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Unitarity Signatures in Diffractive Scattering*

U. Maor†
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Science, Tel Aviv University Tel Aviv, 69978, Israel
E-mail: maor@post.tau.ac.il

E. Gotsman
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Science, Tel Aviv University Tel Aviv, 69978, Israel
E-mail: gotsman@post.tau.ac.il

E. Naftali
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Science, Tel Aviv University Tel Aviv, 69978, Israel
E-mail: erann@post.tau.ac.il

It is claimed that unambiguous unitarity signatures in present high energy hadron-hadron scattering are identified in diffractive LRG channels. These are analyzed in the GLM model emphasizing general results which are, essentially, model independent. We suggest a detailed LHC study of diffraction to be done in small diffracted mass bands, as well as, searching for high P_T diffractive secondaries deficiency. Implications of our study in hard diffraction are briefly discussed.

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1. Introduction

The aim of this presentation is to search and identify unambiguous s-channel unitarity signatures in the high energy scattering data (both soft and hard). To this end, we shall utilize the GLM eikonal model [1, 2, 3] as a convenient platform to present qualitative features that we consider to be general. The utilization and further critical tests of these ideas are expected to be carried out in the LHC small x experiments (see Orava's and Royon's presentations in these Proceedings). It has also important implications for the analysis of Cosmic Rays air shower data, in particular the forthcoming AUGER results. We aim to translate our observations into concrete experimental propositions.

The bulk of the following will be devoted to soft scattering, where the presentation is relatively simple. This will be followed by a brief summary on the extension of the presented analysis to hard diffraction in ep and pp scattering. Special attention will be given to LRG survival probabilities in both soft and hard diffraction.

2. *S*-Channel Unitarity and the Eikonal Model

As a consequence of our inability to execute QCD calculations in the non-perturbative regime, high energy soft scattering is commonly described by the Regge-pole model [4]. Its key leading ingredient is the Pomeron, whose linear t-dependent trajectory is specified by $\alpha_{\mathbb{P}}(0)$ and $\alpha'_{\mathbb{P}}$. Donnachie and Landshoff (DL) have promoted [5] a successful Regge parametrization for total and elastic hadron-hadron cross sections in which $\alpha_{\mathbb{P}}(0) = 1+\varepsilon = 1.08$. The fitted slope [6] is $\alpha'_{\mathbb{P}} = 0.25$ GeV^{-2}. This simple parametrization is bound to eventually violate s-channel unitarity, since σ_{el} grows with energy as $s^{2\varepsilon}$ (modulo logarithmic corrections) while σ_{tot} grows as s^{ε}. The intriguing question is what is the energy at which the unitarity bound becomes significant. It is an easy exercise to check that the DL model, with its fitted global parameters, will violate the unitarity black disk bound (see the end of this Section) at very small impact parameter b, just above the present Tevatron energy. Indeed, CDF estimates [7] $a_{el}(\sqrt{s} = 1800, b = 0) = 0.96 \pm 0.04$. Note, though, that unitarity violations in a given model which are confined to small b have relatively little significance on its σ_{tot} and σ_{el} output. The energy dependence of the experimental SD cross section, in the ISR-Tevatron range, is much weaker than the observed approximate power dependence of σ_{el} [8].

2.1 Single channel eikonal model

The above theoretical difficulties are easily identified and eliminated once we take into account the corrections necessitated by s-channel unitarity. However, enforcing unitarity is a model dependent procedure. Indeed, in this workshop we were exposed to three, conceptually different, approaches to this problem (Goulianos, Selyugin and myself). In the following we shall confine ourselves to a Glauber type eikonal model [9]. In this approximation, the scattering matrix is diagonal and only repeated elastic re-scatterings are summed. Accordingly, we write

$$a_{el}(s,b) = i\left(1 - e^{-\frac{1}{2}\Omega(s,b)}\right). \tag{2.1}$$

Since the scattering matrix is diagonal, the single channel unitarity equation is written as

$$2Im[a_{el}(s,b)] = \mid a_{el}(s,b) \mid^2 + G^{in}(s,b), \tag{2.2}$$

with $G^{in} = 1 - e^{-\Omega(s,b)}$. It follows that $P^S(s,b) = e^{-\Omega(s,b)}$ is the probability that the two initial hadrons reach the final inelastic interaction intact, regardless of their re-scatterings.

Eq. (2.1) is a general solution of Eq. (2.2) as long as Ω is arbitrary. In the eikonal model Ω is real and equals the imaginary part of the iterated input Born amplitude. The eikonalized output amplitude is imaginary, but its real part can be calculated [9]. In a Regge language we substitute $s^{\alpha_{I\!P}} \rightarrow s^{\alpha_{I\!P}} e^{-\frac{1}{2}i\pi\alpha_{I\!P}}$. In the general case, Eq. (2.2) implies a general unitarity bound, $\mid a_{el}(s,b) \mid \leq 2$, obtained when $G^{in} = 0$. This is an extreme option [10] in which asymptotically $\sigma_{tot} = \sigma_{el}$. This is formally acceptable but not very appealing. Assuming that a_{el} is imaginary, we obtain that the unitarity bound coincides with the black disk bound, $\mid a_{el}(s,b) \mid \leq 1$ and $\sigma_{el}(s,b)/\sigma_{tot}(s,b) \leq 1/2$.

3. The GLM model

3.1 Single channel GLM

The GLM model [1] is an eikonal model originally conceived to explain the mild energy dependence of soft diffractive cross sections. In this model we take a DL type Pomeron exchange amplitude input in which $\alpha_{I\!P}(0) = 1 + \Delta_S > 1$. The simplicity of the model derives from the observation that the eikonal approximation with a central Gaussian input (corresponding to an exponential slope of $d\sigma_{el}/dt$) can be calculated analytically. This is, clearly, an over simplification, but it reproduces the bulk of the data well, i.e. the total and the forward elastic cross sections. Accordingly, the eikonal input DL type b-space opacity is

$$\Omega^S(s,b) = \nu_S(s)\,\Gamma^S(s,b)\ , \tag{3.1}$$

where $\nu_S(s) = \sigma^S(s_0)\,(s/s_0)^{\Delta_S}$, $R_S^2(s) = R_0^2 + 4\alpha'_{I\!P}\ln(s/s_0)$ and the soft profile

$$\Gamma^S(s,b) = \frac{1}{\pi R_S^2(s)}\, e^{-b^2/R_S^2(s)}\ ,$$

defined so as to keep the normalization $\int d^2b\,\Gamma^S(s,b) = 1$. One has to distinguish between the model input and output. The key observation is that the Δ_S and ν_S are input information, not bounded by unitarity, and should not be confused with DL input. Obviously, $\Delta_S > \varepsilon$. In a non-screened DL type model with a Gaussian profile the relation $B_{el} = R_S^2(s)/2$ is exact. In a screened model, like GLM, $B_{el} > R_S^2(s)/2$. With this input we obtain [1] analytical expressions for $\sigma_{tot}, \sigma_{el}$ and σ_{in} which are easy to calculate. Note that σ_{el}/σ_{tot} is a single variable function of $\nu_S(s)$. Given this experimental ratio, we can calculate [2] an "experimental" value of $\nu_S(s)$, independent of the free parameters adjustment. The GLM model, with $\Delta_S = 0.10$, provides an excellent reproduction of ISR-Tevatron $\sigma_{tot}, \sigma_{el}$ and B_{el}.

The formalism presented above is extended to diffractive (soft and hard) b-space amplitudes which are suppressed by the probability $\sqrt{P^S(s,b)}$ due to the soft re-scattering of the initial interacting hadrons. We denote the b-space input diffractive amplitude by $M_D(s,b)$. The unitarized output amplitude is $M_D(s,b)e^{-\frac{1}{2}\Omega^S(s,b)}$. A unitarized non-elastic integrated diffractive cross section screened by soft re-scatterings of the initial hadrons is obtained by convoluting its *b*-space input amplitude square with the probability P^S leading to a suppression factor [1, 2]

$$S_D^2 = \frac{\sigma_D^{out}}{\sigma_D^{in}} = \frac{\int d^2b \mid M_D(s,b) \mid^2 P^S(s,b)}{\int d^2b \mid M_D(s,b) \mid^2}. \tag{3.2}$$

Note that the input $M_D(s,b) = \nu_D(s)\Gamma^D(s,b)$, enables us to factor out and eliminate ν_D^2. Associating diffractive processes with LRG signatures, the above suppression factor is identical to Bjorken's LRG survival probability [11]. The approximation just presented underestimates the full screening effects as it neglects the re-scatterings of the final state diffractive products. We consider, thus, σ_D^{out} as an upper limit of the unitarized diffractive cross section. In the GLM model we assume both elastic and diffractive input amplitudes to be central Gaussians. This enables an analytic solution in which S_D^2 depends on just two input parameters ν_S and $a_D(s) = R_S^2(s)/R_D^2(s) > 1$.

$$S_D^2 = \frac{a_D(s)\gamma[a_D(s), \nu_S(s)]}{[\nu_S(s)]^{a_D(s)}}\ , \tag{3.3}$$

where $\gamma(a,x) = \int_0^x z^{a-1}e^{-z}dz$ denotes the incomplete Euler gamma function. In an investigation of a few diffractive channels at a given energy, ν_S and $R_S^2(s)$ are fixed while R_D^2 depends on the investigated channel.

The above has been utilized [1] to calculate single diffraction cross sections in the GLM model. Whereas σ_{tot} and σ_{el}, behave asymptotically as $\ln^2(s/s_0)$, σ_{sd} behaves as $\ln(s/s_0)$, due to the $e^{-\Omega}$ factor. In the high energy limit we have, $\sigma_{sd}/\sigma_{tot} \to 0$ and $\sigma_{el}/\sigma_{tot} \to 1/2$.

3.2 Extension to a multi channel model

The failure of a single channel eikonal model to reproduce the diffractive energy dependence is inherent. This problem is traced to the single channel basic input assumption, $\sigma_D/\sigma_{el} \ll 1$, which is not compatible with the data. This problem is resolved in a multi channel approach [3] where the revised model has improved diffractive (specifically SD) predictions, while maintaining the excellent single channel reproductions of the forward elastic amplitude and its results on LRG survival probabilities [2].

In the simplest approximation we consider diffraction as a single hadronic state. We have, thus, two orthonormal wave functions, Ψ_h the wave function of the incoming hadron and Ψ_D the wave function of the outgoing diffractive system initiated by the incoming hadron, which satisfy $\langle\Psi_h \mid \Psi_D\rangle = 0$. Consider a base of two wave functions Ψ_1 and Ψ_2 which are diagonal with respect to the interaction operator **T**. In this representation, Ψ_h and Ψ_D can be written as

$$\Psi_h = \alpha\Psi_1 + \beta\Psi_2, \tag{3.4}$$

$$\Psi_D = -\beta\Psi_1 + \alpha\Psi_2. \tag{3.5}$$

where, $\alpha^2 + \beta^2 = 1$. The amplitude of the interaction is

$$A_{i,k} = \langle\Psi_i\Psi_k \mid \mathbf{T} \mid \Psi_{i'}\Psi_{k'}\rangle = a_{i,k}\,\delta_{i,i'}\,\delta_{k,k'}. \tag{3.6}$$

The amplitudes of each channel (i,k) satisfy the diagonal unitarity condition, Eq. (2.1) and Eq. (2.2). As in a single channel $a_{i,k}(s,b) = i(1 - e^{-\frac{1}{2}\Omega_{i,k}(s,b)})$, $G_{i,k}^{in} = 1 - e^{-\Omega_{i,k}(s,b)}$ and $\Omega_{i,k} = \nu_{i,k}(s)\,\Gamma_{i,k}(s,b)$,

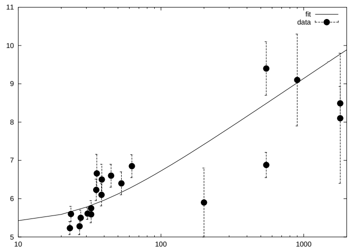

Figure 1: Integrated SD data and a two channel model fit.

with $\nu_{i,k} = \sigma_{i,k}(s_0)\,(s/s_0)^{\Delta}$. The probability of two initial interacting hadrons, i and k, to reach their final inelastic interaction intact, regardless of the initial re-scatterings, is $P_{i,k}(s,b) = e^{-\Omega_{i,k}(s,b)} = [1-a_{i,k}(s,b)]^2$.

Assume an interaction of two hadrons (a,b) for which we consider elastic and diffractive vertices, we obtain four possible re-scattering channels (a,b), (a^*,b), (a,b^*) and (a^*,b^*). For pp or $p\,\bar{p}$ scattering the number of channels is reduced to three. For simplicity we neglect the small double diffraction channel and consider only elastic and SD final states, ending with a two channel model. In this representation we have

$$a_{el}(s,b) \,=\, a_{1,1}(s,b) \,+\, 2\beta^2\left[a_{1,2}(s,b) - a_{1,1}(s,b)\right], \tag{3.7}$$

$$a_{sd}(s,b) \,=\, \alpha\beta\left[a_{1,2}(s,b) - a_{1,1}(s,b)\right]. \tag{3.8}$$

In GLM we assume (different) central Gaussian profiles for $\Omega_{1,1}$ and $\Delta\Omega = \Omega_{1,2} - \Omega_{1,1}$. This enables a successful reproduction of σ_{tot}, σ_{el}, σ_{sd} and B_{el} [3] in the ISR-Tevatron range. In Fig. 1 we show a reproduction of the integrated SD cross sections. Note that the experimental points are too scattered to provide a rigorous phenomenological test. A global analysis of diffractive channels (soft or hard) depends crucially on a reliable and unique algorithm which defines a diffractive system. The diversity of algorithms used by the experimental groups is a severe obstacle in the pursuit of a more reliable analysis.

Following the calculation of S_D^2 in a single channel model, see Eq. (3.2), we can obtain an expression for S_D^2 in a two channel model. However, we are reminded that in a multi channel model each channel of interest has its own dependence on α and β. Let us consider a central exclusive diffractive di-jets (or Higgs) production $(p+p \,\rightarrow\, p+LRG+H+LRG+p)$ for which [12]

$$S^2_{CD}(s) \,=\, \frac{\int d^2b_1 d^2b_2 e^{i(\vec{p}_{1t}\cdot\vec{b}_1+\vec{p}_{2t}\cdot\vec{b}_2)} e^{-\Omega\left((\vec{b}_1+\vec{b}_2)^2\right)} N^2}{\int d^2b_1 d^2b_2 e^{i(\vec{p}_{1t}\cdot\vec{b}_1+\vec{p}_{2t}\cdot\vec{b}_2)} D^2}, \tag{3.9}$$

$$N = \left(1-2\beta^2 a_D\left((\vec{b}_1+\vec{b}_2)^2\right)\right) A_H(p\rightarrow p;b_1)A_H(p\rightarrow p;b_2)$$
$$-2\alpha\beta a_D\left((\vec{b}_1+\vec{b}_2)^2\right)\{A_H(p\rightarrow p;b_1)A_H(p\rightarrow D;b_2)+A_H(p\rightarrow D;b_1)A_H(p\rightarrow p;b_2)\}, \tag{3.10}$$

$$D = (1-2\beta^2)A_H(p\rightarrow p;b_1)A_H(p\rightarrow p;b_2)$$
$$-2\alpha\beta\{A_H(p\rightarrow p;b_1)A_H(p\rightarrow D;b_2)+A_H(p\rightarrow D;b_1)A_H(p\rightarrow p;b_2)\}. \tag{3.11}$$

We follow the philosophy and notation of Ref. [3]

$$a_D(b) = 1 - e^{\frac{1}{2}\Delta\Omega(b)}. \tag{3.12}$$

Ω and $\Delta\Omega$ are parametrized as central Gaussians, with corresponding radii $R^2_{S,p}$ and $R^2_{S,D}$. The hard diffractive amplitudes A_H are approximated as central Gaussians, with radii $R^2_{H,p}$ and $R^2_{H,D}$. We fix the soft parameters from GLM two channel global fit [3] and the hard parameters from J/Ψ elastic and inelastic photoproduction. For details see Ref. [13].

The GLM output for soft scattering is summarized in the upper Table. Its interpretation and consequences will be discussed in the next Section. The survival probabilities listed in the lower Table correspond to Higgs or di-jets produced in hard two LRG exclusive central diffraction.

$\sqrt{s}$[GeV]	σ^{DL}_{tot}[mb]	σ^{GLM}_{tot}[mb]	σ^{GLM}_{el}[mb]	σ^{GLM}_{sd}[mb]	B^{GLM}_{el}[GeV^{-2}]	S^2_{SD}(GLM)
540	60.1	62.0	12.3	8.7	14.9	0.357
1800	72.9	74.9	15.9	10.0	16.8	0.174
14000	101.5	103.8	24.5	12.0	20.5	0.041
30000	114.8	116.3	28.6	12.7	22.0	0.022
60000	128.4	128.7	32.8	13.2	23.4	0.012
90000	137.2	136.5	35.6	13.5	24.3	0.008
120000	143.6	142.2	37.6	13.7	24.9	0.006

We present S^2_{CD} calculated in GLM one and two channels [2, 12], as well as KKMR [14, 13].

$\sqrt{s}$ (GeV)	S^2_{CD}(GLM, 1CH)	S^2_{CD}(GLM, 2CH)	S^2_{CD}(KKMR, 2CH)
540	13.1%	5.1%	6.0%
1800	8.9%	4.4%	4.5%
14000	5.2%	2.7%	2.6%

4. Discussion

We claim that most of the GLM results are general and wish to investigate their experimental consequences. To this end we shall be assisted, when needed, by comparisons of the GLM output with other relevant models predictions. This is carried out with a relative ease in b-space.

1) Even though DL is a model with no, or very weak, unitarity corrections, there is no significant difference between the values of σ_{tot} predicted by DL and GLM up to the top Cosmic Rays energies. As noted, the explanation for this "paradox" is that the DL amplitude violations of s-unitarity are confined, even at super high energies, to small b which does not contribute significantly to the elastic amplitude. The LHC study of σ_{tot} and σ_{el} is not expected, thus, to significantly add to our knowledge on unitarity signatures at high energies. The need for survival probabilities so as to reproduce the experimental soft SD cross section and the hard di-jets rates, is the most compelling existing evidence supporting an observation of unitarization signatures. The study of high energy soft and hard diffraction serves, therefor, as a unique probe of s-channel unitarity. Estimates of S^2 by most existing models are compatible, regardless of their different formulations [13]. We note, though, that S^2 is an integrated observable and stress that an in depth analysis of the role of unitarity

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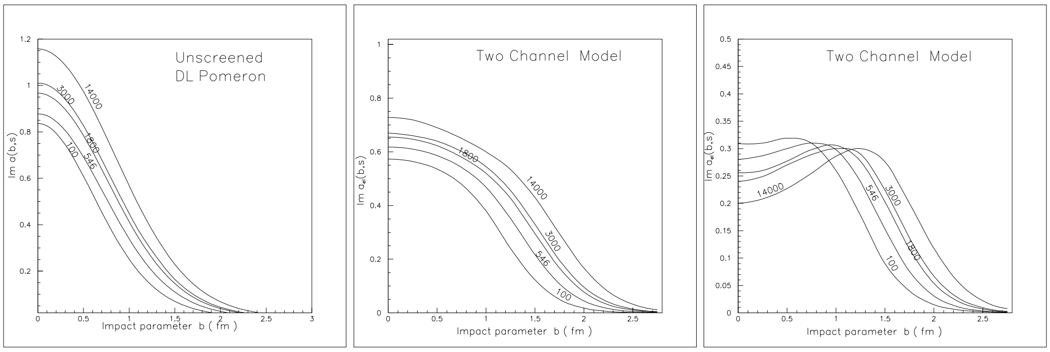

Figure 2: DL elastic $I\!\!P$ (left), and GLM 2Ch elastic (middle) and diffractive (right) amplitude outputs.

in LHC diffraction has to concentrate on differential distributions of the investigated observables.
2) There is a major difference between the unitarized elastic and diffractive b-space amplitudes even though both inputs are central in b. The output suppressed elastic amplitude is different from a Gaussian, but it maintains its b-centrality. On the other hand, the diffractive output changes to a peripheral distribution peaking at higher b. This is a direct consequence of the survival probability b-dependent suppression $e^{-\frac{1}{2}\Omega^S}$, which is exceedingly small at small b and decreases at higher b values. This is shown in Fig. 2. These properties are general and do not depend on the specifics of the eikonal model or its GLM simplification. They are a consequence of the well established b-centrality of the input elastic amplitude. The GLM assumed central Gaussian behavior in b-space for both inputs simplifies the calculation, but does not change the general features of the output. This output is compatible with the Pumplin bound [15], $\sigma_{el}(s,b) + \sigma_{diff}(s,b))/\sigma_{tot}(s,b) \leq 1/2$, regardless of the elastic and diffractive input profile details. Indeed, a check of our results at the Planck scale show $\sigma_{tot} = 1010$ mb with a black disk soft profile, which implies diminishing rates for soft and hard diffraction at exceedingly high energies, well above Cosmic Rays range. This picture is bound to have its effect on Cosmic Rays studies.
3) Goulianos has been promoting his flux renormalization model [8] (for the latest version see these Proceedings). This is a phenomenological procedure which formally does not enforce unitarity, but, rather, a bound of unity on the Pomeron flux in diffractive processes. The Pomeron flux is not uniquely defined so this should be regarded as an ad hoc parametrization. Nevertheless, it has scored an impressive success in reproducing the soft and hard diffractive data in the ISR-Tevatron range. The implied survival probabilities of this procedure are compatible with GLM and KKMR. The model is based on a suppression factor for the diffractive channels which is t (and, thus, b) independent. Even though an output diffractive cross section is reduced relative to its input, there is no change, beside normalization, of the output profile and the Pumplin bound is violated at small b. The model should be tested, and compared to unitarized models, by checking differential observables where unitarity re-scatterings, missing in Goulianos model, can be assessed.
4) We list below two obvious and unique experimental consequences of the small b suppression of the diffractive amplitude.
a) Small b is associated with high $P_\perp$. The small b suppression should induce a reduction in the expected number of high $P_\perp$ secondaries in LHC diffractive channels.

b) Re-scatterings induce dips in $d\sigma/dt$ of elastic and diffractive channels. These dips should be observed in $d\sigma_{sd}/dt$. Since the SD forward slope depends on the diffracted mass, this analysis should be carried out in narrow mass bins. Such dips were not found in the ISR and Tevatron, but the analysis did not have a mass resolution. A similar analysis can be executed in LRG di-jets production [12]. A major obstacle in the way of such analysis is that it is very sensitive to the b-structure of Γ^S. This observation is known from the studies of $d\sigma_{el}/dt$ dips, where small changes in the suggested b-profile Γ^S induce a severe shift of the number of dips and their location. Our studies show this sensitivity also in SD. As a result, we strongly recommend an analysis of SD differential cross section at narrow diffracted mass bands, but we are unable to offer a reliable prediction.

5. Eikonalization in Hard Diffraction

Unitarity plays a dual role in hard diffraction. On the one hand, a hard partonic diffractive scattering process is screened by the soft re-scatterings of the spectator partons. This is the source of the spectator soft survival probability discussed in the previous Sections. In our context, we wish to examine if eikonalization is a viable procedure with which we can assess the role of unitarity in a strictly hard scattering. Note that the suppression due to gluon radiation from the partons participating in the hard scattering is included in the calculation of the hard amplitude through the Sudakov factor. Seemingly, this is an easy problem in which we associate the basic formulas of pQCD in the infinite momentum frame and the target rest frame. This is easily demonstrated in LLA ep DIS calculations.

Following Gribov, the interaction of a virtual photon with the target is written

$$\sigma^H_{tot} \propto \int dz\, d^2r_\perp \mid \Psi^{\gamma^*} \mid^2 \hat{\sigma}(x, r_\perp). \tag{5.1}$$

$\Psi^{\gamma^*}(r_\perp, z)$ is the wave function of the $q\bar{q}$ system within the photon, z and $(1-z)$ are the fractions of the photon energy carried by q and $\bar{q}$ and $r^2_\perp = 4/Q^2$. The SC due to the percolation of a $q\bar{q}$ pair through the target is calculated in the eikonal approximation

$$\hat{\sigma} = 2\int d^2b(1 - e^{-\frac{1}{2}\Omega^H}), \tag{5.2}$$

where, $\Omega^H(x, r_\perp; b_\perp) = \hat{\sigma}_{input}(x, r_\perp)\Gamma^H(b_\perp)$ in which $\hat{\sigma}_{input}(x, r_\perp) = \frac{\pi^2}{3} r^2_\perp \alpha_S(4/r^2_\perp) xG(x, 4/r^2_\perp)$. The suggestive interpretation of the above is to consider the expansion of $2(1-e^{-\frac{1}{2}\Omega^H})$ so that the first term corresponds to the linear DGLAP (or BFKL) evolution, while the following non-linear terms, correspond to the SC induced in the high parton density small x domain.

We aim to obtain a unitarity bound for xG. To this end we start from a bound derived [16] in the dipole picture

$$\frac{\partial^2 xG(x,Q^2)}{\partial y\, \partial \ln Q^2} < \frac{2}{\pi} R^2_H Q^2, \tag{5.3}$$

and combine it with LL DGLAP

$$\frac{\partial^2 xG(x,Q^2)}{\partial y\, \partial \ln Q^2} = \frac{N_c}{\pi} \alpha_S\, xG(x,Q^2). \tag{5.4}$$

At small enough x, we have in the LLA of DGLAP $\partial^2 xG(x,Q^2)/\partial y \partial \ln Q^2 = \frac{N_c}{\pi} \alpha_S xG(x,Q^2)$. Therefore,

$$xG(x,Q^2) < \frac{2}{\pi N_c \alpha_s(Q^2)} R_H^2 Q^2. \tag{5.5}$$

$R_H^2 = 2B_H$ is obtained from the differential cross section of HERA J/Ψ photoproduction.

For a correlated bound of interest, we start from a LLA DGLAP equation

$$\frac{\partial xG(x,Q^2)}{\partial \ln Q^2} = \frac{Q^2}{3\pi^2} xG(x,Q^2). \tag{5.6}$$

In the dipole approximation

$$\frac{\partial xG(x,Q^2)}{\partial \ln Q^2} = \frac{Q^2}{3\pi^3} \sigma_{dipole}(x, r_\perp^2) = \frac{Q^2}{3\pi^2} \int db_\perp^2 Im\, a_{el}^H(x, r_\perp^2). \tag{5.7}$$

a_{el}^H is the hard elastic scattering amplitude of a dipole at a fixed impact parameter. It evolves non-linearly in the domain of high density QCD, bounded by unitarity $\mid a_{el}^H \mid \leq 1$. Setting $a_{el}^H = 1$, enables us to obtain a unitarity bound on $\partial xG(x,Q^2)/\partial \ln Q^2$ which, for very small x, equals $\partial F_2/\partial \ln Q^2$. Fig. 3 displays xG and $\partial F_2/\partial \ln Q^2$ together with a few, relatively old, xG.d.f. and its F_2 logarithmic derivative predictions. The experimental signature implying that xG and $\partial F_2/\partial \ln Q^2$ are getting dangerously close to the unitarity bound, is that their fast increase with decreasing (x,Q^2) is being significantly moderated. This behavior was well reproduced by the GLM eikonal model. Similar success was attained in the calculation of J/Ψ photoproduction.

Despite its phenomenological success, the GLM program was unable to suggest definite unitarity signatures derived from the exceedingly small (x,Q^2) data. The reason for this failure is related to the fact that xG is a phenomenological p.d.f. obtained from a fit to the relevant data. The trigger for our investigations [17] were the early sets of p.d.f.'s, which predicted values for data points of xG and $\partial F_2/\partial \ln Q^2$, at very small (x,Q^2), which were systematically too large. This problem was not present in GLM eikonal predictions for the small (x,Q^2) data. The over estimation at very low (x,Q^2) disappeared in later p.d.f.'s sets, which were obtained from a fit to an enlarged data base, which included the previous problematic experimental points. However, once again, the same problems were found for even smaller (x,Q^2) points which were not included in the fitted data base. The GLM model provided a unitarized reproduction of this data, which was followed by even more advanced, successful p.d.f.'s sets.

At this point it became clear that the existing data could not differentiate between a unitarized and a plain DGLAP evolution interpretation of the data. We conclude that unitarity corrections are, apparently, needed in the exceedingly small (x,Q^2) domain of DGLAP. However, the interpretation of this data should be done directly with the non-linear evolution equation [18] and utilizing more sophisticated modeling than GLM. This has been done by a few groups, GLM included, but is beyond the scope of this contribution. It is an open question if a conventional linear evolution can reproduce the exceedingly small x data which will be measured at the LHC.

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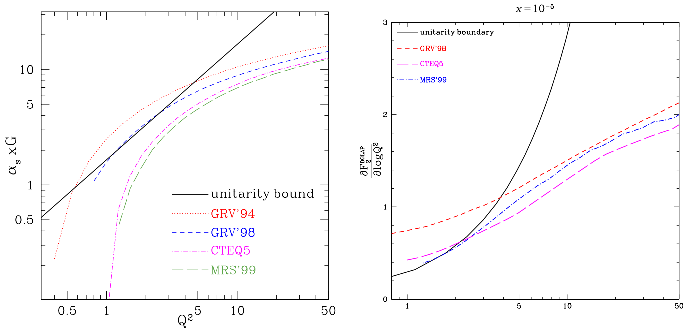

Figure 3: Unitarity bound and p.d.f. predictions for $xG(s,b)$ and $\partial F_2/\partial \ln Q^2$ at $x = 10^{-5}$.

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