PROCEEDINGS OF SCIENCE

Ettore Majorana: genius and mystery

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The geniality of Ettore Majorana is discussed in the framework of the crucial problems being investigated at the time of his activity. These problems are projected to our present days, where the number of space-time dimensions is no longer four and where the unification of the fundamental forces needs the Majorana particle: neutral, with spin ½ and identical to its antiparticle.

The mystery of the way Majorana disappeared is restricted to few testimonies, while his geniality is open to all eminent physicists of the XXth century, who had the privilege of knowing him, directly or indirectly.

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Ettore Majorana's photograph taken from his university card dated 3rd November 1923.Genius and mystery, a two-component insight into Ettore Majorana: the geniality of his contributions to physics and the mystery of his disappearance.

1. Leonardo Sciascia's idea

This great Sicilian writer was convinced that Ettore Majorana decided to disappear because he foresaw that nuclear forces would lead to nuclear explosives (a million times more powerful than conventional bombs) like those that would destroy Hiroshima and Nagasaki. He came to visit me at Erice where we discussed this topic for several days. I tried to change his mind, but there was no hope. Sciascia was too absorbed by an idea that, for a writer, was simply too appealing. In retrospect, after years of reflection on our meetings, I believe that one of my assertions about Majorana's genius actually corroborated Sciascia's idea. At one point in our conversations I assured Sciascia that it would have been nearly impossible – given the state of physics in those days – for a physicist to foresee that a heavy nucleus could be broken to trigger the chain reaction of nuclear fission. Impossible for what Fermi called first-rank physicists, those who were making important inventions and discoveries, I suggested, but not for geniuses like Ettore Majorana. May be this information convinced Sciascia that his idea about Majorana was not just probable, but actually true. A truth that his disappearance only further corroborated.

There are also those who think that his disappearance was related to spiritual faith, and that Majorana retreated to a monastery. This perspective on Majorana as a believer comes from Monsignor Francesco Riccieri, the confessor of Ettore. I met him when he came from Catania to Trapani as Bishop.

Laura Fermi at the Subnuclear Physics School in Erice (1975), lecturing on her recollections of Ettore Majorana.Remarking on his disappearance, Riccieri told me that Ettore had experienced 'mystical crises' and that in his opinion, suicide in the sea was to be excluded. Bound by the sanctity of confessional, he could tell me no more. After the establishment of the Erice Centre, which bears Majorana's name, I had the privilege of meeting Ettore's entire family. No one ever believed it was suicide. Ettore was an enthusiastic and devout Catholic and, moreover, he withdrew his savings from the bank a week before his disappearance. The hypothesis shared by his family and others who had the privilege to know him (Laura Fermi was one of the few) is that he retired to a monastery.

2. Enrico Fermi: Few others in the world could match Majorana's deep understanding of the Physics of the time

When he disappeared, Enrico Fermi said to his wife: 'Ettore was too intelligent. If he has decided to disappear, no one will be able to find him. Nevertheless, we have to consider all possibilities'; in fact, he even tried to get Mussolini himself to support the search. On that occasion, Fermi said: 'There are several categories of scientists in the world; those of second or third rank do their best but never get very far. Then there is the first rank, those who make important discoveries, fundamental to scientific progress. But then there are the geniuses, like Galilei and Newton. Majorana was one of these.' (Roma 1938).

A genius, however, who looked on his own work as completely banal; once a problem was solved, he did his best to leave no trace of his own brilliance. This can be witnessed in the stories of the 'neutron' discovery (half of our weight comes from neutrons) and the hypothesis of the 'neutrinos' that bear his name; we share below two testimonies, one by Emilio Segré and Giancarlo Wick (on the neutron) and the other by Bruno Pontecorvo (on neutrinos). Majorana's comprehension of the Physics of his time, according to Enrico Fermi, had a profundity and completeness that few others in the world could match.

THE UNEXPECTED 'NEGATIVE ENERGY'

Dirac's equation also came up with the seemingly nonsensical prediction of 'negative energies'.

Only a real genius could transform this catastrophic prediction into a formidable new frontier for Science:

the existence of the antielectron and of the 'Dirac sea'.

The proof of this statement is the content of my attempt to illustrate Majorana's scientific work. In the early thirties of the last century, the great novelty was the Dirac equation, which we will illustrate in Chapter 5. This unexpected equation could finally explain why the electron could not be a scalar particle and had to be a particle with spin ½ (in Planck's units: h), the reason being relativistic invariance. The same equation gave as a consequence of the existence of a particle the existence of its antiparticle, thus generating the 'annihilation', i.e. the destruction of both the particle and its antiparticle. We will see the enormous consequences of this new phenomenon.

Ettore Majorana, in his 1932 paper (see Chapter 7) [Majorana 1932], demonstrated that relativity allows any value for the intrinsic angular momentum of a particle. There is no privilege for spin ½. Concerning the necessity for the existence of the antiparticle state, given the existence of a particle, Majorana discovered [Majorana 1938] that a particle with spin ½ can be identical to its antiparticle. We know today that it is not the privilege of spin ½ particles to have their antiparticle and that relativity allows any value for the spin. However, for the physicists of the time, these were topics of great concern.

The Dirac equation was the starting point of the most elaborated description of all electromagnetic phenomena, now called quantum electrodynamics (QED). We also know that

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the fundamental particles are of two types: spin ½ and spin 1. The spin ½ particle (quarks and leptons) are the building blocks of our world. The spin 1 particles are the 'glues', i.e. the quanta of the gauge fields. We do understand the reason why the gauge fields must have spin 1: this is because the fundamental forces of nature originate from a basic principle called local gauge invariance. This principle dictates that the energy density must remain the same if we change something in the mathematical structure that describes the given fundamental forces of nature. For example, if the mathematical structure of the given force is described by a group such as SU(3) (this is the case for the strong force acting between quarks and gluons; the number 3 refers to the number of complex 'intrinsic' dimensions where the group exists) we can operate changes obeying the mathematics of the group SU(3), and the physics must remain the same. By requiring that the physics must remain the same for changes in other 'intrinsic' dimensions, 2, and 1, where other symmetry groups SU(2) and U(1) exist, we get the weak and the electromagnetic forces. It took three quarters of a century to discover that these two forces originate from a mixing between the SU(2) and the U(1) gauge forces. The changes in the 'intrinsic' dimensions 3, 2, 1 can be made at any point in space-time; this is the meaning of 'local' in the gauge invariance. This locality produces the spin 1 for the quantum of the three gauge forces SU(3), SU(2) and U(1), and spin 2 for the gravitational force, because here the 'gauge' invariance refers to the Poincaré symmetry group, which exists in Lorentz space-time dimensions, not in the 'intrinsic' dimensions where the symmetry groups SU(3), SU(2) and U(1) exist. Since all fundamental forces (electromagnetic, weak, strong and gravitational) originate from a local gauge invariance, we understand why the quanta of these forces must have spin 1 and 2. The reason why the building blocks are all with spin ½ remains to be understood.

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THE GOLD MINE OF THE DIRAC EQUATION WAS
THE GREAT CONCERN OF ETTORE MAJORANA

I once had the privilege of speaking to the great Russian physicist Piotr Kapitza, who was at Cambridge with Dirac, where they were both pupils of Rutherford. Every week the pair would attend a lecture. '*No matter what the topic of the seminar*,' Kapitza told me, '*at the end of the lecture I would always address the same question to Dirac: "Paul, where is the antielectron?".*' Kapitza was a great friend of Dirac, but he remained convinced that his equation was only creating trouble. His comments are a reminder that no one at the time took Dirac's equation seriously. No one suspected what a gold mine the equation would turn out to be.

But the problems posed by this equation were of fundamental importance. They were: What is the origin of spin ½ ? Do antiparticles need to exist for all types of spin ½ particles? Can a spin ½ particle exist and be identical to its antiparticle? Can a particle of any spin have its own antiparticle?

These were the problems of which Ettore Majorana wanted to understand how they could be connected to the foundations of physics.

What Majorana proved about the Dirac equation was correct: neither the spin of the electron nor the existence of its antiparticle was a 'privilege' of spin ½ particles. In fact there is no single particle relativistic quantum theory of the sort which Dirac initially was looking for. The combination of relativity and quantum mechanics inevitably leads to theories with unlimited numbers of particles. We do not know why the Standard Model needs only spin ½ and spin 1 particles, plus spin 0 particles associated with imaginary masses. But we know that the Dirac equation led physics to discover that a particle can annihilate with its own antiparticle, thus 'annihilation' must exist. In fact the existence of the antielectron (positron) implies that an electron can annihilate with a positron, with the result that their mass-energy becomes a high energy photon, governed by QED. This photon can also transform into a pair of electron-

Victor F. Weisskopf with Antonino Zichichi (1960).

positron, still governed by QED. But now think of a photon that can also transform into a 'particle–antiparticle' such as quark–antiquark or lepton–antilepton, or (W+W-) pairs. Quark–antiquark pairs are governed by the laws of strong forces, QCD (quantum chromodynamics), by QED and by the laws of the weak forces QFD (quantum flavour dynamics); (W+W-) and lepton–antilepton pairs are governed by the laws of QED and QFD. Each of these pairs can annihilate and form a photon again. The annihilation process allows these three forces, QED, QCD and QFD, to be present in the virtual effects. Without the existence of 'annihilation' these processes could not occur, and the problem of the renormalization of the gauge forces (with or without spontaneous symmetry breaking) would never have been conceived.

Had the renormalization problem not been solved – as was the case in the early 1970s, by the 1999 Nobel prize winners Gerard 't Hooft and Martinus Veltman – we would not have the Standard Model, with its many precise quantitative predictions that have been experimentally validated in labs all over the world. The roots of the Standard Model are in the Dirac equation. Majorana was fascinated by the 'annihilation', but he could not agree with the physics foundations that were at the origin of the 'privileged' spin ½ particles. Let me emphasize the importance of the concept of 'annihilation' in the development of modern Physics.

In fact, the existence of the antielectron (or 'positron' as it has become known) implied that when a particle (of any type) collided with its antiparticle they would annihilate each other, releasing their rest-mass energy as high-energy photons (or other gauge bosons). In the case of a process described purely by QED, a gamma-ray photon can create an electron–positron pair,

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which can transform itself back into a photon. This process, which is called ‘vacuum polarization’, was the first virtual effect to have been theoretically predicted.

The first physicist to compute the vacuum-polarization effects in the hydrogen atom was Victor Weisskopf. He predicted that the 2p½ level in a hydrogen atom should be very slightly higher in energy than the 2s½ level, by some 17 MHz. However, in 1947, Willis Lamb and Robert Retherford discovered that the 2p½ level was in fact lower than the 2s½ level by some

THE NEGATIVE ENERGY AND THE
‘HOLES’ IN THE DIRAC SEA

We now know that ‘negative’ is the energy needed to make an antielectron (positron) and that ‘positive’ energy is what is gained when an electron is destroyed (by annihilation).

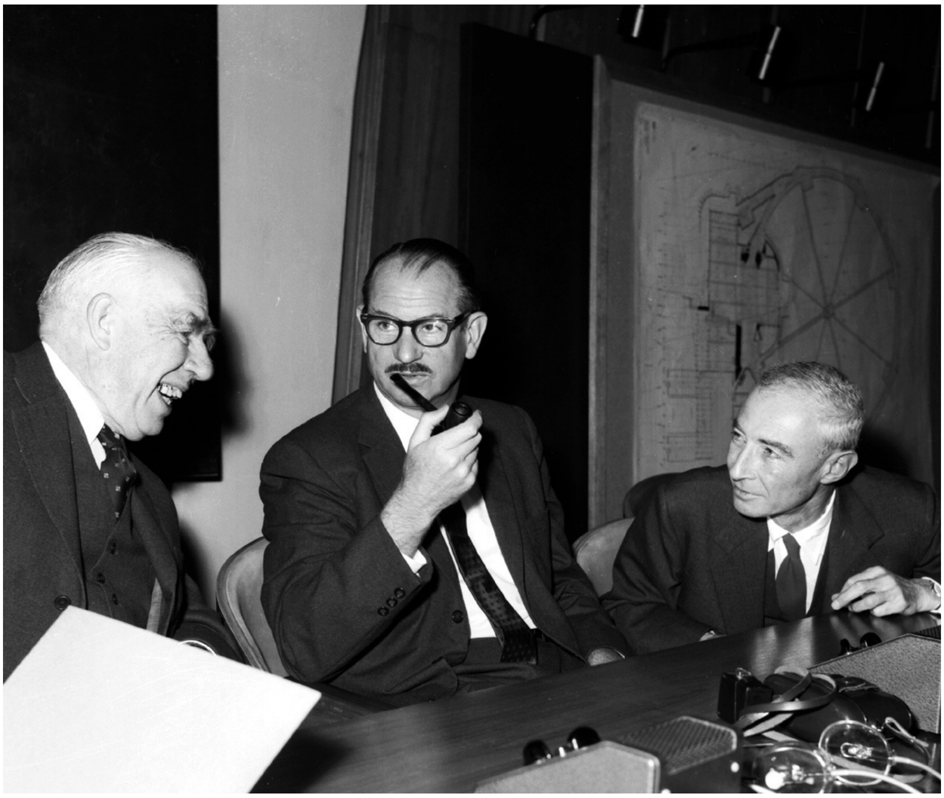
We also know that the ‘Dirac sea’ and its ‘holes’ are of fundamental importance in the science of materials. Transistors and solid-state diodes are the result of clever manipulations of ‘holes’ and electron densities at junctions between different materials. The technology known as PET (positron–electron tomography), LED (light-emitting diodes), solid-state lasers, could not exist without the Dirac sea and its ‘holes’.

1000 ±100 MHz. It was this experimental discovery, now called the Lamb shift, that prompted all theorists, including Weisskopf, Hans Bethe, Julian Schwinger and Richard Feynman, to compute the very simple radiative process in which an electron emits and then absorbs a photon. The ‘vacuum polarization’ is not as simple. Nevertheless, had it not been for the discovery of the positron – and therefore the existence of electron–positron pairs and of their annihilation – no one would have imagined that such simple virtual effects as the one producing the ‘Lamb shift’, could exist in nature. And without ‘virtual effects’, the gauge couplings would not change with energy (in physics jargon this is called ‘running’), no correlation could exist between the different forces and, ultimately, no grand unification of all the fundamental forces and no Standard Model. Of course – and fortunately for us – there are sound reasons to believe that there is a lot of new physics beyond the Standard Model.

The conclusion is that Majorana was right: the electron spin ½ was not a consequence of relativistic invariance, and the concept of antiparticle was not the privilege of spin ½ particles. Nevertheless it is the conceptual existence of particle–antiparticle pairs that sparked the new process called ‘annihilation’, with its far-reaching consequences, which led physics to the Standard Model and Beyond. This took three quarters of a century to be achieved, but it did not start as an equation deprived of immediate successes. Using his equation, Dirac was able to compute the ‘fine structure’ of the hydrogen atom, i.e. the very small energy difference between states that differ only in their total angular momentum, in excellent agreement with experimental data. We will see in Chapter 5 that Dirac was able to show that the gyromagnetic ratio, the famous g factor, had to be 2, as wanted by the experimental data.

The discovery of the antielectron came as a totally unexpected blessing to the ‘prediction’ of the ‘hole’ in the ‘Dirac sea’, with all consequences on positive and negative energy solutions of the Dirac equation. Despite these formidable successes, we now know that there is no relativistic quantum theory of a single particle the sort that Dirac was looking for initially. The combination of relativity and quantum mechanics inevitably leads to theories with unlimited numbers of particles. In such theories, the ‘true dynamical variables’ on which the wave function depends are not the position of one particle or several particles, but ‘fields’, like the electromagnetic field of Maxwell. Particles are quanta – bundles of energy and momentum – of these fields.

Enrico Fermi and Isidor I. Rabi (1952). Isidor Rabi played an essential role in the establishment of the Ettore Majorana Foundation and Centre for Scientific Culture (EMFCSC).



Robert Oppenheimer (right) at CERN with Ed McMillan and Niels Bohr for the inauguration of the Proton Synchrotron (5 February 1960).

A photon is a quantum of the electromagnetic field, with spin 1, while an electron is a quantum of the electron field, with spin ½. So why did Dirac's equation work so well? Because the equation for the 'electron field operator' is mathematically the 'same' as Dirac's equation for the 'wave function'. Therefore the results of the calculation turn out to be the same as Dirac's. This does not diminish the value of Dirac's impact on the development of new physics. Let me just mention an example related to the group where Majorana was working.

In 1932 Enrico Fermi constructed a theory of radiative decay (beta decays), including the neutron decay, by exporting the concepts of QFT far from their origin. Neutron decay corresponds to the destruction of a neutron with the creation of a proton, plus a pair of an electron and an antineutrino. Thus, there exist processes which involve the creation and destruction of protons, neutrons, electrons and neutrinos. Since destruction of a particle means creation of an antiparticle, and destruction of an antiparticle means creation of a particle, none of these processes could have been imagined without the existence of 'annihilation' between a particle and its antiparticle.

To sum up, the 'annihilation' was the seed for 'virtual' physics, the 'running' of the gauge couplings, the correlation between the fundamental forces and their 'unification': in other words, this totally unexpected phenomenon, born with the discovery of the Dirac equation, led physics to the triumph of the Standard Model. Majorana's papers [Majorana 32 and 37] were both in the 'turmoil' of these fundamental developments. Memories of this man had nearly faded when, in 1962, the International School of Physics was established in Geneva, with a branch in Erice. It was the first of the one hundred and twenty schools that now compose the

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Centre for Scientific Culture that bears Majorana's name. The next testimony we turn to is that of an illustrious exponent of XXth century Physics, Robert Oppenheimer.

3. Recollections by Robert Oppenheimer

After suffering heavy repercussions of his opposition to the development of weapons even stronger than those that destroyed Hiroshima and Nagasaki, Oppenheimer decided to get back to physics by visiting the biggest laboratories at the frontiers of scientific knowledge. This is how he came to CERN, the largest European Laboratory for Subnuclear Physics. At a ceremony organized for the presentation of the Erice School dedicated to Ettore Majorana, many illustrious physicists participated. I myself – at the time very young – was entrusted the task of speaking about the Majorana neutrinos. Oppenheimer wanted to voice his appreciation for how the Erice School and the Centre for Scientific Culture had been named. He knew the exceptional contributions Majorana made to physics from the papers he had read. This much, any physicist could do at any time. What would have remained unknown is the episode he told me as a testimony of Fermi's exceptional esteem of '*Ettore*'. He recounted the following episode from the time when the Manhattan Project was being carried out. The Project, in the course of just four years, transformed the scientific discovery of nuclear fission [heavy atomic nuclei can be broken to produce enormous quantities of energy] into a weapon of war. There were three critical turning points during this project. During the executive meeting convened to address the first of these crises, Fermi turned to Wigner and said: 'If only Ettore were here'. The Project

Robert Oppenheimer during an interview at the CERN Library (July 1962).

Giancarlo Wick at Erice (1971).

The 1971 Ettore Majorana International Physics Prize was awarded to Professor Giancarlo Wick, with the following motivation:

‘*Professor Giancarlo Wick is one of the truly outstanding theoretical physicists. His contributions to quantum field theory and scattering theory are both basic and extensive, they have become foundations of these two vast and fruitful areas of research. The Wick theorem and the Wick product are common vocabulary in today's literature, not only in high-energy physics but in solid-state physics and many-body problems as well. His very recent work on a finite theory of quantum electrodynamics is again of fundamental importance*’.

The prize was presented to Professor Giancarlo Wick by the Mayor of Erice. During the official ceremony, held in Erice on the 14th of July 1971, a concert was given by the Sicilian Philharmonic Orchestra, conducted by Ottavio Ziino, in honour of Professor Giancarlo Wick.

seemed to have reached a dead end in the second crisis, during which Fermi exclaimed once more: ‘This calls for Ettore!’. Other than the Project Director himself (Oppenheimer), three people were in attendance at these meetings: two scientists (Fermi and Wigner) and a general of the US armed forces. Wigner worked with nuclear forces, like Ettore Majorana. After the ‘top-secret’ meeting, the general asked the great Professor Wigner who this ‘Ettore’ was, and Wigner replied: ‘*Majorana*’. The general asked where Ettore was, so that he could try to bring him to America. Wigner replied: ‘*Unfortunately, he disappeared many years ago*’.

4. The discovery of the neutron – Recollections by Emilio Segré and Giancarlo Wick

And now a testimony by Emilio Segré and Giancarlo Wick on the discovery of that omnipresent particle: the neutron. By the end of the second decade of last century, Physics had identified three fundamental particles: the photon (quantum of light), the electron (needed to make atoms) and the proton (essential component of the atomic nucleus). These three particles alone, however, left the atomic nucleus shrouded in *mystery*: no one could understand how multiple protons could stick together in a single atomic nucleus. Every proton has an electric charge, and like charges push away from one another. A fourth particle was needed, heavy like the proton but without electric charge, the neutron, and a new force had to exist, the nuclear force, acting between protons and neutrons. But no one knew this yet. Here we will try to explain, in simple terms, what was known in that era about particles, which we present as 'things'.

Only three types exist: doves (photons), motorcycles (electrons) and trucks (protons). The doves – in our example – are white, the motorcycles red and the trucks green. We are substituting 'colour' for electrical charge. Protons are electrically charged (green trucks) with a sign opposite to that of the electrons (red motorcycles). Photons are neutral (white doves). A single dove, even flying at very high velocity, could never move a parked truck. It would require a second truck in motion to move a stationary one. As we know, doves weigh very little, motorcycles are fairly light (relative to trucks), and trucks are very heavy. If a truck is moved from its parking space, then something must have moved it. This is what Frédéric Joliot and Irène Curie discovered. A neutral particle enters matter and expels a proton. Since the 'thing' that enters into matter has no colour, their conclusion was that it must necessarily be a dove,

The President of the Italian Physical Society, Franco Bassani (left), the President of the Associazione Ex-Alumni del Convitto Nazionale 'Amedeo di Savoia', Hon. Francesco Nitto Palma, A. Zichichi and the Mayor of Tivoli.

The inauguration of a bronze dedicated to Emilio Segré (Tivoli, 2003).

because it is the only known 'thing' that has no colour. Ettore Majorana had a different explanation, as Emilio Segré and Giancarlo Wick recounted on different occasions, including during their visits to Erice. Segré and Wick were enthusiasts for what the School and the Centre had become in only a few years, all under the name of the young physicist that Fermi considered a genius alongside Galilei and Newton. Majorana explained to Fermi why that particle had to be as heavy as a proton, even while electrically neutral.

To move a truck from its parking space requires something as heavy as the truck itself. Not a dove, which is far too light, and not a motorcycle because it has a colour. It must be a truck with no colour; white like the doves, but heavy like the green trucks. A fourth 'thing', therefore, must exist: a white truck.

So was born the correct interpretation of what the Joliot-Curie discovered in France: the existence of a particle that is as heavy as a proton but without electric charge. This particle is the indispensable neutron. Without neutrons, atomic nuclei could not exist.

Fermi told Majorana to publish his interpretation of the French discovery right away. Ettore, true to his belief that everything that can be understood is banal, did not bother to do so. The discovery of the 'neutron' is in fact justly attributed to Chadwick (1932) for his beryllium experiments.

Next we turn to the testimony of Bruno Pontecorvo on the neutrinos of Majorana.

Bruno Pontecorvo talking with Antonino Zichichi in Rome (September 1978).

5. The Majorana 'Neutrinos' – Recollections by Bruno Pontecorvo – The Majorana Discovery on the Dirac γ-matrices

Today, Majorana is particularly well known for his ideas about neutrinos. Bruno Pontecorvo, the 'father' of neutrino oscillations, recalls the origin of Majorana neutrinos in the following way: Dirac discovers his famous equation describing the evolution of the electron (in our body there are billions and billions of 'electrons').

Majorana goes to Fermi to point out a fundamental detail: 'I have found a representation where all Dirac γ-matrices are real. In this representation it is possible to have a real spinor, which describes a particle identical to its antiparticle'.

This means that neutrino and antineutrino are identical particles. The starting point is the Dirac equation, which, as we will see later, corresponds to a system of four coupled differential equations. How these equations are related each other is described by the so-called γ-matrices whose 'representation' reported on page 39 was found by Dirac.

This 'representation' is responsible for the existence of the antiparticle property, once the particle is given. Majorana discovered that the γ-matrices could have another totally different 'representation' where these matrices are all 'real'. The consequences are remarkable, since, in this case, we have a spin ½ particle identical to its antiparticle.

For the benefit of the reader we report here the Majorana discovery about his γ-matrix representation.

$$\gamma^0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

THE TOTALLY UNEXPECTED PROPERTY OF SPACE-TIME

In the 1920s the young English physicist Paul Dirac began trying to understand and describe the space-time evolution of the electron, the first elementary particle discovered by J.J. Thomson in 1897.

Dirac was fascinated by an unprecedented property of space-time, discovered by Lorentz in his studies of electromagnetic forces, whereby if space was real, time had to be imaginary, and vice versa.

In other words, space and time had to be a ‘complex’ mixture of two quantities, one real and the other imaginary.

In order to understand the value of Majorana's discovery concerning particles with mass, spin ½, but zero charge, it is necessary to know the deep meaning of the Dirac equation, which is shown in a synthetic form in the Dirac Lecture Hall at the Blackett Institute in Erice.

Let me say a few words of introduction.

During the past decades, thousands of scientists have been in the Dirac Lecture Hall. Very many fellows have repeatedly asked me the same question.

Question: Why in the Aula Magna is there the Dirac equation, and not Einstein's:

$$E = mc^2 \ ?$$

Answer: Because the Dirac equation

$$(i \partial \ + m) \psi = 0 \qquad (1)$$

is the one I like most.

Its origin, its consequences, its impact on human intelligence overpass everybody's imagination, as I will try to explain.

The origin. Dirac was fascinated by the discovery of Lorentz who found that the electromagnetic phenomena, described by the four Maxwell equations, obey an incredible invariance law, now called Lorentz invariance.

The key feature of this invariance is related to a basic property of space and time: if we choose the space to be real, the time must be imaginary, and vice versa.

Contrary to what Kant had imagined, space and time cannot be both 'real' and 'absolute'. The 'absolute' quantities, called 'relativistic invariants' can either be 'space-like' or 'time-like'.

The world we are familiar with is a 'time-like' world, where the sequence of past and future remains the same: no matter the motion of the observer, Napoleon will come after Caesar. There is also a 'space-like' world, where the sequence of past and future, including the simultaneity of events, depend on the observer.

THE UNPRECEDENTED NOVELTY OF THE DIRAC EQUATION

The unprecedented novelty of the Dirac formalism was the introduction of the spinor, which is a mathematical function that has four components. Imagine you want to move in space-time with a bicycle: you need two wheels.

However, you could also move using a unicycle, as an acrobat would do. Similarly, before Dirac came along, mathematics used only one ‘function’ to describe a particle: a scalar function. Dirac’s claim was that to describe an electron, you need a mathematical object made of four components.

In our daily life this would be like saying that to evolve in space-time we need a car with four wheels, not a unicycle with just one.

The old belief that space and time are totally independent is over. No one can isolate space from time.

Whatever happens in the world, it is described by a sequence of **space-time events**.

Not of space and time but of space-time, united and inseparable.

The young Dirac realized that no one had been able to describe the evolution of the first example of ‘elementary particle’, the electron (discovered by J.J. Thomson in 1897), in such a way as to obey the Lorentz condition, i.e. space and time united and drastically different: one real, the other imaginary.

The most successful description of the evolution of the electron in space and time was the celebrated Schrödinger equation, where the charge e, the electromagnetic potential A_{μ}, the mass m, the derivative with respect to the space coordinate

$$\frac{\P}{\P x}$$

and with respect to time,

$$\frac{\P}{\P t},$$

were all present, including the concept of ‘wave function’ whose square was the ‘probability’ for the ‘electron’ to be in a given configuration state. The Lorentz invariance was not there.

The Schrödinger equation describes the evolution in space and time of a numerical quantity, called 'wave function', whose square at any position and time gives the probability, at that time, of finding a particle at that location in space.

How the 'wave function' changes with time and space are not treated in the same way. The rate of change with position is controlled by a second-order derivative, i.e. the rate of change with position of the rate of change of the wave function with position.

But the rate of change with time, of the same function, is computed at the first order, i.e. the rate of change of the wave function with time. The second order would be to compute the rate of change with time of the rate of change of the wave function with time.

These two ways of describing the evolution of the wave function in time (first order) and in space (second order) was in conflict with the condition of putting space and time in a perfectly symmetric way, as requested by relativistic invariance.

Dirac knew that there was an equation, which described the evolution in space and time of a wave function, where the derivatives versus time and space were both of second order. In this equation, discovered by Klein and Gordon, space and time were treated in a symmetric way, as requested by relativity. But the Klein–Gordon equation gave positive and negative probabilities, negative probability being nonsense.

In 1934, this difficulty was shown by Pauli and Weisskopf [Pauli and Weisskopf 1934] to be overcome, since the Klein–Gordon 'wave function' ϕ should not be treated as a 'wave function' describing a single particle, but as an operator in a field equation describing a field of relativistic massive particles having positive and negative electric charges.

Pauli and Weisskopf concluded that positive and negative values should not be attributed to probabilities, but to the net charge densities at any point in space-time.

Victor F. Weisskopf lecturing at Erice (1970).

Let us return to Dirac and his struggle to overcome the difficulties existing with the Schrödinger and Klein–Gordon equations.

ONE OF THE BASIC REASONS WHY THE DIRAC EQUATION ATTRACTED EVERYBODY'S ATTENTION

The study of hydrogen spectra in the 1920s revealed that an electron not only has an orbital angular momentum related to its motion about a nucleus, but also an intrinsic angular momentum or 'spin'.

But where did this spin come from? Why was the spin of the electron only half of the minimum value measured from atomic spectra?

Dirac equation gave the correct answer: the electron must have spin ½h.

Dirac wanted an equation where time and space were treated in a symmetric way, at the first order in the derivative, and obeying the principle that the probability must be positive. Once all the conditions were fulfilled, Dirac discovered that the particle needs an intrinsic angular momentum of ½ in units of Planck's constant.

The two equations existing before Dirac [Schrödinger (can be extended to have spin, but remains non relativistic) and Klein–Gordon (relativistic but no spin)] were both having problems.

And the big question was to understand why the electron was not a scalar particle.

The Dirac equation corresponds to four coupled equations.

Once Lorentz invariance is imposed, the result is that in order to describe the evolution in space-time of the electron, you need four coupled equations.

The Dirac equation (1) corresponds to the following set of equations

$$\begin{pmatrix} i\partial_0+m & 0 & -i(\partial_1+\partial_3) & \partial_2 \\ 0 & i\partial_0+m & -\partial_2 & -i(\partial_1-\partial_3) \\ i(\partial_1-\partial_3) & -\partial_3 & -i\partial_0+m & 0 \\ \partial_2 & i(\partial_1-\partial_3) & 0 & -i\partial_0+m \end{pmatrix} \begin{pmatrix} \psi_e\uparrow(x) \\ \psi_e\downarrow(x) \\ \psi_p\uparrow(x) \\ \psi_p\downarrow(x) \end{pmatrix} = 0 \ ;$$

the wave function that appears in equation (1),

$$\psi\,(x),$$

is made up of four components, and the electron cannot be a scalar particle: it must be a particle with spin ½ . In the four pieces of ψ (x),

$$\psi(x) \equiv \begin{pmatrix} \psi_{e^-}\uparrow(x) \\ \psi_{e^-}\downarrow(x) \\ \psi_{e^+}\uparrow(x) \\ \psi_{e^+}\downarrow(x) \end{pmatrix},$$

each component is a function whose values depend on space and time, as indicated by the

ANOTHER MOTIVATION FOR THE SUCCESS
OF THE DIRAC EQUATION

Why was the 'g factor', i.e. the 'gyromagnetic ratio' of the electron – the magnetic moment divided by its angular momentum –, twice as large as the same ratio measured when the same electron was orbiting around the atomic nucleus? Why was there such a difference in the magnetic field produced when the 'electric charge' is rotating in an orbit (angular momentum) and when it is rotating around an intrinsic axis (spin)?

Dirac equation gave the correct answer:

the electron g factor is 2, not 1.

argument (x). The four components correspond to the following four possible states: electron with spin up, $\psi_{e^-}\uparrow(x)$; electron with spin down, $\psi_{e^-}\downarrow(x)$; positron with spin up, $\psi_{e^+}\uparrow(x)$; positron with spin down $\psi_{e^+}\downarrow(x)$. The totally unexpected result was the need for the existence of the electron antiparticle, called positron, e^+: a particle with the same mass, same spin, but opposite electric charge. This 'antiparticle' had no experimental support. But in favour of Dirac there was another property of the electron. The study of atomic spectra was giving experimental results indicating that the electron, in addition to its spin, has another unexpected property. The electron behaved as if it was a tiny magnet. The magnetic properties required the electron to be like a spinning sphere, but it had to rotate at an extraordinarily rapid rate. So rapid that at its surface the rotation corresponded to a speed higher than that of light.

The model of the spinning electron had been worked out by two Dutch graduate students, Samuel Goudsmit and George Uhlenbeck, who wanted to explain the experimental data of atomic spectra.

Eminent physicists were sceptical about this model, and Wolfgang Pauli tried to dissuade them from publishing their paper since the model they proposed had a quantitative mismatch in the gyromagnetic ratio, the so-called g factor, i.e. the ratio of the magnetic moment divided by the angular momentum.

The electron orbiting around a nucleus has an angular momentum. The same electron, since it is electrically charged, in its orbital motion produces a magnetic field. The ratio of this magnetic field to the angular momentum corresponds to the value $g = 1$. The problem was to understand why intrinsic rotation (spin) produces a magnetic field that is twice stronger than the one produced by the same electron when it is orbiting in an atom: this is the meaning of $g = 2$ and $g = 1$, respectively. In order to agree with the results from atomic spectra, Goudsmit and Uhlenbeck postulated $g = 2$.

THE 'FINE STRUCTURE' IN EXCELLENT AGREEMENT WITH EXPERIMENTAL DATA

Using its equation (2), Dirac was able to compute the very small energy difference that there exists between atomic hydrogen states that differ only in their total angular momentum; this is called the 'fine structure' of the hydrogen atom. Dirac's calculation was in excellent agreement with the experimental data.

The Dirac equation (2) is the starting point of Quantum Electrodynamics, one of the pillars of the Standard Model.

The situation was indeed very complicated. Not only could no one explain why the electron's intrinsic rotation (called spin) had a value of ½ the smallest orbital angular momenta, which was 1 (in units of Planck's constant). This unexpected result was coupled with the value

$$g = 2$$

for the intrinsic magnetic moment, divided by the intrinsic angular momentum.

Dirac finds with his equation that the intrinsic angular momentum of the electron is ½h and the gyromagnetic ratio g = 2. In his celebrated 1928 paper, Dirac [Dirac 1928] simply says: '*The magnetic moment is just that assumed in the spinning electron model*'.

In order to get this formidable result, Dirac needed to introduce in his equation the interaction of the electron with an electromagnetic field; equation (1) thus becomes:

$$\left[\gamma^{\mu}\left(i\frac{\partial}{\partial x^{\mu}} - eA_{\mu}(x)\right) + m\right]\psi(x) = 0. \qquad (2)$$

This equation, as is the case for the free electron, corresponds to a system of four coupled equations shown below:

$$\begin{pmatrix} i\partial_0 - eA_0 + m & 0 & -i(\partial_1 + \partial_3) + e(A_1 + A_3) & \partial_2 + ieA_2 \\ 0 & i\partial_0 - eA_0 + m & -\partial_2 - ieA_2 & -i(\partial_1 - \partial_3) + e(A_1 - A_3) \\ i(\partial_1 - \partial_3) - e(A_1 - A_3) & -\partial_3 - ieA_2 & -i\partial_0 - eA_0 + m & 0 \\ \partial_2 + ieA_2 & i(\partial_1 - \partial_3) - e(A_1 - A_3) & 0 & -i\partial_0 + eA_0 + m \end{pmatrix} \begin{pmatrix} \psi_e\uparrow(x) \\ \psi_e\downarrow(x) \\ \psi_p\uparrow(x) \\ \psi_p\downarrow(x) \end{pmatrix}$$

The great novelty: **the Dirac γ-matrices**

The Dirac equations for a free electron (1) and for an electron interacting with an electromagnetic field (2) correspond, each, to four coupled equations, the coupling being described by the so-called γ-matrices.

These γ-matrices are the unexpected novelty discovered by Dirac in his attempt to describe the evolution of an elementary particle having spin ½,

A GREAT DISCOVERY BY ETTORE MAJORANA

Of course we now know that relativistic invariance allows any value for the intrinsic angular momenta. This was proved in 1932 by Ettore Majorana (see Chapter 7).

Furthermore we now know that also the fact that antiparticles are needed for each particle is not restricted to particles with spin ½h.

The existence of antiparticles is a property linked to all particles which are described by Relativistic Quantum Field Theory, as we will see in Chapter 7.

Furthermore it could also be that particles and antiparticles are identical, as is the case for the Majorana neutrino which we are discussing in the present chapter.

charge e, and mass m. Dirac found the following representation for the γ-matrices:

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

Notice that the γ-matrices represent the correlations that exist between the four coupled Dirac equations, synthetically expressed in terms of a function ψ called spinor, which is composed of four parts. The fact that these correlation matrices $\gamma^0, \gamma^1, \gamma^2, \gamma^3$ are of vectorial nature, thus being of the type γ^μ, allows the construction of the celebrated scalar product with ∂_μ:

$$\Sigma_\mu \gamma^\mu \partial_\mu = ¶ \ .$$

$$\lfloor_{\to} \frac{¶}{¶\, x^m}$$

When we write the four equations in terms of the unique equation

$$(i \partial + m)\psi = 0 \ ,$$

we do make use of the fact that the γ-matrices are four vectors. The symbol ∂ slashed, ¶ , was introduced by Feynman:

$$\partial = \sum_\mu ¿\, \gamma^\mu \frac{\partial}{\partial x^\mu}. \quad ¿$$

The properties of the Dirac γ-functions are the source of the so much wanted properties of particles with spin ½, mass ≠ 0 and charge ≠ 0.

What happens if the charge is zero? Here comes the great discovery of Majorana, now known as the Majorana representation of the γ-matrices (recall Pontecorvo's testimony). This representation of the γ-matrices is responsible for the existence of particles with spin ½, identical to their antiparticles: the Majorana neutrinos prove that it is not a privilege of spin ½ particles to have antiparticles.

The Majorana representation of γ-matrices is not limited to the case D = 4 of our familiar four-dimensional space-time (s = 3, t = 1). In fact the Majorana spinors exist in many space-time dimensions, provided that appropriate conditions are satisfied. These conditions are the number of space-time dimensions

$$D = s + t$$

and the so-called 'signature parameter' ρ = s – t.

For the case of our space-time:

$$\begin{cases} D = s + t = 3 + 1 = 4 \\ \rho = s - t = 3 - 1 = 2 \end{cases} .$$

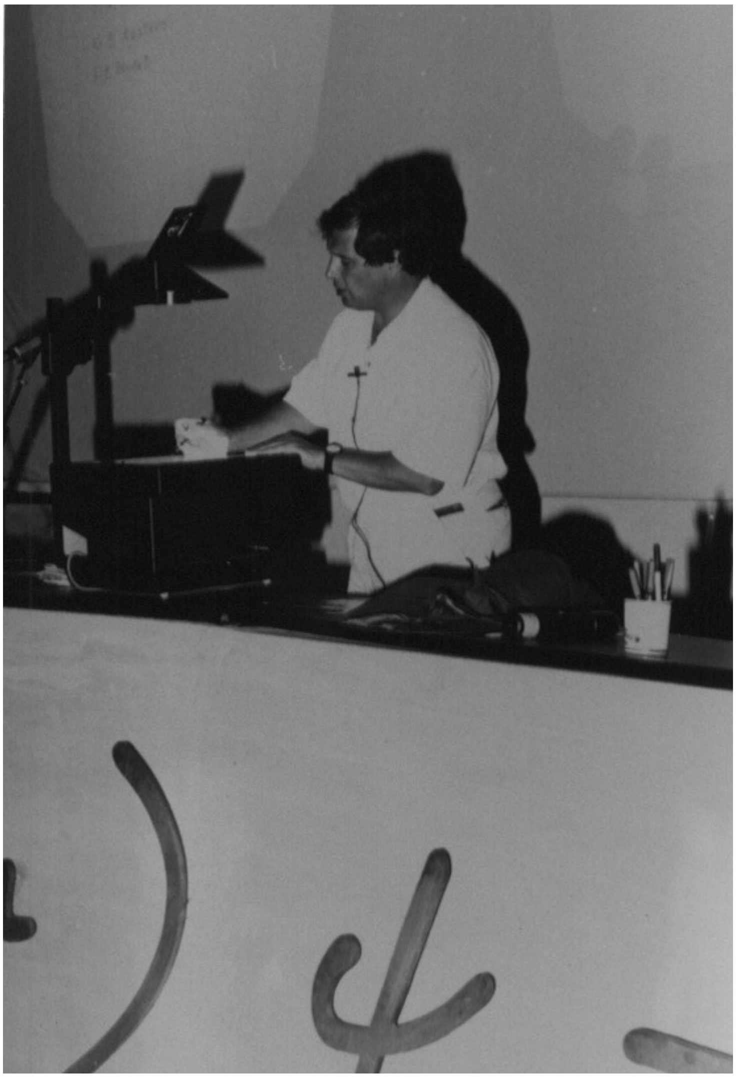
Majorana spinors exist for even and odd numbers of space-time dimensions. If

D even, ρ = 0, 2, 6 modulo 8

and if

D odd, ρ = 1, 7 modulo 8.

For a conventional space-time signature



Sergio Ferrara lecturing at Erice (1988).

$$\rho = D - 2,$$

Majorana spinors exist for

$$D = 2, 3, 4, 8, 9 \text{ modulo } 8.$$

This has enormous consequences for the construction of chiral superstring theories in D = 10 space-time dimensions, as illustrated by L. Andrianopoli and S. Ferrara [Andrianopoli and Ferrara 2006], bringing the Majorana spinors to the most advanced frontier of our physics knowledge. For example, the quantum of the gravitational field, the graviton, has as supersymmetric pattern the gravitino, which is a Majorana spinor, i.e. a particle with mass, spin 3/2 and whose antiparticle is identical to it.

Returning to Dirac, his equation needs 'four' components to describe the evolution in space and time of the simplest of particles, the electron. Majorana jotted down a new equation: for a chargeless particle like the 'neutrino', which is similar to the electron except for its lack of

WEYL AND SCIENCE

In April 1929 Paul Dirac gave an interview to a 'rather obnoxious newspaperman.' Here is the interview.

'*And now I want to ask you something more: They tell me that you and Einstein are the only two real sure-enough high-brows and the only ones who can really understand each other. I won't ask you if this is straight stuff for I know you are too modest to admit it. But I want to know this - Do you ever run across a fellow that even you can't understand?*'

'*Yes*,' says he.

'*This will make a great reading for the boys down at the office,'* says I, '*Do you mind releasing to me who he is?*'

'*Weyl*,' says he.

From W.O. Straub 'Weyl Spinors and Dirac's Electron Equation' March 17, 2005 (www.weylmann.com/weyldirac.pdf).

charge, only two components are needed to describe its movement in space-time. '*Brilliant*' – said Fermi – '*Write it up and publish it*'. Remembering what happened with the 'neutron' discovery, Fermi wrote the article himself and submitted the work, under Ettore Majorana's name, to the prestigious scientific journal 'Il Nuovo Cimento' [Majorana 1937]. Without Fermi's initiative, we would know nothing about the Majorana spinors and the Majorana neutrinos.

A few words to illustrate why the new particle, proposed by Pauli to avoid the violation of energy conservation in β-decay, and named by Enrico Fermi *neutrino*, attracted so much attention. A few years before, Enrico Fermi had given a rigorous formulation of the weak interactions [Fermi 1934], taking for granted the existence of the neutrino. The fact that a spin ½ particle without charge could relativistically be described by a spinor with only two components was indeed very interesting. There was another way of reducing the number of spinor components to two; this had been discovered by Weyl in 1929 [Weyl 1929]. The Dirac equation describes the space-time evolution of a particle with spin, mass and charge. Herman Weyl

Melvin Schwartz, Tsung Dao Lee, Antonino Zichichi and Isidor Isaac Rabi at Erice (1968).

discovered that, if the mass is zero, the four coupled Dirac equations split into two pairs. Each pair needs a spinor with only two components, called

$$\psi_+ \text{ and } \psi_- \ .$$

The original Dirac spinor is the sum of these two spinors

$$\psi_{\text{Dirac}} = \psi_+ + \psi_- \ .$$

Any spinor in even space-time dimensions may be decomposed as $\psi = \psi_+ + \psi_-$. The interest of this decomposition is that it corresponds to two different 'chirality' states, obtained with the complex projection operator

$$P_\pm \equiv \tfrac{1}{2}\,(1 \pm i\,\gamma_5),$$

which produces

$$P_\pm\, \psi = \psi_\pm \ .$$

'*What is now disproved was once thought self-evident*.'

Tsung Dao Lee, Erice 1968

Notice that $(P_{\pm})^* = P_{\mp}$,

and therefore $(\psi_{\pm})^* = \psi_{\mp}$.

The discovery of Weyl implies that, in the Dirac equation, when m = 0 the corresponding particles with spin ½ can have either positive or negative 'chirality'. This paper by Weyl, published in 1929 [Weyl 1929] was ignored for more than a quarter of a century since space inversion (parity operator) reverses chirality and the weak interactions were supposed not to break the law of parity conservation (the symmetry between left and right).

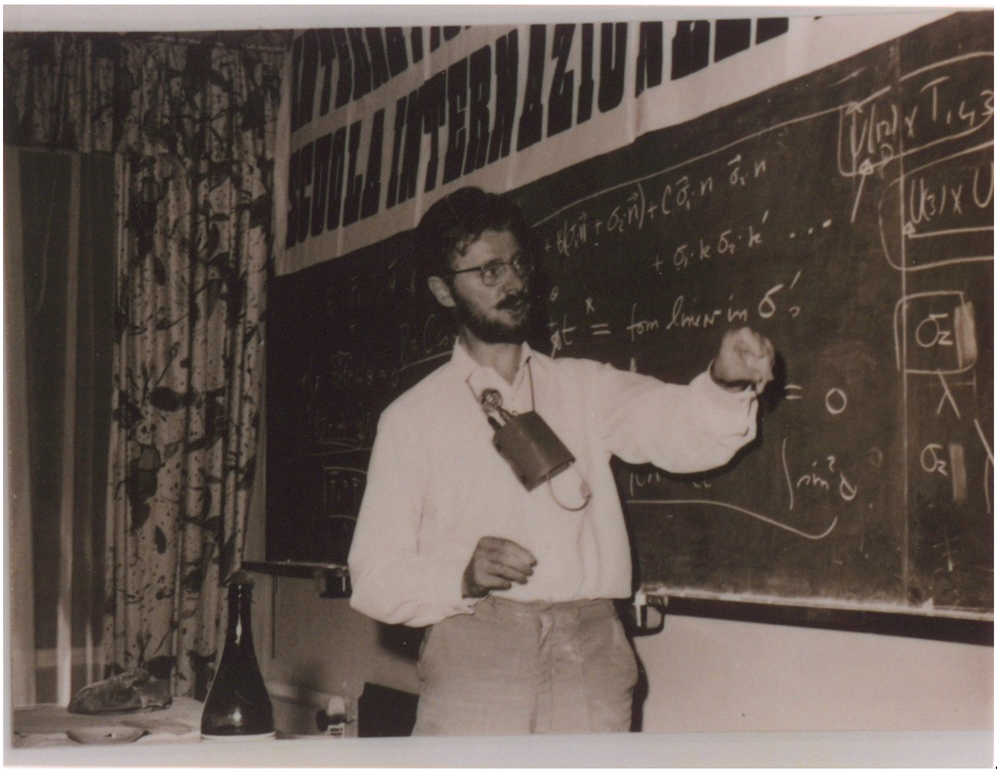
In the middle fifties, it was discovered that parity conservation was violated in the Fermi forces [Lee and Yang 1956], and that only left-handed (negative chirality) neutrinos and right-handed (positive chirality) antineutrinos appear to be coupled to the Fermi forces. The parity objection against the Weyl discovery turned to dust. The physics of the Fermi forces appears to be such that the two chirality states correspond to 'particle' and 'antiparticle' states.

It is as if the property of 'particle' and 'antiparticle' were linked to the property of 'chirality'. The origin of all this is that when m = 0 in the Dirac equation, the Lagrangian becomes invariant under the γ5 rotations, thus acquiring a new global invariance due to the existence of the γ5 matrix:

$$\gamma_5 \equiv \gamma^0 \gamma^1 \gamma^2 \gamma^3 .$$

To sum up: we have seen that if a particle with spin ½ is massless, it can only exist in two different 'chirality' states (Weyl). If a particle with spin ½ has mass, but zero charge, the particle and its antiparticle are the same (Majorana).

And thus the problem arises: What happens if a spin ½ particle has zero mass (Weyl) and zero charge (Majorana). Can Majorana–Weyl spinors exist? In other words, can a neutrino exist with zero mass and be identical to its antineutrino? The answer is no, in our four-dimensional space-time. In fact, the Weyl condition is that the two spinors are



John Stewart Bell at Erice (1963) lecturing on Dirac and Majorana neutrinos

John Stewart Bell at Erice (1975).

$$\psi_+ \text{ and } \psi_- ,$$

but that the antispinor $\psi_{+}^{¿}$ is not equal to the spinor ψ_+, as

$$\psi_{+}^{¿} = \psi_- .$$

Therefore the anti-Weyl spinor, $\psi_{\pm}^{¿}$, is not compatible with the Majorana condition

$$\psi_{\pm}^{¿} = \psi_{\pm} ,$$

which established the equivalence between a particle and its antiparticle.

The existence of spinors with particle–antiparticle equivalence (Majorana) and zero mass (Weyl), i.e. Majorana–Weyl spinors, is allowed in 2, 6, modulo 8 space-time dimensions.

As mentioned before, this is the case of chiral superstring theories in D = 10 space-time dimensions (see the paper by Andrianopoli and Ferrara quoted before). In other words the existence of spinor particles with particle–antiparticle equivalence (Majorana) and zero mass (Weyl) is allowed in the D = 10 space-time dimensions. As was already remarked, the gravitino is a Majorana spinor with mass.

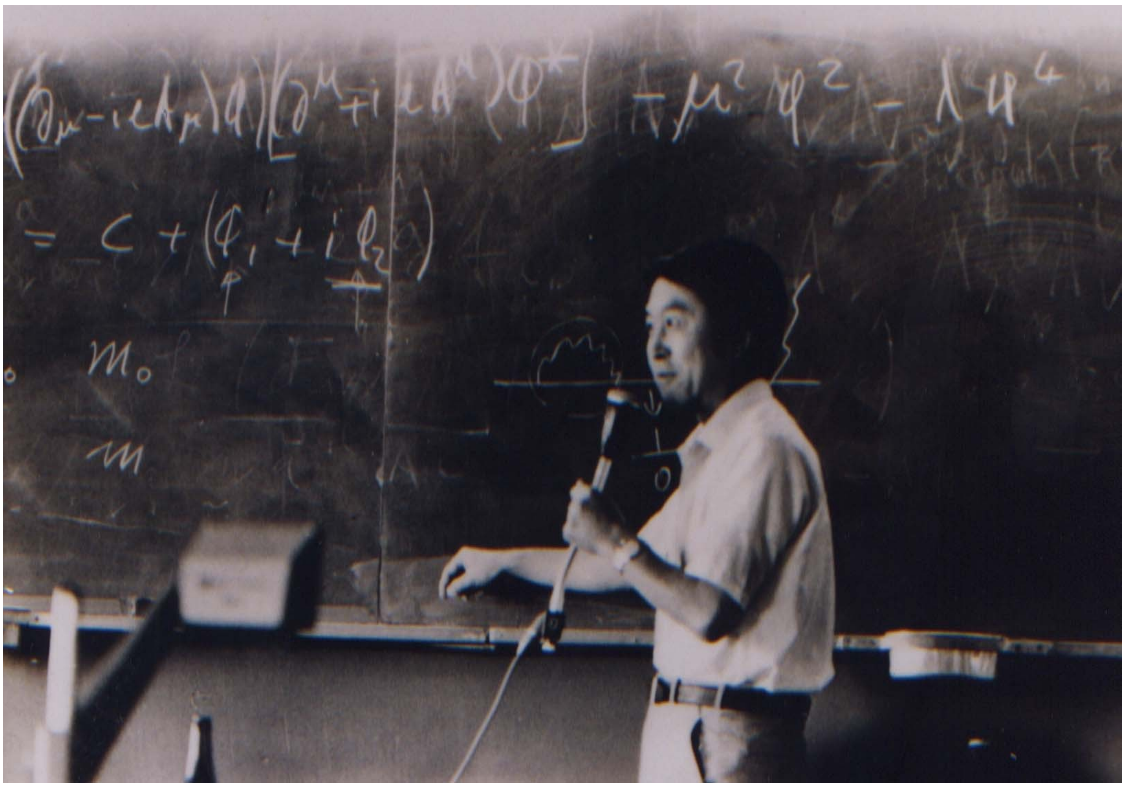
To sum up: in 4 dimensions, a spinor cannot be both Weyl ($m = 0$ and $q \neq \bar{q}$) and Majorana ($m \neq 0$ and $q = \bar{q}$). In 10 dimensions, it can be both. In fact, a Dirac spinor ($m \neq 0$; $q \neq \bar{q}$) in 10 dimensions has 32 degrees of freedom, while a Weyl ($m = 0$) or a Majorana ($q = \bar{q}$) spinor has 16 degrees of freedom.

A Majorana–Weyl ($m = 0$; $q = \bar{q}$) spinor has only 8 degrees of freedom. This 8 exactly matches the number of transverse modes of a vector in 10 dimensions.

This equality, 8 and 8 degrees of freedom, is used to construct supersymmetric theories in 10 dimensions, where the number of fermionic degrees of freedom must be equal to the number of bosonic ones.

6. The first Course of the Subnuclear Physics School (1963): John Bell on the Dirac and Majorana Neutrinos

Yoichiro Nambu at Erice (1972).The great John Bell conducted a rigorous comparison of Dirac's and Majorana's 'neutrinos' in the first year of the Erice Subnuclear Physics School. The detailed version of it can be found in the first chapter of Volume II [A. Zichichi 2006], which is the first of the ten volumes (see Chapter 8) describing the 'highlights' leading to the most formidable synthesis of scientific thought of all times, that we physicists call the 'Standard Model'. This Model has already pushed the frontiers of Physics well beyond what the Model itself first promised, so that the present goal has come to be known as the **SM&B**: **Standard Model and Beyond**.



Going back to the neutrinos of our SM&B, we know today that there exist three types of neutrinos. The first controls the combustion of the Sun's nuclear motor and keeps our Star from overheating. One of the dreams of today's physicists is to prove the existence of Majorana's hypothetical neutral particles, which are needed in the Grand Unification Theory. This is something that no one could have imagined in those years. And no one could have imagined the three conceptual bases needed for the SM&B, as we will discuss in the next chapter.

7. The First Step to Relativistically Describe Particles With Arbitrary Spin

In 1932 the study of particles with arbitrary spin [Majorana 1932] was considered at the level of a pure *mathematical curiosity*.

This paper attracted the interest of mathematically oriented theoretical physicists over many decades and up to now, as discussed by Y. Nambu [Nambu 2006]. The paper remained quasi-unknown in the area of physics, despite its being full of remarkable new ideas. In this paper in fact, there are the first hints of *supersymmetry*, of the *spin–mass correlation,* and of *spontaneous symmetry breaking*: three fundamental conceptual bases of what we now call the SM&B. First hints mean that our conceptual understanding of the fundamental laws of nature were already in Majorana's attempts to describe particles with arbitrary spins in a relativistic invariant way. Majorana starts – as he correctly stated – with the simplest representation of the Lorentz group, which is infinite-dimensional. In this representation the states with integer (bosons) and semi-integer (fermions) spins are treated on equal ground. In other words, the relativistic description of particle states allows bosons and fermions to exist on equal grounds. These two fundamental sets of states (bosons and fermions) are the first hint of supersymmetry.

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THE MASS-SPIN FORMULA

'*One can understand the origin of the Majorana mass-spin formula by considering the Schroedinger limit in perturbation theory starting from the rest frame.*

For a given rest state $|0\rangle$, *the kinetic energy term* $\alpha \cdot p$ *gives rise to a second order energy shift*

$$\Delta E = \Sigma \langle 0 | \alpha \cdot p | i \rangle \langle i | \alpha \cdot p | 0 \rangle / (E_0 - E_i),$$

where the sum is over the components of the neighbouring spin states i.

One expects this to yield the nonrelativistic kinetic energy $p^2 / 2E_0 > 0$, *which requires at least one of the states i to have a lower energy than* E_0.

One sees from this that a descending mass spectrum with an accumulation point is an inevitable feature of a relativistic Hamiltonian which is Hermitian, linear in the momenta and has only positive (nonzero) rest energies.'

From Y. Nambu 'Majorana's Infinite Component Wave Equation' in *Majorana Centenary Celebrations* (A. Zichichi ed, World Scientific Vol. I, 2006).

Another remarkable novelty is the correlation between spin and mass. The eigenvalues of the masses are given by a relation of the type:

$$m = \left(\frac{m_0}{J + \frac{1}{2}} \right),$$

where m_0 is a given constant and J is the spin. The mass decreases with increasing spin, the opposite of what would appear, many decades later, in the study of the strong interactions between baryons and mesons (now known as Chew–Frautschi–Gribov–Regge trajectories).

In this remarkable paper – as a consequence of the description of particle states with arbitrary spins – there is also the existence of imaginary mass eigenvalues. We know today that the only way to introduce real masses – without destroying the theoretical description of nature – is the spontaneous symmetry breaking (SSB) mechanism. But SSB could not exist without imaginary masses. Today, three quarters of a century later, what was considered in 1932 a purely *mathematical curiosity* represents a powerful source of incredibly new ideas, as those three mentioned earlier. There is a further development, which this paper contributed to: the formidable relation between spin and statistics, which was to lead to the discovery of another invariance law, valid for all quantized Relativistic Field Theories, the celebrated PCT theorem.

Majorana's paper shows first of all that the relativistic description of a particle state allows the existence of integer and semi-integer spin values. But it was already known that the electron must obey the Pauli exclusion principle and that the electron has semi-integer spin. Thus the problem arises of understanding if the Pauli principle is valid for all semi-integer spins. If this were the case, it would be necessary to find which properties characterize these two classes of particles, now known as 'fermions' (semi-integer spin) and 'bosons' (integer spin). The first of these properties are of a statistical nature, governing groups of identical fermions and groups of identical bosons. We now know that a fundamental distinction exists and that the bases for the statistical laws governing fermions and bosons are the anticommutation relations for fermions and the commutation relations for bosons.

The 'spin-statistics' theorem has an interesting and long history, whose main actors are some of the most distinguished theorists of the XXth century. The first contribution to the study of the correlation between spin and statistics comes from Markus Fierz, with a paper, *Über die Relativistische Theorie Kräfterfreier Teilchen mit Beliebigem Spin*, where the case of general spin for free fields is investigated [Fierz 1939]. A year later, Wolfgang Pauli comes in with his

Bruno Zumino lecturing at Erice (1969) on the PCT theorem.

paper *On the Connection Between Spin and Statistics* [Pauli 1940]. The first proofs, obtained using only the general properties of relativistic QFT, which include also the microscopic causality (also known as local commutativity), are due to G. Lüders and B. Zumino, *Connection Between Spin and Statistics* [Lüders and Zumino 1958], and to N. Burgoyne, *On the Connection Between Spin and Statistics* [Burgoyne 1958]. Another important contribution to the clarification of the connection between spin and statistics came in 1961, with G.F. Dell'Antonio: *On the Connection Between Spin and Statistics* [Dell'Antonio 1961].

The correlation between spin and statistics had important consequences in understanding the relativistic description of QFTs, whose invariance properties ended in the celebrated PCT theorem.

It certainly cannot be accidental that the first suggestion for the existence of such an invariance law, called PCT, came from the same fellows who were engaged in the study of the 'spin-statistics' theorem: G. Lüders and B. Zumino.

These two outstanding theoretical physicists suggested that if a RQFT obeys the space inversion invariance law, called parity, P, it must also be invariant for the product of charge conjugation (particle–antiparticle) and time inversion, CT. It is in this form that it was proved by G. Lüders in 1954, in the paper On the Equivalence of Invariance under Time Reversal and under Particle Antiparticle conjugation for Relativistic Field Theories [Lüders 1954]. A year later, Pauli proved that the PCT invariance is a universal law, valid for all RQFTs, Exclusion Principle, Lorentz Group and Reflection of Space-Time and Charge [Pauli 1955]. This paper closes a cycle started by Pauli in 1940, with his work on spin and statistics, where he proved already what is now considered the 'classical' PCT invariance, since it was derived using free non-interacting fields. The validity of PCT invariance for QFTs was obtained by Julian Schwinger (a great admirer of Ettore Majorana) in 1951, with his work On the Theory of Quantized Fields I [Schwinger 1951].

It is interesting to see what Arthur Wightman, another Ettore Majorana's enthusiastic supporter, wrote about this Schwinger paper in his book *PCT, Spin and Statistics, and All That* [Wightman 1964]: '*Readers of this paper did not generally recognize that it stated or proved the PCT theorem*'. Something similar to those who, reading Majorana's paper on arbitrary spins, have not found the imprint of the original ideas that we have discussed in the present short review.

SHOULD WE BELIEVE IN QUANTUM FIELD THEORY?

ARTHUR S. WIGHTMAN
SUBNUCLEAR PHYSICS SCHOOL - ERICE 1977
(The first two pages (983 and 984) of the lectures in the volume *The Whys of Subnuclear Physics* A. Zichichi ed., Plenum, New York and London (1979)

SHOULD WE BELIEVE IN QUANTUM FIELD THEORY?

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The following is a straightforward attempt to answer the question stated in the title and posed by Professor Zichichi in his opening lecture of the School. Its perspective is to some extent historical as is appropriate, in my opinion, for part of an attempt to assess where we are and where we are going in the study of subnuclear matter. If there are readers who are not aware of the inherent fallibility of such attempts to learn from history, they should be reminded that the method involved is somewhat reminiscent of that used by the celebrated wise men of Chelm, who, when they wished to record the location of an especially good fishing spot, carefully marked the place on the boat from which they were fishing.

WHAT DOES IT MEAN TO BELIEVE IN A PHYSICAL THEORY

In answering this question it is useful to consider a family of examples: the old quantum theory, the Boltzmann equation in the theory of dilute gases, Maxwell's (classical) theory of electricity and magnetism, and quantum mechanics. These examples have the virtue that they are fully mature theories with their main features

understood. (That is not the case for relativistic quantum field theory as I will discuss in detail.) There are several useful distinctions illustrated by this list.

First, unlike the others, the old quantum theory is internally inconsistent. It combines Maxwell classical theory of electricity and magnetism and Newtonian or relativistic mechanics with quantization conditions. There is a blatant contradiction between its description of the emission of radiation in terms of quantum jumps and that predicted by its equations of motion. It need not be emphasized to students of the parton model that an internally inconsistent theory can, in spite of its inconsistency, be exceedingly useful in the development of physical understanding. Certainly, the history of quantum theory illustrates the point. The main problem of atomic mechanics, as seen by those who were to create modern quantum mechanics, was to obtain a theory that would give the successes of the old quantum theory (discrete spectral lines, Balmer formula for the spectrum of hydrogen-like atoms, etc.), without its inconsistencies. Whatever the usefulness of internally inconsistent theories, one hopes and expects that in the end they will be replaced by internally consistent theories. The rest of the list are such examples.

The Boltzmann equation in the theory of dilute gases illustrates a second distinction. It defines a model which is internally consistent but externally inconsistent in the sense that it is inconsistent with the laws of statistical mechanics, except in a certain limit in which the density of the gas goes to zero but the range of interaction of the molecules increases as the reciprocal square root of the density. Thus, as applied to actual gases, the Boltzmann equation is inconsistent with the laws of statistical mechanics.

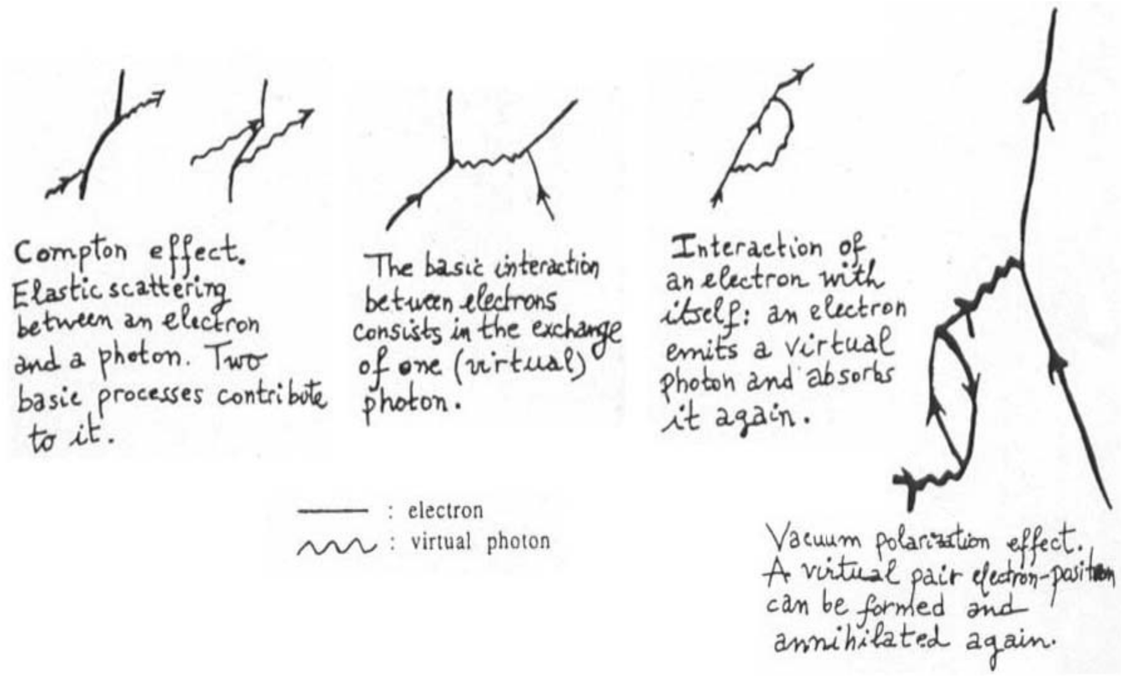
THE FEYNMAN LECTURE HALL

in Erice

Original diagrams drawn by Feynman, reproduced in iron and fixed on the walls of the Lecture Hall.

8. The Centennial of the birth of a genius – A homage by the International Scientific Community

This year is the hundredth anniversary of the birth of Ettore Majorana, Enrico Fermi's young student whom, on the occasion of his mysterious disappearance during a boat trip from Palermo to Naples, he referred to as 'a genius of the order of Galilei and Newton'. The President of the Sicilian Government and the Mayor of Erice have decided to launch a series of



Each line in a Feynman diagram describes the path of a particle; when a particle breaks into two, its line divides as well. A mathematical expression is associated with each line and vertex in a Feynman diagram. The product of these expressions gives the amplitude of the probability that the depicted process occurs.

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initiatives intended to make known not only Majorana's contributions to the advancement of Physics, but also the tribute expressed for decades in the unbending determination of the international scientific community. Through the International School of Subnuclear Physics, since 1963, this community has striven to provide the most prestigious protagonists of the most advanced frontiers of Galilean Science today with the best qualified new talents from all over the world, unrestricted by any ideological, political or racial barriers.

The twelve volumes that comprise this work [A. Zichichi 2006] are intended to provide everyone – not only those who have attended Courses at the Erice Subnuclear Physics School (with lecturers of the caliber of Richard Feynman), but also those who have remained in their own Universities or Laboratories – with a faithful account of the crucial steps that led up to the most formidable synthesis of scientific thought of all times, known in physics jargon as the Standard Model and Beyond (SM&B).

The first volume starts with recollections about Ettore Majorana that I have gathered from those who knew him both directly (Laura Fermi, Bruno Pontecorvo, Emilio Segré, Giancarlo Wick, Eugene Wigner, Paul Dirac, Werner Heisenberg) and indirectly (Robert Oppenheimer, John Bell, Isidor Rabi, Patrick Blackett, Victor Weisskopf, Monsignor Francesco Ricceri and Leonardo Sciascia). These are followed by accounts from illustrious exponents of todays

Opening of the Celebrations of the Ettore Majorana Centenary in Rome (Pietro da Cortona Hall of the Capitol Hill), on February 28, 2006. From left to right: Samuel C.C. Ting, Bruno Maraviglia, Antonino Zichichi, Athos De Luca, Giovanni Bornia, Giuseppe Ducrot (hidden), Renato Guarini.

Physics, who recognize links to the work of Ettore Majorana in their own work: Sergio Ferrara, David Gross, Leon Lederman, Tsung Dao Lee, Yoichiro Nambu, Samuel C.C. Ting, Gerardus 't Hooft and Frank Wilczek. I then offer my own description of the SM&B in order to give the reader a concrete idea of how far we have come since the times in which Majorana was working.

In this first volume, we also wanted the contribution of two 'best students': the first one (1963), Haim Harari, and the other for the year 1980, Serguey Petcov, who has devoted his activity to a study of many consequences of Majorana's original ideas. For example the possibility of Majorana CP-violating phases playing the role of leptogenesis CP-violating parameters, which determine the baryon asymmetry of the Universe. Petcov has also investigated the neutrino oscillations in matter, which do not respect PCT invariance and the absolute neutrino mass scale. Of course the key problem in this field is to understand the origin of the leptonic mixing phenomenon, which remains totally open, as unknown is the mixing in the quark sector. These two 'best students' are examples of our activity devoted to new talents in order to give them a chance to be recognized by the international scientific community. This is why in the first volume we have also the work of the best young participants in the 'New Talents' competition as a testimony that honours the spirit of Ettore Majorana on the occasion of his hundredth anniversary.

The ten volumes, from second to eleventh, are dedicated to the ten steps that have led us to the formidable synthesis of SM&B.

The twelfth volume, '*The Glorious Days of Physics and Erice*', is dedicated to such eminent figures of XXth century Physics as Gilberto Bernardini, Patrick M.S. Blackett, Richard H. Dalitz, Paul A.M. Dirac, Enrico Fermi, Richard P. Feynman, Robert Hofstadter, Gunnar Källen, Giuseppe P.S. Occhialini, Wolfang Paul, Bruno Pontecorvo, Isidor I. Rabi, Bruno Rossi, Julian S. Schwinger, Bruno Touschek, Victor F. Weisskopf and Eugene P. Wigner, who, through their participation in the Erice School of Subnuclear Physics, have made this school *the most prestigious post-university institution in the world* (these are the words of Isidor I. Rabi in Erice, July 1975).

In the past, our scientific community has proposed, in the mythical City of Venus, to dedicate lecture halls, streets, diplomas, squares, discussion halls, courts, cloisters, and institutes to these illustrious physicists, in recognition of their link with the activities of the Erice School of Subnuclear Physics.

The President of Sicily and the Mayor of Erice have decided – on the occasion of the Majorana Centenary – to make official these dedications to our fellows whose inventions and discoveries have carried modern Physics into an era of scientific glory. The twelfth volume is devoted to an illustration of the scientific value of these fellows, who all had great admiration for the geniality of Ettore Majorana.

The opening of the Majorana Centenary Celebrations took place in the Pietro da Cortona Hall (p. 56) in the Capitol Hill of Rome on 28 February 2006; it was attended by a large audience, in particular by members of the Majorana family. The ceremony was presided by Senator Athos.

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De Luca, Chair of the Panisperna Committee. After welcoming the whole audience and describing the achievements and activities of the Comitato Panisperna, he introduced the people present at the desk, among whom, besides Antonino Zichichi (President of the Enrico Fermi Centre), the Nobel Laureate Samuel C.C. Ting, the Rector of the University of Rome 'La Sapienza', Renato Guarini, and the 'Assessore alla Cultura' of the City of Rome, Giovanni Bornia. Senator De Luca then gave the floor to Professor Zichichi for his official lecture, 'The Majorana geniality according to Enrico Fermi' (*La genialità di Ettore Majorana vista da Enrico Fermi*). Zichichi recalled that the 'Ettore Majorana Foundation and Centre for Scientific Culture' (EMFCSC) was founded at CERN in 1963 (the instituting act was signed by J.S. Bell, P.M.S. Blackett, I.I. Rabi, V.F. Weisskopf and A. Zichichi) and named after the outstanding

Clay bust of Ettore Majorana by young sculptor Giuseppe Ducrot

Julian Schwinger at Erice during a discussion session devoted to Anomalies in Quantum Field Theory.

Sicilian physicist, at the time almost unknown. The EMFCSC, located in the ancient and enchanting City of Erice, in Sicily, was and still is intended both to expand the impact of science at the highest level and to establish a permanent reminder of the role of Majorana.

Many other interesting points were stressed in this stimulating event. Ting, at the end of Zichichi's lecture, expressed his opinion about the great contributions of Italian physicists to science and recalled the basic discovery of the antideuteron and the first search for the heavy lepton, both performed at CERN by Zichichi and collaborators, at the time when CERN was starting to compete with the major laboratories in the US.

The ceremony was followed by the presentation to the public of the new clay bust of Ettore Majorana (p. 58) made by the young sculptor Giuseppe Ducrot. From this bust, a bronze cast will be made and its location will be at the Enrico Fermi Centre in Rome.

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Julian Schwinger celebrating his 70th birthday in Erice during the 26th Subnuclear Physics School. From left: Sheldon Glashow, Mrs Manci Dirac, Sergio Ferrara, Michael Duff (1988).

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