

Continuation of the dual amplitude with Mandelstam analyticity off mass shell

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A model for the Q^2 -dependent modified dual amplitude with Mandelstam analyticity (M-DAMA) is proposed. M-DAMA preserves all the attractive properties of DAMA, such as its pole structure and Regge asymptotics, and leads to a generalized dual amplitude $A(s, t, Q^2)$. This amplitude can be checked in the known kinematical limits, i.e. it should reduce to onshell hadronic scattering amplitude for $Q^2 = 0$ and can be related to the nuclear structure function for $t = 0$. In such a way we complete a unified "two-dimensionally dual" picture of strong interaction. By comparing the structure function F_2 , resulting from M-DAMA, with phenomenological parameterizations, we fix the Q^2 -dependence in M-DAMA. In all studied regions, i.e. large and low x limits as well as for the resonance region, the results of M-DAMA are in qualitative agreement with the experiment. The new feature of the M-DAMA is the possible existence, according to vector meson dominance, of Q^2 poles in negative Q^2 region, accessible in e^+e^- annihilation.

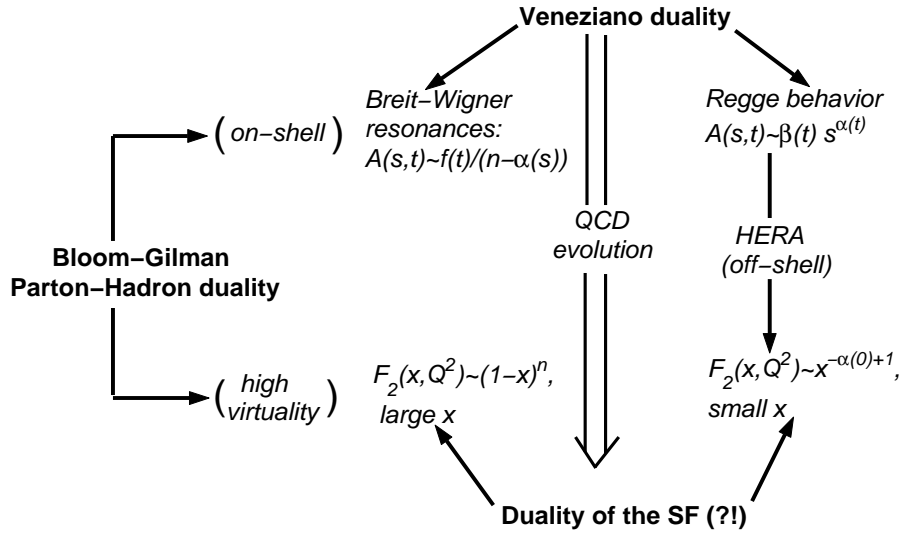
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Our aim in this work is a construction of an explicit dual model combining direct channel resonances, Regge behaviour typical for hadrons and scaling behaviour typical for the partonic picture. Such a model would produce a generalized Q^2 -dependent dual amplitude $A(s, t, Q^2)$. This amplitude, a function of three variables, should have correct known limits, i.e. it should reduce to the on shell hadronic scattering amplitude on mass shell, and to the nuclear structure function (SF) when $t = 0$. In such a way we could complete a unified "two-dimensionally dual" picture of strong interaction [1, 2, 3, 4, 5]:



About thirty years ago Bloom and Gilman [7] observed that the prominent resonances in inelastic $e^- p$ scattering do not disappear with increasing photon virtuality Q^2 , but fall at roughly the same rate as background. Furthermore, the smooth scaling limit proved to be an accurate average over resonance bumps seen at lower Q^2 and s , this is so called Bloom-Gilman or hadron-parton duality. Since the discovery, the hadron-parton duality was studied in a number of papers [8] and the new supporting data has come from the recent experiments [9, 10].

First attempts to combine resonance (Regge) behaviour with Bjorken scaling were made [11, 12, 13] at low energies (large x), with the emphasis on the right choice of the Q^2 -dependence, such as to satisfy the required behaviour of form factors, vector meson dominance (the validity (or failure) of the (generalized) vector meson dominance is still disputable) with the requirement of Bjorken scaling. Similar attempts in the high-energy (low x) region became popular recently stimulated by the HERA data [14, 15, 16].

A consistent treatment of the problem requires the account for the spin dependence, which we ignore in this paper for the sake of simplicity. Our goal is rather to check qualitatively the proposed new way of constructing the "two-dimensionally dual" amplitude.

In Refs. [5] a modified definition of dual model with Mandelstam analyticity (M-DAMA) with Q^2 -dependence was proposed. M-DAMA preserves all the attractive features of DAMA, such as pole decompositions in s and t , Regge asymptotics etc., yet it gains the Q^2 -dependent form factors, correct large and low x behaviour for $t = 0$ etc. The M-DAMA integral reads [5]:

$$D(s, t, Q^2) = \int_0^1 dz \left(\frac{z}{g} \right)^{-\alpha_s(s') - \beta(Q^2) - 1} \left(\frac{1-z}{g} \right)^{-\alpha_t(t'') - \beta(Q^2) - 1}, \quad (1)$$

where $a' = a(1-z)$, $a'' = az$, and g is a free parameter, $g > 1$, and $\alpha_s(s)$ and $\alpha_t(t)$ stand for the Regge trajectories in the s - and t -channels. The $\beta(Q^2)$ is a smooth dimensionless function of Q^2 , which will be specified later on from studying different regimes of the above integral.

The on mass shell limit, $Q^2 = 0$, leads to the shift of the s - and t -channel trajectories by a constant factor $\beta(0)$ (to be determined later), which can be simply absorbed by the trajectories and, thus, M-DAMA reduces to DAMA. In the general case of the virtual particle with mass M we have to replace Q^2 by $(Q^2 + M^2)$ in the M-DAMA integral.

Now all the machinery developed for the DAMA model (see for example [17]) can be applied to the M-DAMA integral. Below we shall report briefly only some of its properties, relevant for the further discussion.

The dual amplitude $D(s, t, Q^2)$ is defined by the integral (1) in the domain $\Re(\alpha_s(s') + \beta(Q^2)) < 0$ and $\Re(\alpha_t(t'') + \beta(Q^2)) < 0$. For monotonically decreasing function $\Re \beta(Q^2)$ (or non-monotonic function with maximum at $Q^2 = 0$) and for increasing or constant real parts of the trajectories these equations, applied for $0 \leq z \leq 1$, mean $\Re(\alpha_s(s) + \beta(0)) < 0$ and $\Re(\alpha_t(t) + \beta(0)) < 0$. To enable us to study the properties of M-DAMA in the domains $\Re(\alpha_s(s') + \beta(Q^2)) \geq 0$ and $\Re(\alpha_t(t'') + \beta(Q^2)) \geq 0$, which are of the main interest, we have to make an analytical continuation of M-DAMA. This leads to the appearance of two moving poles

$$\alpha_s(s(1-z_n)) + \beta(Q^2 z_n) = n \quad \text{and} \quad \alpha_t(t z_m) + \beta(Q^2(1-z_m)) = m, \quad n, m = 0, 1, 2, \dots \quad (2)$$

The singularities of the dual amplitude are generated by pinches which occur in the collisions of the above mentioned moving and fixed singularities of the integrand $z = 0, 1$ [17].

It was shown in Refs. [5], that in this way for the s poles (and similarly for t poles) we obtain the following expression for the pole term:

$$D_{s_n}(s, t, Q^2) = g^{n+1} \sum_{l=0}^n \frac{[\beta'(0)Q^2 - s\alpha'_s(s)]^l C_{n-l}(t, Q^2)}{[n - \alpha_s(s) - \beta(0)]^{l+1}}, \quad (3)$$

where

$$C_l(t, Q^2) = \frac{1}{l!} \frac{d^l}{dz^l} \left[\left(\frac{1-z}{g} \right)^{-\alpha_t(tz) - \beta(Q^2(1-z)) - 1} \right]_{z=0}. \quad (4)$$

The modifications with respect to DAMA [17] are the shift of the trajectory $\alpha_s(s)$ by the factor of $\beta(0)$; and the coefficients C_l are now Q^2 -dependent and can be associated with the form factors.

The new thing here is a possibility of having Q^2 poles, defined by

$$\alpha_s(0) + \beta(Q_n^2) = n, \quad \alpha_t(0) + \beta(Q_m^2) = m. \quad (5)$$

In this sense we can think about $\beta(Q^2)$ as of a kind of trajectory. As we will see later with a proper choice of $\beta(Q^2)$ we can avoid unphysical poles in positive Q^2 region.

Below, for the first time, the calculations for the Q^2 -pole terms for the last case in eqs. (5) are presented.

The point z_m is a solution of the second equation in system (2) and for the pole condition, $z_m \rightarrow 0$, it gives

$$\alpha_t(0) - t \alpha'_t(0) z_m + \beta(Q^2) - \beta'(Q^2) Q^2 z_m = m \quad \Rightarrow \quad z_m = \frac{m - \alpha_t(0) - \beta(Q^2)}{t \alpha'_t(0) - Q^2 \beta'(Q^2)}. \quad (6)$$

Then residue at the pole z_m is equal to:

$$\begin{aligned} 2\pi i \text{Res}_{z_m} &= \frac{1}{t \alpha'_t(0) - Q^2 \beta'(Q^2)} \left(\frac{z_m}{g} \right)^{-\alpha_s(s(1-z_m)) - \beta(Q^2 z_m) - 1} \left(\frac{1-z_m}{g} \right)^{-m-1} \\ &= \frac{g^{m+2+\alpha_s(s)+\beta(0)} [t \alpha'_t(0) - Q^2 \beta'(Q^2)]^{\alpha_s(s)+\beta(0)}}{[m - \alpha_t(0) - \beta(Q^2)]^{\alpha_s(s)+\beta(0)+1}} F(z_m), \end{aligned} \quad (7)$$

where the non-pole function is

$$\begin{aligned} F(z_m) &= \left(\frac{z_m}{g} \right)^{-\alpha_s(s(1-z_m)) + \alpha_s(s) - \beta(Q^2 z_m) + \beta(0)} \cdot \frac{1}{(1-z_m)^{m+1}} \\ &\approx e^{\ln\left(\frac{z_m}{g}\right) z_m \kappa} \frac{1}{(1-z_m)^{m+1}}, \quad z_m \ll 1, \quad \kappa = s \alpha'_s(s) - Q^2 \beta'(0). \end{aligned} \quad (8)$$

The κ can be either positive or negative. Now as we have done above we would like to expand $F(z_m)$ in a series for $z_m \ll 1$, but there is a problem, since the derivatives $\left. \frac{d^n F(z_m)}{dz_m^n} \right|_{z_m=0}$ are divergent for any $n \geq 1$. On the other hand, we can expand F in a power series of $y = \ln\left(\frac{z_m}{g}\right) z_m$ ($y \rightarrow 0$, $z_m \rightarrow 0$). The inverse function $z_m = Z_m(y)$ is very (infinitely) smooth, i.e. $\left. \frac{d^n Z_m(y)}{dy^n} \right|_{y=0} = 0$ for $n = \overline{1, \infty}$. Thus,

$$F(z_m) = \sum_{l=0}^{\infty} \frac{1}{l!} \left(\ln\left(\frac{z_m}{g}\right) z_m \kappa \right)^l. \quad (9)$$

Inserting (9) into (7) we obtain the following expression for the pole term:

$$\begin{aligned} D_{Q_m^2}(s, t, Q^2) &= g^{m+2+\alpha_s(s)+\beta(0)} \sum_{l=0}^{M(s)+1} \frac{[t \alpha'_t(0) - Q^2 \beta'(Q^2)]^{\alpha_s(s)+\beta(0)-l} [s \alpha'_s(s) - Q^2 \beta'(0)]^l}{l!} \\ &\quad \cdot \frac{\ln^l \left[\frac{m - \alpha_t(0) - \beta(Q^2)}{g(t \alpha'_t(0) - Q^2 \beta'(Q^2))} \right]}{[m - \alpha_t(0) - \beta(Q^2)]^{\alpha_s(s)+\beta(0)-l+1}}, \end{aligned} \quad (10)$$

where

$$M(s) = [\alpha_s(s) + \beta(0)], \quad (11)$$

i.e. the integer part of $\alpha_s(s) + \beta(0)$. Comparing this expression with that for s -poles, eq. (3), we can say that the power of the Q^2 -pole is s -dependent and not always integer and also distorted by the logarithms. Contrary to eq. (3), the number of multipoles - $M(s) + 2$ - is the same for all m . $M(s)$ is growing with s , so the Q^2 -poles become more prominent for larger s .

The SF is related to the imaginary part of the scattering amplitude in the following way

$$F_2(x, Q^2) = \frac{4Q^2(1-x)^2}{\alpha(s - m_N^2)(1 + 4m_N^2 x^2/Q^2)^{3/2}} \mathcal{I} m A(s(x, Q^2), t=0, Q^2), \quad (12)$$

where α is the fine structure constant, m_N is a nucleon mass.

In the low x limit: $x \rightarrow 0$, $t = 0$, $Q^2 = \text{const}$, $s = Q^2/x \rightarrow \infty$, $u = -s$ M-DAMA gives the following expression - see Refs. [5] for details:

$$\mathcal{I} m A(s, 0, Q^2)|_{s \rightarrow \infty} \sim s^{\alpha(t)+\beta(0)+1} g^{\beta(Q^2)}. \quad (13)$$

According to the two-component duality picture [19], both the scattering amplitude A and the structure function F_2 are the sums of the diffractive and non-diffractive terms. At high energies both terms are of the Regge type. For $\gamma^* p$ scattering only the positive-signature exchanges are allowed. The dominant ones are the Pomeron and f Reggeon, respectively. The relevant scattering amplitude is as follows:

$$B(s, Q^2) = i \sum_k R_k(Q^2) \left(\frac{s}{m_N^2} \right)^{\alpha_k(0)}, \quad (14)$$

where α_k and R_k are Regge trajectories and residues and k stands either for the Pomeron or for the Reggeon. In the phenomenological models which are used nowadays to fit F_2 data [15, 16, 9, 10, 20, 3, 4] the Q^2 -dependence is introduced "by hands", via residues, which are fitted to the data. Now we have a model which contains Q^2 -dependence from the very beginning and automatically gives a correct behaviour of the residues.

Let us now come back to M-DAMA results. Using eqs. (12,13) we get:

$$F_2 \sim s^{\alpha_t(0)+\beta(0)} Q^2 g^{\beta(Q^2)}. \quad (15)$$

We propose $\beta(Q^2)$ in the following form [5]

$$\beta(Q^2) = -1 - \frac{\alpha_t(0)}{\ln g} \ln \left(\frac{Q^2 + Q_0^2}{Q_0^2} \right). \quad (16)$$

where Q_0^2 is some characteristic Q^2 scale. So finally we obtain

$$F_2(x, Q^2) \sim x^{1-\alpha_t(0)} \left(\frac{Q^2}{Q^2 + Q_0^2} \right)^{\alpha_t(0)}, \quad (17)$$

where slowly varying factor $\left(\frac{Q^2}{Q^2 + Q_0^2} \right)^{\alpha_t(0)}$ is typical for the Bjorken scaling violation (for example [16]).

Now let us turn to the large x limit. In this regime $x \rightarrow 1$, s is fixed, $Q^2 = \frac{s-m^2}{1-x} \rightarrow \infty$ and correspondingly $u = -2Q^2$. Following [5] we obtain qualitatively correct behaviour

$$F_2 \sim (1-x)^2 Q^4 g^{2\beta(Q^2/2)} \sim (1-x)^{2\alpha_t(0) \ln 2g / \ln g}. \quad (18)$$

Let us now study F_2 given by M-DAMA in the resonance region. In the vicinity of the resonance $s = s_{Res}$ only the resonance term $D_{Res}(s, 0, Q^2)$ is important in the scattering amplitude and correspondingly in the SF. Using $\beta(Q^2)$ in the form (16) we obtain [5]:

$$C_1(Q^2) = \left(\frac{gQ_0^2}{Q^2 + Q_0^2} \right)^{\alpha_t(0)} \left[\alpha_t(0) + \ln g \frac{Q^2}{Q^2 + Q_0^2} - \frac{\alpha_t(0)}{\ln g} \ln \left(\frac{Q^2 + Q_0^2}{Q_0^2} \right) \right]. \quad (19)$$

The term $\left(\frac{Q_0^2}{Q^2 + Q_0^2} \right)^{\alpha_t(0)}$ gives the typical Q^2 -dependence for the form factor (the rest is a slowly varying function of Q^2). If we calculate higher orders of C_n for subleading resonances, we will see that the Q^2 -dependence is still defined by the same factor $\left(\frac{Q_0^2}{Q^2 + Q_0^2} \right)^{\alpha_t(0)}$.

As we saw above the appearance and properties of Q^2 poles depend on the particular choice of the function $\beta(Q^2)$, and for our choice, given by eq. (16), the unphysical Q^2 poles can be avoided.

We have chosen $\beta(Q^2)$ to be a decreasing function, then, according to conditions (5), there are no Q^2 poles in M-DAMA in the domain $Q^2 \geq 0$, if

$$\Re \beta(0) < -\alpha_s(0), \quad \Re \beta(0) < -\alpha_t(0). \quad (20)$$

We have already fixed $\beta(0) = -1$ and, thus, we see that indeed we do not have Q^2 poles, except for the case of supercritical Pomeron with the intercept $\alpha_P(0) > 1$. Such a supercritical Pomeron would generate one unphysical pole at $Q^2 = Q_{pole}^2$ defined by equation

$$-1 - \frac{\alpha_P(0)}{\ln g} \ln \left(\frac{Q^2 + Q_0^2}{Q_0^2} \right) + \alpha_P(0) = 0 \quad \Rightarrow \quad Q_{pole}^2 = Q_0^2 \left(g^{\frac{\alpha_P(0)-1}{\alpha_P(0)}} - 1 \right). \quad (21)$$

Therefore we can conclude that M-DAMA does not allow a supercritical trajectory - what is good property from the theoretical point of view, since such a trajectory violates the Froissart-Martin limit [21].

There are other phenomenological models which use dipole Pomeron with the intercept $\alpha_P(0) = 1$ and also fit the data [14, 22, 23]. This is a very interesting case - ($\alpha_t(0) = 1$) - for the proposed model. At the first glance it seems that we should anyway have a pole at $Q^2 = 0$. It should result from the collision of the moving pole $z = z_0$ with the branch point $z = 0$, where $\alpha_t(0) + \beta(Q^2(1 - z_0)) = 0$ in our case. Then, checking the conditions for such a collision:

$$\alpha_t(0) - t \alpha_t'(0) z_0 + \beta(Q^2) - \beta'(Q^2) Q^2 z_0 = 0 \quad \Rightarrow \quad z_0 = \frac{-\alpha_t(0) - \beta(Q^2)}{t \alpha_t'(0) - Q^2 \beta'(Q^2)},$$

we see that for $t = 0$ and for $\beta(Q^2)$ given by eq. (16) the collision is simply impossible, because $z_0(Q^2)$ does not tend to 0 for $Q^2 \rightarrow 0$. Thus, for the Pomeron with $\alpha_P(0) = 1$ M-DAMA does not contain any unphysical singularity.

On the other hand, a Pomeron trajectory with $\alpha_P(0) = 1$ does not produce rising SF, as required by the experiment. So, we need a harder singularity and the simplest one is a dipole Pomeron. A dipole Pomeron produces poles of the second power - $D_{dipole}(s, t_m) \propto \frac{C(s)}{(m - \alpha_P(t) + 1)^2}$, see for example Ref. [22] and references therein.

So far we have discussed DIS, where Q^2 is positive. However, in e^+e^- annihilation we can access the negative Q^2 region, and according to vector meson dominance here we should have Q^2 poles corresponding to vector mesons. The behaviour of $\beta(Q^2)$ in this region is under investigation now.

Conclusions: A new model for the Q^2 -dependent dual amplitude with Mandelstam analyticity is proposed. The M-DAMA preserves all the attractive properties of DAMA, such as its pole structure and Regge asymptotics, but it also leads to generalized dual amplitude $A(s, t, Q^2)$ and in this way realizes a unified "two-dimensionally dual" picture of strong interaction [1, 2, 3, 4]. This amplitude, when $t = 0$, can be related to the nuclear SF, and in this way we fix the function $\beta(Q^2)$, which introduces the Q^2 -dependence in M-DAMA, eq. (1). Our analysis shows that for both large and low x limits as well as for the resonance region the results of M-DAMA are in qualitative agreement with the experiment. In this work as well as in [5] only the positive Q^2 region was studied. The vector meson dominance suggests the possibility of having new features in the model - Q^2 poles in the negative Q^2 region, which is accessible in e^+e^- annihilation. This study is in progress now.

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