PROCEEDINGS OF SCIENCE

E. Majorana researcher and teacher: the latest achievements

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"In the world there are various categories of scientists: people of secondary or tertiary standing, who do their best but do not go very far. There are also those of high standing, who come to discoveries of great importance. But then there are geniuses like Galileo and Newton. Well, Ettore was one of them. Majorana had what no one else in the world has...". In this talk we try to put some light on this quite unusual statement by Enrico Fermi about Ettore Majorana, by exploring mainly personal notes left unpublished by the great sicilian physicist. Some emphasis is given on recent achievements about Majorana as a research scientist as well as a teacher in Theoretical Physics.

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1. Introduction

Ettore Majorana [1] began his academic career as a student of the Faculty of Engineering at the University of Rome, where he entered in 1923 at the age of 17 and frequented until the beginning of the fifth year. In the spring of 1927 Orso Mario Corbino, the director of the Institute of Physics at Rome, launched a famous appeal to the students of the Faculty of Engineering, where he lectured in General Physics, to entice the most brilliant young minds into studying Physics. Emilio Segrè and his friend Edoardo Amaldi rose to the challenge, joining Enrico Fermi and Franco Rasetti's group and telling them of his friend Ettore's exceptional gifts. After some encouragement from Segrè and Amaldi, Majorana eventually decided to meet Fermi in the autumn of that year.

At that time, Fermi was working on the statistical model of atoms, later to be known as the Thomas-Fermi model, whose main idea is that of considering the electrons around the atomic nucleus as a gas of particles, obeying the Pauli exclusion principle, at the absolute zero of temperature. The model involves a complicated non-linear differential equation, whose analytical solution was then unknown, but Fermi had managed to obtain a numerical table of approximate values for it, in order to have some estimates of the atomic energy levels.

According to Amaldi [2], when Majorana went for the first time in the Fermi's office, "Fermi gave a broad outline of the model and showed some reprints of his recent works on the subject to Majorana, in particular the table showing the numerical values of the so-called Fermi universal potential. Majorana listened with interest and, after having asked for some explanations, left without giving any indication of his thoughts or intentions. The next day, towards the end of the morning, he again came into Fermi's office and asked him without more ado to draw him the table which he had seen for few moments the day before. Holding this table in his hand, he took from his pocket a piece of paper on which he had worked out a similar table at home in the last twenty-four hours, transforming, as far as Segrè remembers, the second-order Thomas-Fermi non-linear differential equation into a Riccati equation, which he had then integrated numerically". Once established the agreement between the two tables, Majorana noted that Fermi's table was correct and left the Institute with no further comment.

What seems nothing more than an amusing anecdote on the switching of Majorana from Engineering to Physics, as recalled by Rasetti, Segrè and Amaldi, has since been carefully tested on scientific grounds by studying previously unpublished study notes [3]. Majorana, in fact, first of all reduced the Fermi equation to an Abel equation (rather than a Riccati equation, as confused by Segrè) by using a peculiar method invented by him, which can be applied to an entire class of mathematical problems [4]. Probably, this was done in order to apply known theorems on the existence and uniqueness of the solution of the Abel equation to the Thomas-Fermi case, thus anticipating by several year some results of renowned mathematicians [3]. However, it is remarkable that none of the Majorana's colleagues and friends was aware of the fact that the precise numerical values for the Fermi function were obtained by Majorana by solving another first-order differential equation, different from the considered Abel one. This is well documented in the Majorana's notebooks [5] and in some other unpublished papers. Majorana, in fact, transformed again the original Thomas-Fermi equation into another first-order differential equation, whose series solution was explicitly obtained in terms of only one quadrature. From this solution Majorana built on the table of numerical values that he compared with the analogous one by Fermi, establishing the accuracy of the given numerical values. It is also remarkable that such an analytic solution of the Thomas-Fermi equation has not been obtained by anyone else since.

As if satisfied that Fermi had passed his "examination", Majorana then decided to leave Engineering and join what became known as the "via Panisperna boys", the Fermi's influential group that was named after the street in Rome where the Physics department was located.

Majorana made substantial theoretical contributions to this group's research, and in 1928 - while still an undergraduate - published his first paper and, at the end of that same year, Fermi invited Majorana to give a talk at the General Meeting of the Italian Physical Society just on some applications of the Thomas-Fermi model. Several other papers followed, accounting for a total of nine published ones, but due to Majorana's peculiar character, which made him averse to publicizing his work, it is unfortunately a recurring feature of his professional life that a very minor part of his work is adequately known by the broader physics community and, in some cases, by even his colleagues and friends.

In the following we concentrate just on the papers left unpublished by Majorana that are currently under study, pointing out the peculiarities of his research in Theoretical Physics and of his teaching.

2. Unpublished researches

As recalled above, the largest part of the Majorana's work was left unpublished. We are now left [6] with his Master thesis on "The quantum theory of radioactive nuclei", 5 notebooks ("Volumetti"), 18 booklets ("Quaderni"), 12 folders with spare papers, and the set of the lecture notes for the course on Theoretical Physics held at the University of Naples.

In the following we report on only few examples of the works performed by Majorana and contained in his unpublished notes.

2.1 Anticipating Feynman Q.E.D.

In one of the sections of Volumetto II [5], Majorana made an attempt to find a relation between the fundamental constants e, h, c. The interest for this paper is not related to the particular mechanical model used by the author (that, indeed, leads to a result ($e^2 \simeq hc$) far from the truth, as noted by Majorana himself), but rather on the interpretation of the electromagnetic interaction in terms of particles exchange. As it is easily recognized, the space around charged particles is quantized, and two electrons interact between them by means of the exchange of particles from one to another. Such an interpretation substantially coincides with that introduced by Feynman in the Quantum Electrodynamics: the space around the charged particles will be identified with the QED vacuum, while the particles exchanged are then assumed to be the photons.

2.2 Anticipating Fano quasi-stationary states

Majorana was the first to study Nuclear Physics in Rome, at least since 1929 when he defended (on 6 July) his Master thesis. However, quite unexpectedly, he continued to research on such topics for several years, till his famous theory of nuclear exchange forces of 1933 [7]. In the study of (α, p) reactions on light nuclei, whose experimental results were interpreted by Chadwick and Gamov, in 1930 Majorana elaborated a dynamical theory (in Volumetto IV, see Ref. [5]) for these processes describing the energy states resulting from the superposition of a continuous spectrum and a discrete level. A complete theory for the artificial disintegration of nuclei by means of α particles (with and without α absorption) was in fact provided by Majorana. He approached the problem by considering the simplest case with an unstable state of the system formed by a nucleus plus an α particle, which spontaneously decays with the emission of an α particle or a proton. The explicit expression for the integrated cross section is explicitly obtained, thus rendering his theory fully accessible to experimental investigation. Note that the peculiarity of the Majorana theory was the introduction in it of quasi-stationary states, which were later (in 1935) considered by U. Fano in a completely different context.

2.3 Anticipating Feynman path integral approach to Quantum Mechanics

In some notes, probably prepared for a seminar at the University of Naples [8], Majorana gave a physical interpretation of Quantum Mechanics which anticipated of several years the Feynman approach in terms of path integral, independently of the underlying mathematical formulation.

The starting point in Majorana's paper was to search for a meaningful and clear formulation of the concept of quantum state. This was achieved by considering some sets of "solutions that differ for the initial conditions" which, in the Feynman language of 1948, corresponded precisely to the different integration paths. In fact, the different initial conditions were, in any case, always referred to the same initial time, while the determined quantum state corresponded to a fixed end time. The crucial point in the Feynman formulation of Quantum Mechanics, namely that of considering not only the paths corresponding to classical trajectories, but all the possible paths joining the initial point with the end one, was introduced in the Majorana manuscript after a discussion on an interesting example of the harmonic oscillator. Quite explicitly, in fact, the author pointed out that the wave function "corresponds in Quantum Mechanics to any possible state of the electron". Such a reference, which only superficially could be interpreted, in the common acceptation, that all the information on the physical systems is contained in the wave function, should instead be considered in the meaning given by Feynman, according to the comprehensive discussion made by Majorana on the concept of state. Finally, we also stress the key role played by the symmetry properties of the physical system in the Majorana analysis; a feature which is quite common in papers of this author.

2.4 Quantizing the Dirac's hole theory

In a recently retrieved text written by Majorana in French [9], the author dealt with a peculiar topic in Quantum Electrodynamics. It is instructive, for this topic, just to quote directly from the Majorana's paper.

"Let us consider a system of p electrons and put the following assumptions: 1) the interaction between the particles is sufficiently small allowing to speak about individual quantum states, so that we may consider that the quantum numbers defining the configuration of the system are good quantum numbers; 2) any electron has a number $n > p$ of inner energetic levels, while any other level has a much greater energy. We deduce that the states of the system as a whole may be divided into two classes. The first one is composed of those configurations for which all the electrons belong to one of the inner states. Instead the second one is formed by those configurations in which at least one electron belongs to a higher level not included in the n levels already mentioned. We will also assume that it is possible, with a sufficiently degree of approximation, to neglect the interaction between the states of the two classes. In other words we will neglect the matrix elements of the energy corresponding to the coupling of different classes, so that we may consider the motion of the p particles in the n inner states, as if only these states exist. Then, our aim is to translate this problem into that of the motion of $n-p$ particles in the same states, such new particles representing the holes, according to the Pauli principle."

Majorana, thus, by following a track left by Heisenberg, applied the formalism of field quantization to the Dirac's hole theory, obtaining the general expression for the Q.E.D. hamiltonian in terms of anticommuting holes quantities. We also point out the peculiar justification of the use of anticommutators for fermionic variables given by Majorana; such use, in fact, "cannot be justified on general grounds, but only by the particular form of the hamiltonian. In fact, we may verify that the equations of motion are satisfied to the best by these last exchange relations rather than by the Heisenberg ones".

2.5 Photon wave function

In the second (and third) part of the same manuscript mentioned just above [9], Majorana also considered a reformulation of Q.E.D. in terms of a photon wave function, a topic which was particularly studied even elsewhere by him [10]. Following Oppenheimer, if the Maxwell theory of electromagnetism should be viewed as the Wave Mechanics of the photon, then it should be possible to write the Maxwell equations as a Dirac-like equation for a suitable wave function. Majorana then reformulated Q.E.D. by introducing a two-component wave function for the photon, corresponding to the only two different polarization states of this particle. The basic postulates of the theory were just the experimental properties of the photon:
"1) the photons move at the speed of light;
2) the energy and the momentum are related each other by the very simple relation: $W = cp$;
3) given the strength and the direction of the momentum, two possible polarization states are present;
4) the photon has a spin that may assume the values $\pm h/2\pi$ along the propagation axis."

3. Teaching theoretical physics

As we have seen, Majorana contributed significantly to some theoretical researches which were considered as the frontier topics in the 1930s. However, his own peculiar contribution ranged also on the basics and the applications of Quantum Mechanics, and Majorana's lectures on Theoretical Physics very effectively give evidence of this contribution. He delivered such lectures in 1938, when he obtained a position as a full professor in Naples (just before he mysteriously disappeared) "for high and well deserved repute, independently of the competition rules". It was the first chair of Theoretical Physics ever established in Italy since the one Fermi obtained in 1927. A recent analysis [11] showed that Majorana's 1938 course was very innovative for that time, and this has been confirmed by the retrieval (on September 2004) of a faithful transcription of the whole set of Majorana's lecture notes (the so-called "Moreno lecture notes") comprising 6 lectures not included in the original collection [12].

PoS(EMC2006)006

The first part of his course on Theoretical Physics dealt with the phenomenology of the atomic physics and its interpretation in the framework of the old quantum theory of Bohr-Sommerfeld. This part presents strict analogy with the course given by Fermi in Rome (1927-28) followed by the student Majorana.

The second part starts, instead, with the classical radiation theory, reporting explicit solutions of the Maxwell equations, scattering of the solar light and some other applications. It then continues with the theory of relativity: after the presentation of the corresponding phenomenology, a complete discussion of the mathematical formalism required by the theory is given, ending with some applications as the relativistic dynamics of the electron. Then a discussion of important effects for the interpretation of Quantum Mechanics, such as the photoelectric effect, the Thomson scattering, the Compton effects and the Franck-Hertz experiment, follows.

The last part of the course, more mathematical in nature, treats explicitly on Quantum Mechanics, both in the Schrödinger and in the Heisenberg formulation. This part does not follow the Fermi approach, but rather refers to previous personal studies by Majorana, also following the original Weyl's book on Group Theory and Quantum Mechanics [11].

As realized only recently, Majorana revealed a genuine interest in advanced Physics teaching starting from 1933, just after that he obtained (at the end of 1932) the professorship degree of "libero docente" (analogous to the German privatdozent). Due to this position, he proposed some academic courses at the University of Rome, as testified by the programs of three courses he would have given between 1933 and 1937. These documents are quite important, since they cover a period of time that has been referred to as Majorana's gloomy years by the testimonies of that epoch. Although Majorana never effectively delivered such three courses, probably due to the lacking of students, they are particularly interesting and informative due to a very careful choice of the topics he would have treated in his courses, which we report in the following.

The first course (academic year 1933-34) proposed by Majorana was that of Mathematical Methods of Quantum Mechanics; the program for it contained the following topics:

1) Unitary geometry. Linear transformations. Hermitian operators. Unitary transformations. Eigenvalues and eigenvectors. 2) Phase space and the quantum of action. Modifications to classical kinematics. General framework of Quantum Mechanics. 3) Hamiltonians which are invariant under a transformation group. Transformations as complex quantities. Non compatible systems. Representations of finite or continuous groups. 4) General elements on abstract groups. Representation theorems. The group of spatial rotations. Symmetric groups of permutations and other finite groups. 5) Properties of the systems endowed with spherical symmetry. Orbital and intrinsic momenta. Theory of the rigid rotator. 6) Systems with identical particles. Fermi and Bose-Einstein statistics. Symmetries of the eigenfunctions in the center-of-mass frames. 7) The Lorentz group and the spinor calculus. Applications to the relativistic theory of the elementary particles.

The second course (academic year 1935-36) was instead that of Mathematical Methods of Atomic Physics and the corresponding arguments are:

Matrix calculus. Phase space and the correspondence principle. Minimal statistical sets or elementary cells. Elements of the quantum dynamics. Statistical theories. General definition of symmetry problems. Representations of groups. Complex atomic spectra. Kinematics of the rigid body. Diatomic and polyatomic molecules. Relativistic theory of the electron and the foundations of electrodynamics. Hyperfine structures and alternating bands. Elements of Nuclear Physics.

Finally, the third course (academic year 1936-37) was that of Quantum Electrodynamics, whose main topics were:

Relativistic theory of the electron. Quantization procedures. Field quantities defined by commutability and anticommutability laws. Their kinematical equivalence with sets with an undetermined number of objects obeying the Bose-Einstein or Fermi statistics, respectively. Dynamical equivalence. Quantization of the Maxwell-Dirac equations. Study of the relativistic invariance. The positive electron and the symmetry of charges. Several applications of the theory. Radiation and scattering processes. Creation and annihilation of opposite charges. Collisions of fast electrons.

As it appears clear from what briefly discussed above, even in the case of "standard" or well-known topics, they were never faced off by Majorana in an obvious way. His writings are still a goldmine of seminal new physical and mathematical ideas and suggestions, all still quite stimulating and useful for present-day research.

4. Conclusion

We end our presentation of few of the unknown works performed by Majorana by quoting again the words by Fermi in describing the peculiar abilities of one of his greatest colleagues in Theoretical Physics:

"Able at the same time to develop audacious hypothesis and criticize acutely his work and that of others; very skilled calculating man, a deep-routed mathematician that never loses the very essence of the physical problem behind the veil of numbers and algorithms, Ettore Majorana has at the highest level that rare collection of abilities which form the theoretical physicist of very first-rank. Indeed, in the few years during which his activity has been carried out, until now, he has been able to outclass the attention of scholars from all over the world, who recognized, in his works, the stamp of one of the greatest mind of our times and the promise of further conquests".

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PoS(EMC2006)006

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