One of the frontiers of QCD which are intensely investigated in high energy experiments is the high energy (small \(x\)) regime, where we expect to observe the non-linear behavior of the theory. In this regime, the growth of the parton distribution should saturate, forming a Color Glass Condensate (CGC). In this contribution we investigate the saturation physics in diffractive deep inelastic electron-ion scattering. In particular, we present our results for the nuclear dependence of the ratio \(\sigma^{\text{diff}}/\sigma^{\text{tot}}\) and \(\beta\) behavior of the distinct contributions for the nuclear diffractive structure function. We show that saturation physics predicts that approximately 37\% of the events observed at eRHIC should be diffractive.
1. Introduction

Significant progress in understanding diffraction has been made at the ep collider HERA (See, e.g. Refs. [1, 2, 3]). Currently, there exist many attempts to describe the diffractive part of the deep inelastic cross section within pQCD (See, e.g. Refs. [4, 5, 6, 7]). One of the most successful approaches is the saturation one [4] based on the dipole picture of DIS [8]. It naturally incorporates the description of both inclusive and diffractive events in a common theoretical framework, as the same dipole scattering amplitude enters in the formulation of the inclusive and diffractive cross sections. In the studies of saturation effects in DDIS, non-linear evolution equations for the dipole scattering amplitude have been derived [9, 10, 11], new measurements proposed [12, 13, 14] and the charm contribution estimated [15]. However, as shown in Ref. [5], current data are not yet precise enough, nor do they extend to sufficiently small values of $x_F$, to discriminate between different theoretical approaches.

Other source of information on QCD dynamics at high parton density is due to nuclei which provide high density at comparatively lower energies. This expectation can easily be understood if we assume the empirical parameterization $Q^2_s = A^4 \times Q^2_0 (\frac{t}{x^0})^\lambda$, with the parameters $Q^2_0 = 1.0 \text{ GeV}^2$, $x_0 = 0.267 \times 10^{-4}$ and $\lambda = 0.253$ as in Ref. [20]. In Fig. 1 we present the $A$ and $x$ dependence of the saturation scale. We can observe that, while in the proton case we need very small values of $x$ to obtain large values of $Q^2_s$, in the nuclear case a similar value can be obtained for values of $x$ approximately two orders of magnitude greater. Recently, in Ref. [16], we have estimated a set of inclusive observables which could be analyzed in a future electron-ion collider [17]. Our results have demonstrated that the saturation physics cannot be disregarded in the kinematical range of eRHIC. In Ref. [18] we have extended this analyzes for diffractive processes. Our main goal was to understand to what extend the saturation regime of QCD manifests itself in diffractive deep inelastic eA scattering. In particular, we have studied the energy and nuclear dependence of the ratio between diffractive and total cross sections ($\sigma_{\text{diff}} / \sigma_{\text{tot}}$). Moreover, we have made predictions for more detailed diffractive properties, such as those embodied in the diffractive structure function $F^{D(3)}_2(Q^2, \beta, x_F)$. Here we present a brief review of our main results.

2. Basic Formulae

In the rest frame of the target, the QCD description of DIS at small $x$ can be interpreted as a two-step process. The virtual photon (emitted by the incident electron) splits into a $q\bar{q}$ dipole which subsequently interacts with the target. In the color dipole approach, the total diffractive cross sections take on the following form (See e.g. Refs. [3, 4, 8])

$$\sigma_{T,L}^D = \int_{-\infty}^{0} dt e^{B_D t} \frac{d\sigma_{T,L}^D}{dt} \bigg|_{t=0} \frac{1}{B_D} \frac{d\sigma_{T,L}^D}{dt} \bigg|_{t=0}$$

(2.1)

where

$$\frac{d\sigma_{T,L}^D}{dt} \bigg|_{t=0} = \frac{1}{16\pi} \int d^2 \mathbf{r} \int_0^1 d\alpha |\Psi_{T,L}(\alpha, \mathbf{r})|^2 \sigma_{dip}^2(x, r^2),$$

(2.2)

and we have assumed a factorizable dependence on $t$ with diffractive slope $B_D$. 


The diffractive process can be analyzed in more detail studying the behavior of the diffractive structure function $F_{2}^{D(3)}(Q^2, \beta, x_{P})$. In Refs. [4, 8] the authors have derived expressions for $F_{2}^{D(3)}$ directly in the transverse momentum space and then transformed to impact parameter space where the dipole approach can be applied. Following Ref. [4] we assume that the diffractive structure function is given by

$$F_{2}^{D(3)}(Q^2, \beta, x_{P}) = F_{q\bar{q}, L}^{D} + F_{q\bar{q}, T}^{D} + F_{q\bar{q}g, T}^{D}$$  \hspace{1cm} (2.3)$$

where $T$ and $L$ refer to the polarization of the virtual photon. For the $q\bar{q}g$ contribution only the transverse polarization is considered, since the longitudinal counterpart has no leading logarithm in $Q^2$. The computation of the different contributions was made in Refs. [4, 8, 21] and here we quote only the final results:

$$x_{P}F_{q\bar{q}, L}^{D}(Q^2, \beta, x_{P}) = \frac{3Q^6}{32\pi^4 \beta B_{D}} \sum f e_{f}^2 \int_{\alpha_0}^{1/2} d\alpha \alpha^3 (1-\alpha)^3 \Phi_0,$$  \hspace{1cm} (2.4)$$

$$x_{P}F_{q\bar{q}, T}^{D}(Q^2, \beta, x_{P}) = \frac{3Q^4}{128\pi^4 \beta B_{D}} \sum f e_{f}^2 \int_{\alpha_0}^{1/2} d\alpha \alpha (1-\alpha) \left\{ \varepsilon^2 \left[ \alpha^2 + (1-\alpha)^2 \right] \Phi_1 + m_f^2 \Phi_0 \right\}$$  \hspace{1cm} (2.5)$$

where the lower limit of the integral over $\alpha$ is given by $\alpha_0 = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{4m_f^2}{M^2}} \right)$ and we have introduced the auxiliary functions [5]:

$$\Phi_{0,1} \equiv \left( \int_{0}^{\infty} rdrK_{0,1}(\varepsilon r)\sigma_{dip}(x_{P}, r)J_{0,1}(kr) \right)^2.$$  \hspace{1cm} (2.6)$$

For the $q\bar{q}g$ contribution we have [21, 4, 22]

$$x_{P}F_{q\bar{q}g, T}^{D}(Q^2, \beta, x_{P}) = \frac{81\beta \alpha_0}{512\pi^2 \beta B_{D}} \sum f e_{f}^2 \int_{\beta}^{1} \frac{dz}{(1-z)^3} \left[ \left( \frac{1-\beta}{z} \right)^2 + \left( \frac{\beta}{z} \right)^2 \right]$$  \hspace{1cm} (2.7)$$
\[ \times \int_0^{(1-x)Q^2} dk_t^2 \ln \left( \frac{(1-x)Q^2}{k_t^2} \right) \left[ \int_0^\infty u du \sigma_{dip}(u/k_t,x_F)K_2 \left( \sqrt{\frac{x}{1-x}}u^2 \right) J_2(u) \right]^2. \]

We use the standard notation for the variables \( \beta = Q^2/(M_X^2 + Q^2) \), \( x_F = (M_X^2 + Q^2)/(W^2 + Q^2) \) and \( x = Q^2/(W^2 + Q^2) = \beta x_F \), where \( M_X \) is the invariant mass of the diffractive system and \( W \) the total energy of the \( \gamma' p \) (or \( \gamma' A \)). When extending (2.4), (2.5) and (2.7) to the nuclear case we need to change the slope to the nuclear slope parameter, \( B_A \). In what follows we assume that \( B_A \) may be approximated by \( B_A = \frac{R_A^2}{4} \), where \( R_A \) is given by \( R_A = 1.2A^{1/3} \) fm [23].

In the color dipole approach the behavior of the diffractive cross sections, as well as the diffractive structure functions, is strongly dependent on the dipole cross section, which is determined by the QCD dynamics. Consequently, in the dipole picture the inclusion of saturation physics is quite transparent and straightforward, as the dipole cross section is closely related to the solution of the QCD non-linear evolution equations (For recent reviews see, e.g. Refs. [24, 25])

\[ \sigma_{dip}(x,r) = 2 \int d^2 b N(x,r,b), \tag{2.8} \]

where \( N \) is the quark dipole-target forward scattering amplitude for a given impact parameter \( b \) which encodes all the information about the hadronic scattering, and thus about the non-linear and quantum effects in the hadron wave function. In what follows we will disregard the impact parameter dependence [\( \sigma_{dip} = \sigma_0 N(x,r) \)] and consider the phenomenological model proposed in Ref. [20], in which a parameterization of \( N(x,r) \) was constructed so as to reproduce two limits analytically under control: the solution of the BFKL equation for small dipole sizes, \( r \ll 1/Q_s(x) \), and the Levin-Tuchin law [26] for larger ones, \( r \gg 1/Q_s(x) \). Here, \( Q_s \) denotes the saturation momentum scale, which is the basic quantity characterizing the saturation effects, being related to a critical transverse size for the unitarization of the cross section, and is an increasing function of the energy \( [Q_s^2 = Q_0^2(\ln \frac{x}{Q_0})^\lambda] \). Following Ref. [16], we generalize the IIM model for nuclear collisions assuming the following basic transformations: \( \sigma_0 \to \sigma_0^A = A^{\frac{2}{3}} \times \sigma_0 \) and \( Q_s^2(x) \to Q_s^{2,A} = A^{\frac{2}{3}} \times Q_s^2(x) \). As already emphasized in that reference, more sophisticated generalizations for the nuclear case are possible. However, as our goal in Ref. [18] was to obtain a first estimate of the saturation effects in these processes, our choice was to consider a simplified model which introduces a minimal set of assumptions.

3. Results

We now present a qualitative analysis of the A and energy dependence of the ratio \( \sigma_{dip}/\sigma_{tot} \) using the IIM model generalized for nuclear targets. Following Ref. [4] and assuming that \( \sigma_{dip} \) in the saturation regime can be approximated by \( \sigma_0 \), the transverse part of the inclusive and diffractive cross sections, in the kinematical range where \( Q^2 > Q_s^2 \), can be expressed as

\[ \sigma_T \approx \int_0^{4/Q_s^2} \frac{dr^2}{r^2} \sigma_0 [\frac{r^2 Q_s^2}{4}]^{\lambda_{eff}} + \int_0^{4/Q_s^2} \frac{dr^2}{r^2} \left( \frac{1}{Q_s^2 r^2} \right) \sigma_0 [\frac{r^2 Q_s^2}{4}]^{\lambda_{eff}} + \int_{4/Q_s^2}^{\infty} \frac{dr^2}{r^2} \left( \frac{1}{Q_s^2 r^2} \right) \sigma_0 \tag{3.1} \]
In order to obtain an approximated expression for the ratio we will disregard the $r$-dependence of the effective anomalous dimension, i.e. $\gamma_{\text{eff}} = \gamma = \text{cte}$. In this case, we obtain $\sigma_{\text{diff}} / \sigma_{\text{tot}} \approx \left( \frac{Q^2}{Q_s^2} \right)^{1-\gamma}$. Assuming $\gamma = 0.84$, as in Ref. [20], we predict that the ratio decreases with the photon virtuality and presents a weak energy dependence. However, analyzing the $A$-dependence, we expect a growth of approximately 30% when we increase $A$ from 2 to 197. In the kinematical range where $Q^2 < Q_s^2$ the ratio of cross sections presents a similar behavior. The main difference is that in the asymptotic regime of very large energies the cross section for diffraction reaches the black disk limit of 50% of the total cross section. In Fig. 2 we show the ratio $\sigma_{\text{diff}} / \sigma_{\text{tot}}$ as a function of $W$ for different values of $A$ and $Q^2$. The black disk limit, $\sigma_{\text{diff}} / \sigma_{\text{tot}} = 1/2$, is also presented in the figure. We can see that the ratio depends weakly on $W$ but is strongly suppressed for increasing $Q^2$. This suggests that in the deep perturbative region, diffraction is more suppressed. This same behavior was observed in diffractive $e\,D$ data [19]. Moreover, the energy dependence of the ratio is remarkably flat, increasing with $A$, becoming 37% (30%) larger for gold in comparison to proton (deuteron). The appearance of a large rapidity gap in 37% of all $e\,D$ scattering events would be a striking confirmation of the saturation picture.

In Fig. 3 we show our predictions for the diffractive structure functions $x_F F_0^{D(3)}(x_F, \beta, Q^2)$ as a function of $\beta$ and different nuclei. We can see that the normalization of $x_F F_2^{D(3)}$ is strongly reduced increasing the atomic number. Moreover, although the photon wavefunction determines the general structure of the $\beta$-spectrum [4, 21], the $q\bar{q}g$ component, which dominates the region of small $\beta$, has its behavior modified by saturation effects and changes the behavior of $x_F F_2^{D(3)}$ in this region. Moreover, the diffractive structure function becomes almost flat at intermediate values of $\beta$ and large $A$. Finally, we can observe that another interesting feature of diffraction off nuclear targets emerges, namely, the relative reduction of the $q\bar{q}g$ component with respect to the $q\bar{q}$ one.
4. Summary

Diffractive physics in nuclear DIS experiments could be studied at the electron-ion collider eRHIC. Hence it is interesting to extend the current $ep$ predictions to the corresponding energy and targets which will be available in this collider. In this contribution we have presented a brief review of the main results obtained in Ref. [18], where we addressed nuclear diffractive DIS and compute observable quantities like $\sigma_{\text{diff}}/\sigma_{\text{tot}}$ and $F_2^{D(3)}$ in the dipole picture. In particular, we have investigated the potential of $eA$ collisions as a tool for revealing the details of the saturation regime. Since $\sigma_{\text{diff}}$ is proportional to $\sigma_{\text{dip}}^2$, diffractive processes are expected to be particularly sensitive to saturation effects. Moreover, due to the highly non-trivial $A$ dependence of $\sigma_{\text{dip}}$, diffraction off nuclear targets is even more sensitive to non-linear effects. Using well established definitions of $\sigma_{\text{diff}}$ and $F_2^{D(3)}$ and a recent and successful parametrization of $\sigma_{\text{dip}}$, we have studied observables which may serve as signatures of the Color Glass Condensate. Without adjusting any parameter, we have found that the ratio $\sigma_{\text{diff}}/\sigma_{\text{tot}}$ is a very flat function of the center-of-mass energy $W$, in good agreement with existing HERA data. Extending the calculation to nuclear targets, we have shown that this ratio remains flat and increases with the atomic number. At larger nuclei we predict that approximately 37% of the events observed at eRHIC should be diffractive.

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References


