

## Neutrino oscillations

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We briefly review the history of the neutrino oscillations. With a clear demonstration that neutrinos are not massless, neutrino oscillations are renewing the interest in the historic paper of Ettore Majorana on the nature of the neutrino.

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## 1. The beginnings

Neutrinos first appeared in a letter<sup>1</sup> sent by W. Pauli on Dec. 4, 1930 to his friends, that starts:

Dear radioactive ladies and gentlemen,

I have come upon a desperate way out regarding the 'wrong' statistics of the N- and the Li 6-nuclei, as well as to the continuous  $\beta$  spectrum, in order to save the 'alternation law' of statistics and the energy law.

Pauli discussed his idea with Fermi in 1931, during the Rome conference on nuclear physics, and on this occasion Fermi suggested that the correct name was not "neutron", but "neutrino", more suited to a very light particle. While Pauli readily adopted Fermi's name, Fermi himself kept using the name proposed by Pauli. In his talk[2] at the International Conference on Electricity (Paris, July 1932) he said: "We could think that, according to Pauli's suggestion, the atomic nucleus contains neutrons which are emitted together with the beta particles".

The truth is probably more complicated. The discovery of Chadwick's neutron had been discussed in Rome; Majorana had jumped to the correct conclusion that this particle was essentially a neutral proton, and that nuclei were composed of protons and neutrons. Characteristically he did not feel ready to publish what he considered an unpolished idea, and had not allowed Fermi to discuss this idea in Paris. I will here quote from Segrè's introduction[2] to Fermi's conference:

The neutron had barely been observed, and while there was still uncertainty on the interpretations of the experiments of Bothe and Curie and Joliot, Ettore Majorana ... had immediately understood that there was what he called a "neutral proton". Majorana preceeded then to develop a model of a nucleus built of protons and "neutral protons" only, and proceeded considerably far in the description of the forces between these particles. He told Fermi and several of his friends of this work. ... Fermi asked for permission to report Majorana's results at the Paris conference ... giving him credit for the new ideas. Majorana answered that he would give permission only if the ideas were attributed to an old professor of Electrical Engineering ... for these reasons Majorana's ideas were made known only later, when they had been discovered independently by other physicists.

From Majorana's ideas, and the similar proposal by Heisenberg stems Fermi's theory of beta decay, where the neutrino as we know it appears for the first time[3]. As Rasetti writes, "... the idea of the neutrino had remained up to that time a rather vague hypothesis, while the construction of a formal theory had never been attempted. ... "

In 1937 Ettore Majorana [4] posed the question whether the neutrino is a normal Dirac particle, with its antiparticle, or an intrinsically neutral particle like the photon. Majorana's paper on a "Symmetrical Theory of the electron and positron" was not only a major contribution to neutrino theory but also to the development of quantum field theories, and clarified the relationship, better the lack of it, between Dirac theory and the existence of antiparticles. This paper in fact contains the first clear statement of charge symmetry, a member of the C-P-T triad that has be at the center

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<sup>1</sup>Reproduced in[1], which contains a more extensive discussion of the early history of the neutrino.

of elementary particle physics over the last half century. We recall that Majorana had strongly disliked Dirac's solution to the problem of negative energy states, and had attempted an alternative theory for particles of arbitrary spin[5]. In the 1937 paper he succeeded in getting rid of negative energy states within Dirac's theory, but without any recourse to a negative energy "Dirac's sea". Majorana neutrinos have an essential role in the modern understanding of the origin of neutrino masses, and of neutrino mixing in the frame of unified theories.

The modern developments of neutrino physics, that I will discuss, are in more than one way related to Bruno Pontecorvo. His monumental work on the neutrino starts in 1946 with the proposal[6] of using inverse beta decay processes for establishing the physical existence of the neutrino. In this paper he proposed the radiochemical method for detecting neutrinos, singling out the  $^{37}\text{Cl} \rightarrow ^{37}\text{Ar}$  transition, later used in the Davis solar neutrino experiments, as one of the most promising. In the same paper he discusses the sun, nuclear reactors and material irradiated in reactors as suitable intense neutrino sources<sup>2</sup>. With the experiment by Reines and Cowan[7] the status of the neutrino changed drastically. Not anymore an hypothesis, a mere theoretical construct, but a very solid fact.

It is interesting to note that Pontecorvo's 1946 paper was written in the frame of Majorana's neutrino theory: he proposed the  $^{37}\text{Cl} \rightarrow ^{37}\text{Ar}$  transition not only for solar neutrinos, but also for reactor neutrinos. In Majorana's theory the same kind of neutrino would be emitted in both cases, while in the Dirac case the reactor neutrinos are in fact antineutrinos. In 1955 R. Davis used a chlorine detector to prove that reactor neutrinos are in fact antineutrinos[8].

We must finally remember Pontecorvo's work on neutrino oscillations, which started in 1958 with the proposal[9] of  $\nu \rightarrow \bar{\nu}$  oscillations, seen as the possible analogue of  $K \rightarrow \bar{K}$  oscillations, and described as the mixing of two Majorana neutrinos. In 1967 he returned to the subject and examined[10] the different types of possible oscillations, including  $\nu_e \rightarrow \nu_\mu$ . A substantial part of this paper is devoted to the impact of neutrino oscillations on the solar neutrino experiments then being planned.

Neutrino oscillations have in the following years become the principal interest of Pontecorvo and his collaborators. With V. Gribov[11] he reexamined in 1969 the oscillations of solar neutrinos in view of the early results by Davis, and in 1975 he discussed[12] the analogy between neutrino oscillations and quark mixing. This paper signals a transition to the Standard Model description of neutrino oscillations, and concludes the historical part of my talk.

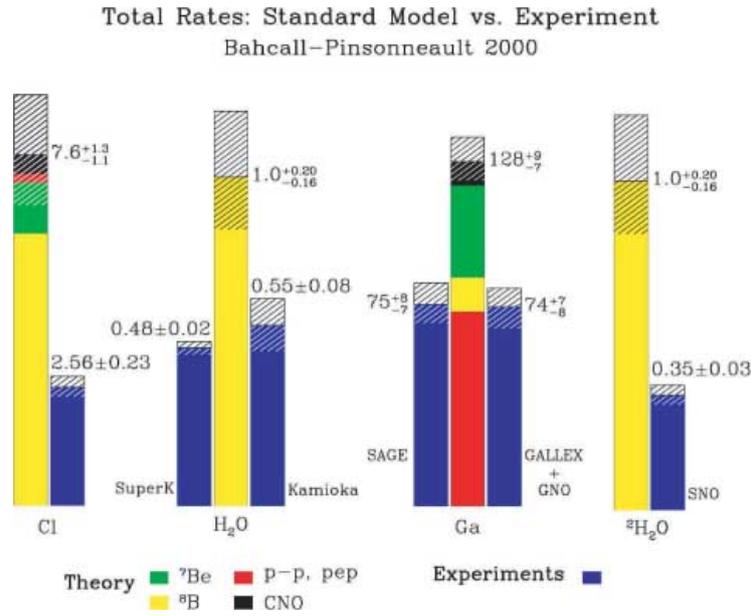
## 2. Solar neutrinos and neutrino oscillations

Up to the end of last century we did not know for certain that neutrino oscillations are a real phenomenon. We suspected it for many years, since the first results of the Homestake Mine solar neutrino experiment led by Ray Davis started to indicate<sup>3</sup> that the observed flux of electron neutrinos from the Sun is smaller than the theoretical expectations. New experiments — Kamiokande and Superkamiokande, Gallex, also in its new reincarnation as GNO, and Sage — and improved

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<sup>2</sup>This paper was classified — in discussing the neutrino fluxes arising from reactors it probably revealed still secret details of their operation — but must have been widely available to the many physicists then engaged in classified research in the United States and Canada.

<sup>3</sup>For recent results of the solar neutrino experiments, see [13].



**Figure 1:** The solar neutrino deficit: is it oscillations?

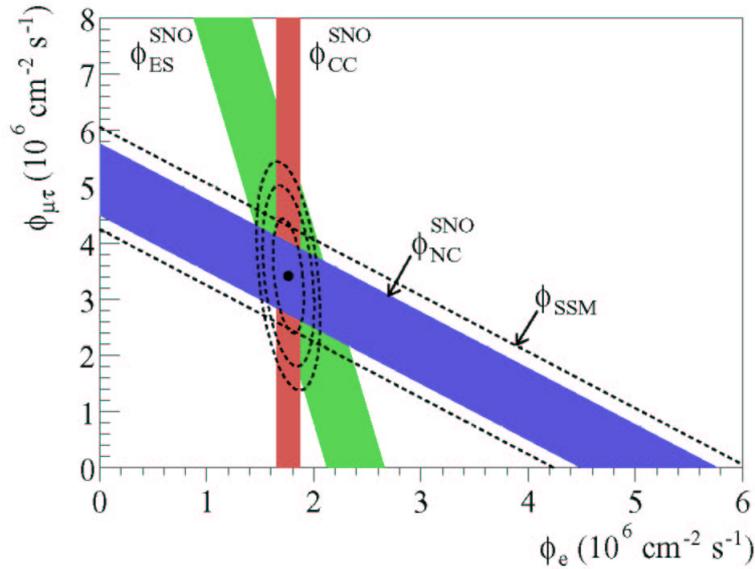
theoretical models of the Sun and its neutrino flux [14] have over the years demonstrated that the “neutrino deficit” (see Fig. 1) is real, and neutrino oscillations, proposed by Bruno Pontecorvo even before the first solar neutrino data were available, offered the simplest interpretation of the experimental data.

For a long time neutrino oscillations were not the only possible explanation. An interesting alternative was the proposal that the neutrino has a magnetic moment which interacts with the powerful magnetic fields in the sun and causes a spin flip, essentially changing a neutrino into an anti-neutrino which would become invisible to many of the solar neutrino experiments. Just after the first Davis results I had even proposed, in a paper [15] with John Bahcall, that massive neutrinos could decay on the way from the Sun to the Earth. This particular proposal was neatly killed by experiments, such as Gallex/Sage, that are sensitive to a wider range of energies than is the case in the chlorine experiment: the deficit should have become much larger instead of smaller at lower energies. Other exotic proposals, such as the oscillation into sterile neutrinos, remained a possibility.

In 2001 the first results of SNO demonstrated that the deficit in solar electron neutrinos is balanced by particles which have the neutral current interactions expected for muon and tau neutrinos. The central idea of the SNO experiment consists in observing three types of reactions, each of which measures a different combination of the two fluxes,

<i>CC</i>	$\nu + \text{Nucleus} \rightarrow \text{Nucleus}' + e^-$	$\Phi_{CC} = \Phi_e$
<i>ES</i>	$\nu + e^- \rightarrow \nu + e^-$	$\Phi_{ES} \approx \Phi_e + 0.14\Phi_{\mu\tau}$
<i>NC</i>	$\nu + D \rightarrow P + N + \nu$	$\Phi_{NC} = \Phi_e + \Phi_{\mu\tau}$

At the Lepton-Photon conference in 2001 SNO presented [16] their first results, a precise determi-

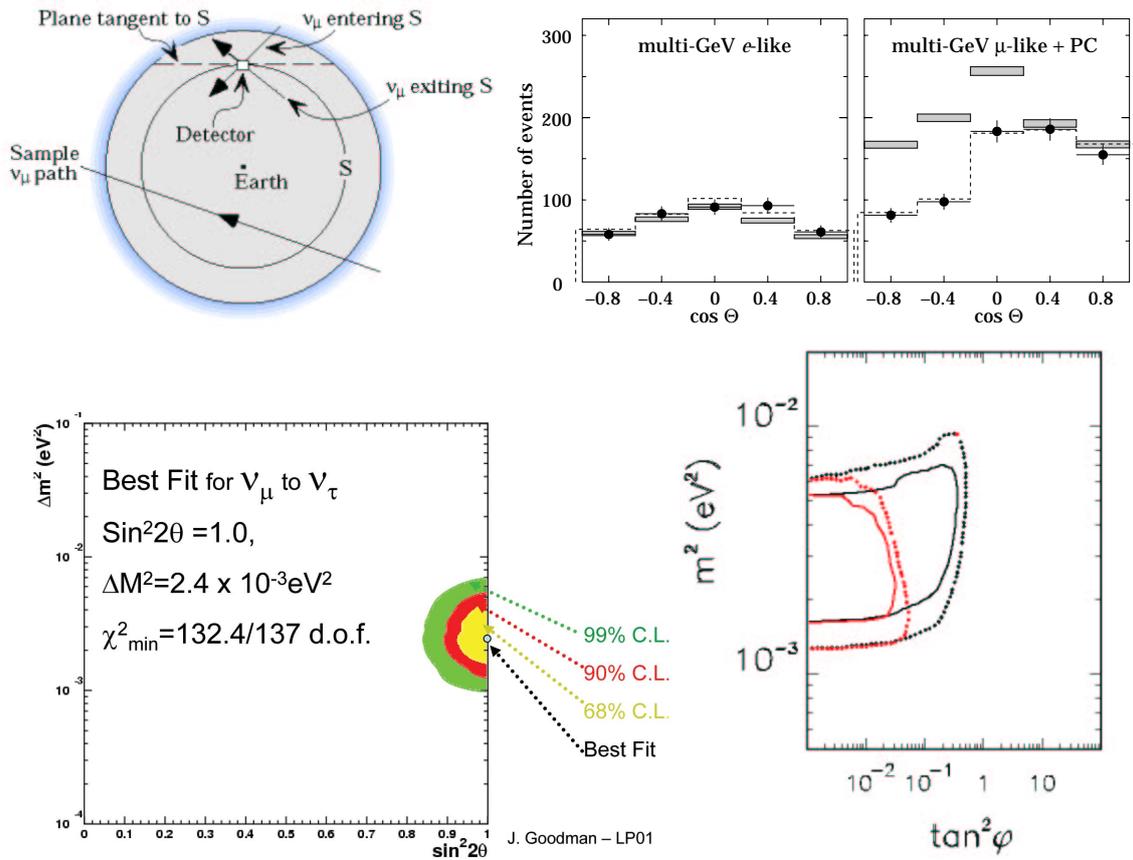


**Figure 2:** The SNO results[17]: the fluxes determined by charged current, neutral current, and electron scattering reactions overdetermine the fluxes of  $\nu_e$  and  $\nu_{\mu,\tau}$ . The result agrees with the total neutrino flux predicted by the Standard Solar Model (dashed band).

nation of the  $\Phi_e$  flux from charged-current (CC) events, shown at the right of Fig. 1. The flux they observed was smaller than that obtained with great precision at Super-Kamiokande from electron scattering (ES), also shown in Fig. 1. If the difference of the two is attributed to the contribution in ES of muon and tau neutrinos one obtains a value for the total flux which is in excellent agreement with the prediction of the current solar models. Since the two experiments are sensitive to neutrinos in the same energy range, they can be safely compared.

The results from the SNO group[17] now cover the three reactions, which are reproduced in Fig. 2 and beautifully converge to a single determination for the pair  $\Phi_e, \Phi_{\mu\tau}$ . The total flux results in excellent agreement with the current solar models,  $\Phi_{\text{SSM}}$ . With no further analysis this result shows that a total neutrino flux compatible with solar models reaches the earth as a mixture of electron and  $\mu - \tau$  neutrinos, as would be expected in the presence of neutrino oscillations.

The SNO results obviously say much more, as we can combine them with the flux measurements from chlorine and gallium experiments to identify the oscillation parameters,  $\Delta m^2, \theta$  which characterize the solar neutrino oscillations. The result nearly exclusively identifies a solution with a relatively large mixing angle,  $\tan^2 \theta \approx 0.35$  and a very small “frequency”,  $\Delta m_{\odot}^2 \approx 10^{-4} eV^2$ . This is fully confirmed by the recent results on reactor electron antineutrinos by the KAMLAND experiment. The KAMLAND values [18] are in excellent agreement with the best fit to the solar neutrino data, but are more accurate. KAMLAND also removed a residual ambiguity of the solar neutrino data which could marginally fit a much smaller value of  $\Delta m^2$ , in the range  $10^{-7} - 10^{-8} eV^2$ .



**Figure 3:** The Super-Kamiokande discovery of  $\nu_\mu$  oscillations. Above-left: Principle of the experiment. Above-right: electron and muon neutrino fluxes at SK (dots) compared with the no-oscillation hypothesis (boxes). Below-left: Two neutrino oscillation fit to the SK  $\nu_\mu$  data. Below-right: Chooz limits to the oscillation of electron neutrino.

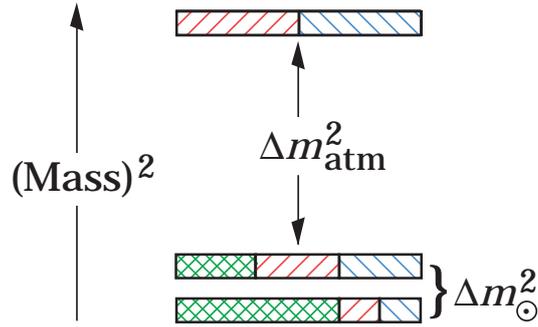
### 3. The oscillation of atmospheric muon neutrinos

A different kind of neutrino oscillation has been discovered at Super-Kamiokande from a study of the angular distribution of high-energy neutrinos produced in the atmosphere by cosmic rays.

The principle of the experiment is illustrated at the upper-left of Fig 3<sup>4</sup>: the isotropy of the incoming cosmic radiation — an excellent approximation for multi-GeV primaries — implies that in the absence of oscillations the flux of neutrinos coming from above at an angle  $\theta$  from the vertical direction should be equal to the flux coming from below at an angle  $180^\circ - \theta$ . The first however have been produced at a short distance, few km, from the detector, while the second have crossed a distance of thousands of kilometers.

The SK results[20] indicate that while the flux of downward moving  $\nu_\mu$  agrees with the predictions based on the cosmic ray measurements, the flux of upward moving  $\nu_\mu$  is substantially lower. Since no effect is seen for electron neutrinos of comparable energy, we must conclude that the effect observed at SK is mainly due to  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations.

<sup>4</sup>The top-left image in this figure and figure 4 are taken from B. Kayser's review of neutrino oscillations in PDG[19].



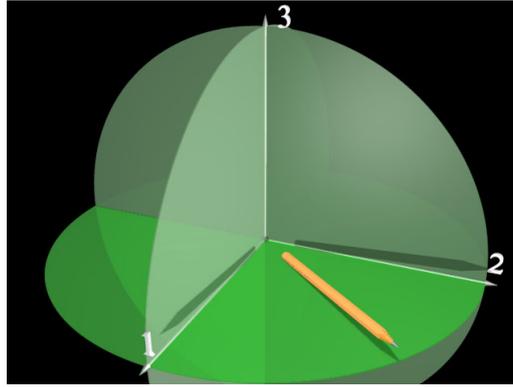
**Figure 4:** A possible neutrino spectrum

The fit indicates a large mixing angle,  $\psi \approx 45^\circ$  and a “frequency”  $\Delta m_{\text{atm}}^2 = 2.4 \cdot 10^{-3} eV^2$ . Super-Kamiokande gives a bound on the oscillations of electron neutrinos at the frequency observed in atmospheric muon neutrinos, but an even better bound was given by reactor experiments, e.g. at Chooz[21], that with a flight-path of 1 Km for MeV antineutrinos have an  $L/E$  ratio which is comparable to that of the atmospheric neutrinos in the SK experiment.

Combining these results we have a first picture of the neutrino spectrum, composed of a close doublet, whose separation  $\Delta m_{\odot}^2 \sim 10^{-4} eV^2$  determines the frequency of solar neutrino oscillations, and a singlet at a distance  $\Delta m_{\text{atm}}^2 = 2.4 \cdot 10^{-3} eV^2$ . A possible spectrum is represented in Fig. 4, borrowed from B. Kayser’s review of neutrino oscillations in PDG[19]. There is a residual ambiguity: the singlet could be the lightest state (an inverted spectrum), instead of the heaviest. This ambiguity can in principle be resolved by future long-baseline accelerator experiments

The configuration in Fig. 4 could appear the most natural, since it mimics the spectrum of charged leptons and that of both  $Q = 2/3$  and  $Q = -1/3$  quarks, where we find two relatively light particles and a much heavier one. We have however no real understanding of these mass patterns, and the neutrino masses could arise through a different mechanism from those of charged particles. An inverted spectrum should not be a-priori considered less probable than a normal one.

The singlet-doublet separation establishes a lower limit,  $m \geq 0.05 eV$  for the *heaviest* neutrino, and puts the neutrino masses in an interesting range which can be explored in future double beta decay experiments. Since the electron neutrino is now known to be mainly a mixture of the two components of the doublet, the detection of neutrino-less double-beta decay is possible on two conditions: the mass-generating mechanism must be of the Majorana type, and the doublet must not be too light. In order for the doublet not to be too light one would either require an inverted spectrum (doublet on top) or the whole triplet must be displaced to higher masses. The detection of neutrino-less double beta decay would establish the Majorana nature of the neutrino mass. Furthermore if long baseline accelerator experiments demonstrate that the spectrum is inverted, a negative double beta decay result could establish the Dirac nature of the neutrino mass. Either result would constitute a major step in our understanding of mass generation, one of the major problem areas in the Standard Model. The less satisfactory situation would be that of a negative result of the future double beta decay experiments combined with a “normal” neutrino spectrum, with the singlet on top, a case where no conclusion could be reached on the Dirac/Majorana alternative unless the



**Figure 5:** The electron neutrino as a mixing of three massive states

sensitivity of double beta decay experiments could be pushed to masses below 0,01 eV.

The recent results point to a situation where neutrinos undergo mixing in much the same way as quarks, as proposed by Bruno Pontecorvo in a two-family framework. Like in the case of quarks, the mixing is described by a unitary matrix of which we have started to pin down some elements, and we have fairly accurate values for two of the mixing angles, both of them large. Although two of the mixing angles are large, the third mixing angle,  $\theta_{13}$ , is known to be small. While Super Kamiokande has observed oscillations of atmospheric muon neutrinos, no similar effect was observed for electron neutrinos, as shown in the center-top graph of Fig. 3. The best limit  $\tan(\theta_{13})^2 \leq 0.02$  was obtained by the Chooz reactor experiment, that with a baseline of  $\sim 1$  Km was sensitive to oscillations at the “atmospheric” frequency  $\Delta m_{\text{atm}}^2$ , but not at the lower “solar” frequency  $\Delta m_{\odot}^2$ .

The situation for the electron neutrino can be visualized as in Fig. 5, which shows  $\nu_e$  as a vector in the space of the three mass states,  $\nu_1, \nu_2$ , the doublet, and  $\nu_3$ , the singlet. The smallness of  $\theta_{13}$  means that the electron neutrino is essentially a mixing of the two states which form the doublet with a small component  $\propto \theta_{13}$  along the singlet. In this situation, as demonstrated by the Chooz and Superkamiokande results, the oscillation of the electron neutrino at the higher frequency  $\Delta m_{\text{atm}}^2$  is suppressed.

#### 4. CP Violation in the lepton sector

Thanks to the recent results we know that two of the mixing angles are large, close to maximal ( $45^\circ$ ) mixing. This is very different from the situation of the quark mixing, which is dominated by powers of  $\lambda = \sin(\theta_C) \approx .022$ , with  $\theta_C$  my original mixing angle.

In 1978 I noted [22] that, as in the case of quark mixing, one expects the mixing matrix to contain complex elements, leading to the possibility of  $CP$  and  $T$  violation effects<sup>5</sup> in neutrino oscillation.

<sup>5</sup>For a review of  $CP$  and  $T$  violation in neutrino oscillations, see[23].

The effects one could observe are

$$\begin{aligned} P(\nu_a \Rightarrow \nu_b) &\neq P(\bar{\nu}_a \Rightarrow \bar{\nu}_b) && CP \text{ violation} \\ P(\nu_a \Rightarrow \nu_b) &\neq P(\nu_b \Rightarrow \nu_a) && T \text{ violation} \end{aligned}$$

while one would not expect a violation of  $CPT$ , so that

$$P(\nu_a \Rightarrow \nu_b) = P(\bar{\nu}_b \Rightarrow \bar{\nu}_a) \quad CPT \text{ theorem}$$

These effects can only appear when one observes the oscillation of one neutrino type into a *different* one, the so-called *appearance* experiments; disappearance experiments are necessarily  $T$  conserving and  $CPT$  symmetry guarantees their  $CP$  invariance. Let me note in passing that in principle  $T$  violation can be demonstrated by the presence of time-odd terms in the oscillation in an “appearance” experiment. In practice the capability of comparing  $\nu$  and  $\bar{\nu}$  beams in the same detector could however prove to be essential for controlling the systematic errors and obtaining a convincing demonstration of  $CP$  violation.

The possibility of observing  $CP$  and  $T$  violation effects depends on the existence of a large phase parameter in the mixing matrix, but also requires that the  $\theta_{13}$  mixing angle, the one on which Chooz gives such a stringent bound, is not too small. In fact the Chooz limit means that these effects will be small. Their observation will require entirely new experiments in a much more controlled setup. Together with the requirement of studying appearance processes, e.g  $\nu_e \leftrightarrow \nu_\mu$  or  $\nu_\mu \leftrightarrow \nu_\tau$ , this means that the new experiments must be based on high-energy neutrino beams on long baselines. The CERN to Gran Sasso neutrino oscillation experiment is the first to explore this entirely new field. In spite of the very exciting progress of the recent years, neutrino oscillation physics is just now moving its first steps.

## 5. The return of the Majorana neutrino

The Davis reactor experiments [8] demonstrated that neutrinos and antineutrinos are different particles, and the interest in Majorana neutrinos waned. With the discovery of parity violation, it became clear that neutrinos are left-handed, and antineutrinos right-handed. A consequence of this discovery was that the original proposal by Pontecorvo [9] of a neutrino-antineutrino oscillation could not be maintained as it clashed with the conservation of angular momentum. The left-handed neutrino and right-handed antineutrino could however be interpreted as the two spin components of a Majorana neutrino, but this interpretation would have been devoid of experimental consequences if the neutrinos were, as was then believed, exactly massless.

Neutrino oscillations imply that neutrinos have a non-zero mass, and this reopens the question of the Majorana vs Dirac nature of the three neutrinos. Massive spin 1/2 particles must have both a left (L) and a right (R) component and two scenarios are possible: in the first (Dirac) each neutrino flavour would have a left component  $\nu_L$  that takes part in weak interactions, and a non-weak-

interacting right component<sup>6</sup>  $n_R$ , thus

$$\text{Dirac neutrino} \quad \{v_L, n_R\} \quad (5.1)$$

$$\text{Dirac antineutrino} \quad \{\bar{n}_R, \bar{v}_L\} \quad (5.2)$$

while in the Majorana case one would have ( $\bar{v}_L$  is *right-handed*)

$$\text{Majorana} \quad \{v_L, \bar{v}_L\} \quad (5.3)$$

Exact conservation of the lepton number remains possible in the Dirac case, while the very presence of mass terms would violate lepton number conservation in the the Majorana case, giving rise, for instance, to the possibility of neutrinoless beta decay, an exciting experimental test which will be discussed by Ettore Fiorini at this conference. From the theoretical point of view Majorana neutrinos are preferred in the sense that even if the  $n_R$  particles required by the Dirac scheme are present, one would still expect the observable neutrinos to behave as Majorana particles through the so-called see-saw mechanism. In fact, since the  $n_R$  particles are invariant (singlets with zero hypercharge) under the Standard Model symmetry group  $SU(3) \times SU(2) \times U(1)$ , nothing would forbid the presence of a mass term  $M$  that directly links the  $n_R$  and  $\bar{n}_R$ . If  $m_D$  is the Dirac-like mass that links  $v_L$  to  $n_R$  (obtained from the usual Higgs mechanism), one would obtain a Majorana-like link from  $v_L$  to  $\bar{v}_L$ ,

$$v_L \xrightarrow{m_D} n_R \xrightarrow{M} \bar{n}_R \xrightarrow{m_D} \bar{v}_L \quad (5.4)$$

that would endow the neutrino with a Majorana mass

$$m_\nu = \frac{m_D^2}{M}. \quad (5.5)$$

Since the value of the Majorana mass  $M$  is not constrained by any of the symmetries of the Standard Model one would expect its value to be very large, thus offering a natural explanation for the smallness of the effective neutrino mass  $m_\nu$ . It is thus clear that the presence of a right-handed  $n_R$  would not alter the final result with respect to the structure of low-lying states: the Majorana neutrino is definitely back!

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<sup>6</sup>It is convenient to use different names to emphasize the difference between the weak interacting  $v_L$  and the non interacting  $n_R$

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