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DIS Spin Structure Functions at small x

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Explicit expressions for the non-singlet and singlet spin-dependent structure function g_1 in the small-x region are obtained. They include the total resummation of the double- and single- logarithms of x and account for the running QCD coupling α_s effects. Both the non-singlet and singlet structure functions are Regge behaved asymptotically, with the intercepts predicted in agreement with experiments. A detailed comparison with the DGLAP evolution equations for different values of x and Q^2 is performed. The role played by singular terms in DGLAP fits for the initial quark densities is discussed and explicitly shown to mimic the resummation of leading logarithms at small-x. Finally, explicit expressions for the singlet g_1 at small x and small Q^2 are obtained with the total resummation of the leading logarithmic contributions. It is shown that g_1 practically does not depend on x in this kinematic region.

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1. Introduction

In the standard theoretical approach for investigating the DIS structure function $g_1(x,Q^2)$, namely DGLAP [1], g_1^{DGLAP} is a convolution of the coefficient functions C_{DGLAP} and the evolved parton distributions. The latter are also expressed as a convolution of the splitting functions P_{DGLAP} and initial parton densities, which are fitted from experimental data at large x, $x \sim 1$ and $Q^2 \sim 1$ GeV^2. However there is an obvious asymmetry in treating the Q^2- and x- logarithmic contributions in DGLAP. Indeed, the leading Q^2- contributions, $\ln(Q^2)$, are accounted to all orders in α_s whereas $C_{DGLAP}(x)$ and $P_{DGLAP}(x)$ are known in first two orders of the perturbative QCD. On the other hand, in the small-x region the situation looks opposite: logarithms of x, namely double logarithms (DL), i.e. the terms $(\alpha_s \ln^2(1/x))^k$, and single logarithms (SL), the terms $(\alpha_s \ln(1/x))^k$,with $k = 1,2,..$, are becoming quite sizable and should be accounted to all orders in α_s. The total resummation of DL terms was first done [2] in the fixed α_s approximation, and led to a new expression g_1^{DL}, for g_1, that in the small-x asymptotics was of the Regge (power-like) form and much greater than the well-known small-x asymptotics of g_1^{DGLAP}.

Strictly speaking, the results of Refs. [2] could not be compared in a straightforward way with DGLAP because instead of the running α_s, with the parametrization

$$\alpha_s^{DGLAP} = \alpha_s(Q^2), \tag{1.1}$$

Refs. [2] had used α_s fixed at an unknown scale. A closer investigation of this matter [5] led us to conclude that the DGLAP-parametrization of Eq. (1.1) can be a good approximation at x not far from 1 only. Instead, a new parametrization was suggested, where the argument of α_s in each of the ladder rungs of the involved Feynman diagram is the virtuality of the horizontal gluon (see Ref. [5] for detail). Indeed this parametrization works well both for small and large x. It converges to the DGLAP-parametrization at large x but differs from it at small x, and it allowed us to obtain in Refs. [7] the expressions for g_1 accounting for all-order resummations of DL and SL terms, including the running α_s effects [1]. This led us to predict the numerical values of the intercepts of the singlet and non-singlet g_1. These results were then confirmed [8] by several independent groups who have analyzed the HERMES data and extrapolated them at small x.

On the other hand, it is well known that, despite missing the total resummation of $\ln x$, DGLAP works quite successfully at $x \ll 1$. This might suggest that the total resummation of DL contributions performed in Refs. [7] should not be relevant at available values of x and might be of some importance at extremely small x reachable in the future. In Ref. [9] we made a detailed numerical analysis and explained why DGLAP fits can be successful at small x. Indeed in order to describe the available experimental data, singular expressions (see for example Refs. [10, 11]) are introduced for the initial parton densities. These singular factors (i.e. the factors which $\to \infty$ when $x \to 0$) introduced in the fits mimic the total resummation of Refs. [7]. Then using the results of Ref. [7] for incorporating the total resummation of $\ln x$ allows to simplify the rather sophisticated structure of the standard DGLAP fits down to a normalization constant at small x.

[1] The parametrization of Ref. [5] was used later in Refs. [6] for studying the small-x contribution to the Bjorken sum rule.

2. Difference between DGLAP and our approach

In DGLAP, g_1 is expressed through convolutions of the coefficient functions and evolved parton distributions. As convolutions look simpler in terms of integral transforms, it is convenient to represent g_1 in the form of the Mellin integral. For example, the non-singlet component of g_1 can be represented as follows:

$$g_{1\ DGLAP}^{NS}(x,Q^2) = (e_q^2/2)\int_{-\imath\infty}^{\imath\infty}\frac{d\omega}{2\imath\,\pi}(1/x)^{\omega}C_{DGLAP}(\omega)\delta q(\omega)\exp\Big[\int_{\mu^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\gamma_{DGLAP}(\omega,\alpha_s(k_\perp^2))\Big]\ , \tag{2.1}$$

with $C_{DGLAP}(\omega)$ being the non-singlet coefficient functions, $\gamma_{DGLAP}(\omega,\alpha_s)$ the non-singlet anomalous dimensions and $\delta q(\omega)$ the initial non-singlet quark densities in the Mellin (momentum) space. The expression for the singlet g_1 is similar, though more involved. Both γ_{DGLAP} and C_{DGLAP} are known in first two orders of the perturbative QCD. Technically, it is simpler to calculate them at integer values of $\omega = n$. In this case, the integrand of Eq. (2.1) is called the n-th momentum of g_1^{NS}. Once the moments for different n are known, g^{NS} at arbitrary values of ω is obtained by interpolation. The expressions of the initial quark densities are obtained from phenomenological consideration, by fitting the experimental data at $x \sim 1$. Eq. (2.1) shows that γ_{DGLAP} governs the Q^2- evolution whereas C_{DGLAP} evolves $\delta q(\omega)$ in the x-space from $x \sim 1$ into the small x region. When in the x-space the initial parton distributions $\delta q(x)$ are regular in x, i.e. do not $\to \infty$ when $x \to 0$, the small-x asymptotics of $g_{1\ DGLAP}$ is given by the well-known expression:

$$g_{1\ DGLAP}^{NS},\ g_{1\ DGLAP}^{S} \sim \exp\Big[\sqrt{\ln(1/x)\ln\Big(\ln(Q^2/\mu^2)/\ln(\mu^2/\Lambda_{QCD}^2)\Big)}\ \Big]. \tag{2.2}$$

On the contrary, when the total resummation of the double-logarithms and single-logarithms of x is done[5], the Mellin representation for g_1^{NS} is

$$g_1^{NS}(x,Q^2) = (e_q^2/2)\int_{-\imath\infty}^{\imath\infty}\frac{d\omega}{2\pi\imath}(1/x)^{\omega}C_{NS}(\omega)\delta q(\omega)\exp\big(H_{NS}(\omega)\ln(Q^2/\mu^2)\big)\ , \tag{2.3}$$

with new coefficient functions C_{NS},

$$C_{NS}(\omega) = \frac{\omega}{\omega - H_{NS}^{(\pm)}(\omega)}\ , \tag{2.4}$$

and anomalous dimensions H_{NS},

$$H_{NS} = (1/2)\Big[\omega - \sqrt{\omega^2 - B(\omega)}\Big]\ , \tag{2.5}$$

where

$$B(\omega) = (4\pi C_F(1+\omega/2)A(\omega) + D(\omega))/(2\pi^2)\ . \tag{2.6}$$

$D(\omega)$ and $A(\omega)$ in Eq. (2.6) are expressed in terms of $\rho = \ln(1/x)$, $\eta = \ln(\mu^2/\Lambda_{QCD}^2)$, $b = (33 - 2n_f)/12\pi$ and the color factors $C_F = 4/3$, $N = 3$:

$$D(\omega) = \frac{2C_F}{b^2 N}\int_0^\infty d\rho e^{-\omega\rho}\ln\big(\frac{\rho+\eta}{\eta}\big)\Big[\frac{\rho+\eta}{(\rho+\eta)^2+\pi^2} \mp \frac{1}{\eta}\Big]\ , \tag{2.7}$$

$$A(\omega) = \frac{1}{b}\Big[\frac{\eta}{\eta^2+\pi^2} - \int_0^\infty \frac{d\rho e^{-\omega\rho}}{(\rho+\eta)^2+\pi^2}\Big]. \quad (2.8)$$

H_S and C_{NS} account for DL and SL contributions to all orders in α_s.

When $x \to 0$,

$$g_1^{NS} \sim \left(x^2/Q^2\right)^{\Delta_{NS}/2},\ g_1^S \sim \left(x^2/Q^2\right)^{\Delta_S/2} \quad (2.9)$$

where the non-singlet and singlet intercepts are $\Delta_{NS} = 0.42,\ \Delta_S = 0.86$. The x- behaviour of Eq. (2.9) is much steeper than the one of Eq. (2.2). Obviously, the total resummation of logarithms of x leads to a faster growth of g_1 when x is decreasing, compared to what is predicted by DGLAP, provided the input initial parton density δq in Eq. (2.1) is a regular function of ω at $\omega \to 0$.

3. Role of the initial parton densities

There are various forms in the literature for $\delta q(x)$, but all available fits include both a regular and a singular factor when $x \to 0$ (see e.g. Refs. [10, 11] for detail). For example, one of the fits from Ref. [10] is given by

$$\delta q(x) = N\eta x^{-\alpha}\left[(1-x)^\beta (1+\gamma x^\delta)\right], \quad (3.1)$$

with N, η being normalization factors, $\alpha = 0.576$, $\beta = 2.67$, $\gamma = 34.36$ and $\delta = 0.75$. In the ω-space Eq. (3.1) is a sum of pole contributions:

$$\delta q(\omega) = N\eta\left[(\omega-\alpha)^{-1} + \sum m_k(\omega+\lambda_k)^{-1}\right], \quad (3.2)$$

with $\lambda_k > 0$, and the first term in Eq. (3.2) corresponds to the singular factor $x^{-\alpha}$ of Eq. (3.1). When Eq. (3.1) is substituted in Eq. (2.1), the singular factor $x^{-\alpha}$ affects the small-x behavior of g_1 and changes its asymptotics Eq. (2.2) for g_1 for the Regge asymptotics. Indeed, the small-x asymptotics is governed by the leading singularity $\omega = \alpha$, so

$$g_{1\ DGLAP} \sim C(\alpha)(1/x)^\alpha \left((\ln(Q^2/\Lambda^2))/(\ln(\mu^2/\Lambda^2))\right)^{\gamma(\alpha)}. \quad (3.3)$$

Obviously, the actual DGLAP asymptotics of Eq. (3.3) is of the Regge type, and differs a lot from the conventional DGLAP asymptotics of Eq. (2.2). Indeed it looks similar to our asymptotics given by Eq. (2.9), namely by incorporating the singular factors into DGLAP initial parton densities it leads to the steep rise of g_1^{DGLAP} and therefore to a successful description of DGLAP at small x. In Ref. [9] it is shown that without the singular factor $x^{-\alpha}$ in the fit of Eq. (3.1), DGLAP would not be able to work successfully at $x \leq 0.05$. In other words, the singular factors in DGLAP fits mimic the total resummation of logarithms of x of Eqs. (2.3), (2.9). To be more specific, although both (3.3) and (2.9) predict the Regge asymptotics for g_1, there is a numerical difference in the intercepts: Eq. (3.3) predicts that the intercept of g_1^{NS} should be $\alpha = 0.57$, a value which is greater than our predicted non-singlet intercept $\Delta_{NS} = 0.42$. Therefore the non-singlet g_1^{DGLAP} grows, when $x \to 0$, faster than our predictions. Such a rise however is too steep and contradicts the results obtained in Refs. [7] and confirmed in Refs. [8].

4. Expressions for the singlet g_1 at small Q^2

More recently, in Ref. [13], we have extended the results of Ref. [7] for the small-x behavior of the singlet g_1 in a more general framework. In particular, we have given a special attention to the kinematic region where not only x but also Q^2 are small. On one hand, this kinematics has been investigated experimentally by the COMPASS collaboration, see Ref. [12]. On the other hand, the region of small Q^2 is clearly beyond the reach of the standard approach. We have suggested that in this kinematics g_1 can be practically independent of x even for $x \ll 1$. Also we obtain that g_1, being positive at small values of the invariant energy $2(pq)$, can turn negative when $2(pq)$ increases. The position of the turning point is sensitive to the ratio between the initial quark and gluon densities. Then we have also shown that, in spite of the presence of large factors providing g_1 with the Regge behavior at small x, the interplay between initial quark and gluon densities might keep g_1 close to zero even at small x, regardless of the values of Q^2. Explicit expressions for the singlet g_1 at small Q^2 can be found in Ref. [13].

5. Conclusions

We have explicitly shown, by direct comparison of Eqs. (2.2) and (3.3), that the singular factor $x^{-\alpha}$ in the Eq. (3.1) for the initial quark density converts the exponential DGLAP-asymptotics into the Regge one. On the other hand, comparison of Eqs. (2.9) and (3.3) also shows that this singular factor in the DGLAP fits mimics the total resummation of logarithms of x. This type of factors can be dropped when the total resummation of logarithms of x performed in Ref. [7] is taken into account. The remaining terms, which are regular in x in the DGLAP fits (the terms in squared brackets in Eq. (3.1)), can obviously be simplified or even dropped at small x and replaced by constants. A more detailed analysis as well as a suggestion to combine the leading logarithms resummation at small x with DGLAP can be found in Ref. [9]. The above results lead to an interesting conclusion: the expressions for the initial parton densities δq used in DGLAP analysis have been commonly believed to be related to non-perturbative QCD effects; indeed they actually mimic the contributions of the perturbative QCD, so the whole impact of the non-perturbative QCD effects on g_1 at small x is not large and can be approximated by a normalization constant. We have also shown that the study of g_1 at small-Q^2 could be as interesting as in the large-Q^2 region. An explicit expression is given in Ref. [13] which describes the singlet g_1 at small x and arbitrary values of Q^2, generalizing both the standard approach and our previous results.

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