, $T(x, \xi, t_{min})$, in which we assume that the meson is built from a quark with a single flavour, $\rho_L^q(k_2) = \bar{q}q$. The vector $P = 1/2(q_2 + k_2) \approx n_2$ in our kinematics, and the ray-vector of coordinates is oriented along the light-cone vector $n = n_1/(n_1.n_2)$. The computation of the TDA gives

$$\int \frac{dz^-}{2\pi}\, e^{ix(P.z)}\, \langle \rho_L^q(k_2) | \bar{q}(-z/2) \hat{n}\, e^{-ieQ_q \int_{z/2}^{-z/2} dy_\mu A^\mu(y)} q(z/2) | \gamma^*(q_2) \rangle$$

$$= \frac{e\, Q_q\, f_\rho}{P^+} \frac{2}{Q_2^2} \varepsilon_\nu(q_2) \left((1+\xi) n_2^\nu + \frac{Q_2^2}{s(1+\xi)} n_1^\nu \right) T(x, \xi, t_{min}) \,, \tag{4.5}$$

in which we explicitly show the electromagnetic Wilson-line assuring the abelian gauge invariance of the non-local operator. We omit the Wilson line required by the non-abelian QCD invariance since it does not play any role in this case. Note that the factor $(1+\xi)n_2^\nu + \frac{Q_2^2}{s(1+\xi)} n_1^\nu = p_2^\nu + \frac{Q_2^2}{s} p_1^\nu$ corresponds to a part of the tensorial structure of the second term in Eq. (2.3). The perturbative calculation of the matrix element in (4.5) leads to:

$$T(x,\xi,t_{min}) \equiv N_c \left[\Theta(1 \geq x \geq \xi)\, \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1)\, \phi\left(\frac{1+x}{1-\xi}\right) \right] . \tag{4.6}$$

In particular, the contribution to the rhs of (4.5) proportional to the vector n_1^ν (or p_1^ν) corresponds to the contribution coming from the expansion of the electromagnetic Wilson line. Putting all factors together and restoring the flavour structure of the ρ^0, we obtain the factorized form (up to corrections of order Q_2^2/Q_1^2) involving a TDA, of Eq. (2.5) as

$$T^{\alpha\beta} p_{2\alpha} p_{1\beta} = -i f_\rho^2 e^2 (Q_u^2 + Q_d^2) g^2 \frac{C_F}{8N_c} \int_{-1}^{1} dx \int_0^1 dz_1 \left[\frac{1}{\bar{z}_1(x-\xi)} + \frac{1}{z_1(x+\xi)} \right] T(x,\xi,t_{min}) . \tag{4.7}$$

5. conclusion

In conclusion, we have shown that the perturbative analysis of the process $\gamma^*\,\gamma^* \to \rho_L^0 \rho_L^0$ in the Born approximation leads to two different types of QCD factorization which are dictated not only by the kinematics but also by the polarization states of the photons. The arbitrariness in choosing values of photon virtualities shows that there may exist an intersection region where both types of factorization are simultaneously valid.

Acknowledgments

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References

[1] B. Pire, M. Segond, L. Szymanowski and S. Wallon, Phys. Lett. **B639** (2006) 642.

[2] B. Pire, L. Szymanowski and S. Wallon, Eur. Phys. J. C **44** (2005) 545 and hep-ph/0410108.

[3] R. Enberg, B. Pire, L. Szymanowski and S. Wallon, Eur. Phys. J. C **45** (2006) 759; D.Yu. Ivanov and A. Papa, Nucl. Phys. **B732** (2006) 183 and hep-ph/0510397.

[4] S. J. Brodsky and G. P. Lepage, Phys. Lett. **B87** (1979) 359 and Phys. Rev. D **24** (1981) 1808; A.V. Efremov and A.V. Radyushkin, Phys. Lett. **B94** (1980) 245.

[5] M. Diehl, T. Feldmann, P. Kroll and C. Vogt, Phys. Rev. D **61** (2000) 074029.

[6] D. Müller, D. Robaschik, B. Geyer, F. M. Dittes, J. Hořejši, Fortsch. Phys. **42** (1994) 101.

[7] M. Diehl *et al.*, Phys. Rev. Lett. **81** (1998) 1782 and Phys. Rev. D **62** (2000) 073014 ; I. V. Anikin, B. Pire and O. V. Teryaev, Phys. Rev. D **69** (2004) 014018.

[8] B. Pire and L. Szymanowski, Phys. Rev. D **71** (2005) 111501; Phys. Lett. **B622** (2005) 83; J. P. Lansberg, B. Pire and L. Szymanowski, Phys. Rev. D **73** (2006) 074014.

[9] X. Ji, Phys. Rev. Lett. **78**, 610 (1997) and Phys. Rev. **D55**, 7114 (1997); A. V. Radyushkin, Phys. Rev. D **56**, 5524 (1997); J. Blümlein, B. Geyer and D. Robaschik, Phys. Lett. **B406**, 161 (1997); For a review, see M. Diehl, Phys. Rept. **388**, 41 (2003) and references therein.