BRANES

M. J. Duff

Blackett Laboratory, Imperial College London, Prince Consort Road,
London SW7 2BZ, UK

ABSTRACT

This lecture provides an elementary introduction to the theory of branes.

Contents

1 BRANES

1.1 History

Membrane theory has a strange history which goes back even further than strings. The idea that the elementary particles might correspond to modes of a vibrating membrane was put forward originally in 1962 by Dirac. When string theory came along in the 1970s, there were some attempts to revive the membrane idea but without much success. Things did not change much until 1986 when Hughes, Liu and Polchinski showed that it was possible to combine membranes with supersymmetry: the *supermembrane* was born.

Consequently, while all the progress in string theory was going on, a small splinter group was posing the question: Once you have given up 0-dimensional particles in favor of 1-dimensional strings, why not 2-dimensional membranes or in general p-dimensional objects (inevitably dubbed p*-branes*)? Just as a 0-dimensional particle sweeps out a 1-dimensional *worldline* as it evolves in time, so a 1-dimensional string sweeps out a 2-dimensional *world-sheet* and a p-brane sweeps out a d-dimensional *worldvolume*, where $d = p + 1$. Of course, there must be enough room for the p-brane to move about in spacetime, so d must be less than or equal to the number of spacetime dimensions D. In fact, as we shall see in section (1.4) supersymmetry places further severe restrictions both on the dimension of the extended object and the dimension of the spacetime in which it lives. We can represent these as points on a graph where we plot spacetime dimension D vertically and the p-brane dimension $d = p + 1$ horizontally. This graph is called the *brane scan*. See Table 1. In the early eighties Green and Schwarz had shown that spacetime supersymmetry allows classical superstrings moving in spacetime dimensions $3, 4, 6$ and 10. (Quantum considerations rule out all but the ten-dimensional case as being truly fundamental. Of course some of these ten dimensions could be curled up to a very tiny size in the way suggested by Kaluza and Klein. Ideally six would be compactified in this way so as to yield the four spacetime dimensions with which we are familiar.) It was now realized, however, that these 1-branes in $D = 3, 4, 6$ and 10 should now be viewed as but special cases of this more general class of supersymmetric extended object.

Curiously enough, the maximum spacetime dimension permitted is eleven, where Bergshoeff, Sezgin and Townsend found their supermembrane which couples to eleven-dimensional supergravity. (The 3-form gauge field of $D = 11$ supergravity had long been suggestive of a membrane interpretation). Moreover, it was then possible to show by simultaneous dimensional reduction of the spacetime and worldvolume that the membrane looks like a string in

$D \uparrow$													
11	.			S			T						
10	.	V	S/V	V	V	V	S/V	V	V	V	V		
9	.	S				S							
8	.				S								
7	.			S			T						
6	.	V	S/V	V	S/V	V	V						
5	.	S		S									
4	.	V	S/V	S/V	V								
3	.	S/V	S/V	V									
2	.	S											
1	.												
0	.	.	.	.	.	.	.	.	.	.	.	.	
	0	1	2	3	4	5	6	7	8	9	10	11	$d \rightarrow$

Table 1: The brane scan, where S, V and T denote scalar, vector and antisymmetric tensor multiplets.

ten dimensions. In fact, it yields precisely the Type IIA superstring. This suggested that the eleven-dimensional theory was perhaps the more fundamental after all.

Notwithstanding these and subsequent results, the supermembrane enterprise was, until recently, largely ignored by the mainstream physics community. Those who had worked on eleven-dimensional supergravity and then on supermembranes spent the early eighties arguing for *spacetime* dimensions greater than four, and the late eighties and early nineties arguing for *worldvolume* dimensions greater than two. The latter struggle was by far the more bitter!

In this chapter we shall review the progress reached over the last two decades and see how it fits in with recent results in string duality, D-branes and M-theory.

1.2 Bosonic p-branes

Consider some extended object with 1 time and $(d-1)$ space dimensions moving in a spacetime with 1 time and $(D-1)$ space dimensions. We shall demand that its dynamics is governed by minimizing the worldvolume which the object sweeps out

$$S = -T_d \int d^d\xi \ \{-det\partial_i X^M \ \partial_j X^N \eta_{MN}\}^{1/2} \tag{1.1}$$

where we have introduced worldvolume coordinates ξ^i (i = 0, ..., (d-1)) and spacetime coordinates X^M $(M = 0, \ldots, (D-1))$. To begin with, we assume spacetime is flat with Minkowski metric η_{MN} and signature $(-, +, \ldots, +)$. The tension of the object is given by the constant T_d which renders the action S dimensionless. This action was first introduced by Dirac in the case of a membrane ($d = 3$) and later by Nambu and Goto in the case of a string ($d = 2$).

The classical equations of motion that follow from (1.1) may equivalently be obtained from the action

$$S = T_d \int d^d\xi \Big(-\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\partial_i x^M \partial_j x^N \eta_{MN} + \frac{1}{2}(d-2)\sqrt{-\gamma} \Big) \tag{1.2}$$

where we have introduced the auxiliary field $\gamma_{ij}(\xi)$. γ denotes its determinant and γ^{ij} its inverse. Varying with respect to γ_{ij} yields the equation of motion

$$\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\gamma^{k\ell}\partial_k X^M \partial_\ell X^N \eta_{MN} - \sqrt{-\gamma}\partial_k x^M \partial_\ell X^N \gamma^{ik}\gamma^{j\ell}\eta_{MN} = \frac{1}{2}(d-2)\sqrt{-\gamma}\gamma^{ij} \quad . \tag{1.3}$$

Taking the trace, we find for $d \neq 2$, that

$$\gamma^{k\ell}\partial_k X^M \partial_\ell X^N \ \eta_{MN} = d \tag{1.4}$$

and hence that γ_{ij} is just the induced metric on worldvolume

$$\gamma_{ij} = \partial_i X^M \partial_j X^N \eta_{MN}. \tag{1.5}$$

Varying (1.2) with respect to X^M yields

$$\partial_i \left(\sqrt{-\gamma}\gamma^{ij}\partial_j X^N \eta_{MN} \right) = 0. \tag{1.6}$$

Thus equations (1.5) and (1.6) are together equivalent to the equation of motion obtained by varying (1.1) with respect to X^M.

Note that the case $d = 2$ is special. Here, the worldvolume cosmological term drops out and (1.2) displays a conformal symmetry

$$\begin{aligned} \gamma_{ij}(\xi) &\to \Omega^2(\xi)\gamma_{ij}(\xi) \\ X^M(\xi) &\to X^M(\xi) \end{aligned} \tag{1.7}$$

where Ω is some arbitrary function of ξ. In this case γ_{ij} and $\partial_i X^M \partial_j X^N \eta_{MN}$ are related only up to a conformal factor. The actions (1.1) and (1.2) are, however, equivalent for all d, at least classically.

There are two useful generalizations of the above. The first is to go to curved space by replacing η_{MN} by $g_{MN}(X)$; the second is to introduce an antisymmetric tensor field $B_{MN...P}(X)$ of rank d which couples via a Wess-Zumino term. The action (1.2) then becomes

$$S = T_d \int d^d\xi \Big[-\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\partial_i X^M \partial_j X^M g_{MN}(X) + \frac{1}{2}(d-2)\sqrt{-\gamma}$$

$$+\frac{1}{d!}\epsilon^{i_1 i_2 .. i_d}\partial_{i_1} X^{M_1}\partial_{i_2} X^{M_2} \ldots \partial_{i_d} X^{M_d} B_{M_1 M_2 .. M_d}(X)\Big] \tag{1.8}$$

and the equations of motion are

$$\partial_i \left(\sqrt{-\gamma}\gamma^{ij}\partial_j X^N\right) g_{MN} + g_{MP}\Gamma^P{}_{KL}\partial_i X^K \partial_j X^L \gamma^{ij}$$

$$= \frac{1}{d!} F_{MNT\cdots S}\epsilon^{ij\cdots k}\partial_i X^N \partial_j X^T \ldots \partial_k X^S \tag{1.9}$$

and

$$\gamma_{ij} = \partial_i X^M \partial_j X^N g_{MN}(X) \tag{1.10}$$

where the field-strength F is given by

$$F = dB \tag{1.11}$$

and hence obeys the Bianchi identity

$$dF = 0. \tag{1.12}$$

The virtue of these generalizations is that they now permit a straightforward transition to the supermembrane.

Our experience with string theory suggests that there are two ways of introducing supersymmetry into membrane theory. The first is to look for a *supermembrane* which has manifest spacetime supersymmetry but no supersymmetry on the worldvolume. The second is to look for a *spinning membrane* which has manifest worldvolume supersymmetry but no supersymmetry in spacetime. An early attempt at spinnning membranes by Howe and Tucker encountered the problem that the worldvolume cosmological term does not permit a supersymmetrization using the usual rules of $d = 3$ tensor calculus without the introduction of an Einstein-Hilbert term. Indeed, these objections have been elevated to the status of a *no-go theorem* for spinning membranes. Attempts to circumvent this no-go theorem have been made starting from the conformally invariant action, but is is fair to say that the spinning membrane approach never really caught on. Recently, there has been some success in formalisms with both worldvolume and spacetime supersymmetry. The last ten years

Dimension (D or d)	Minimal Spinor (M or m)	Supersymmetry (N or n)
11	32	1
10	16	2, 1
9	16	2, 1
8	16	2, 1
7	16	2, 1
6	8	4, 3, 2, 1
5	8	4, 3, 2, 1
4	4	8, ..., 1
3	2	16, ..., 1
2	1	32, ..., 1

Table 2: Minimal spinor components and supersymmetries.

of supermembranes, however, has been dominated by the approach with spacetime supersymmetry and worldvolume kappa symmetry. At first, progress in supermembranes was hampered by the belief that kappa symmetry, so crucial to Green-Schwarz superparticles ($d = 1$) and superstrings ($d = 2$) could not be generalized to membranes. The breakthrough came when Hughes, Liu and Polchinski showed that it could.

1.3 Super p-branes

Following Bergshoeff, Sezgin and Townsend, let us introduce the coordinates Z^M of a curved superspace

$$Z^M = (x^\mu, \theta^\alpha) \tag{1.13}$$

and the supervielbein $E_M{}^A(Z)$ where $M = \mu, \alpha$ are world indices and $A = a, \alpha$ are tangent space indices. We also define the pull-back

$$E_i{}^A = \partial_i Z^M E_M{}^A \quad . \tag{1.14}$$

We also need the super-d-form $B_{A_d \ldots A_1}(Z)$. Then the supermembrane action is

$$\begin{aligned} S = T_d \int d^d\xi \Big[& -\frac{1}{2}\sqrt{-\gamma}\gamma^{ij} E_i{}^a E_j{}^b \eta_{ab} + \frac{1}{2}(d-2)\sqrt{-\gamma} \\ & + \frac{1}{d!}\epsilon^{i_1 \ldots i_d} E_{i_1}{}^{A_1} \cdots E_{i_d}{}^{A_d} B_{A_d \ldots A_1}\Big]. \end{aligned} \tag{1.15}$$

As in (1.8) there is a kinetic term, a worldvolume cosmological term, and a Wess-Zumino term. The action (1.15) has the virtue that it reduces to the Green-Schwarz superstring action when $d = 2$.

The target-space symmetries are superdiffeomorphisms, Lorentz invariance and d-form gauge invariance. The worldvolume symmetries are ordinary diffeomorphisms and kappa invariance which we now examine in more detail. The transformation rules are

$$\delta Z^M E^a{}_M = 0, \quad \delta Z^M E^\alpha{}_M = \kappa^\beta (1+\Gamma)^\alpha{}_\beta \tag{1.16}$$

where $\kappa^\beta(\xi)$ is an anticommuting spacetime spinor but worldvolume scalar, and where

$$\Gamma^\alpha{}_\beta = \frac{(-1)^{d(d-3)/4}}{d!\sqrt{-\gamma}} \epsilon^{i_1..i_d} E_{i_1}{}^{a_1} E_{i_2}{}^{a_2} \dots E_{i_d}{}^{a_d} \Gamma_{a_1..a_d} \quad . \tag{1.17}$$

Here Γ_a are the Dirac matrices in spacetime and

$$\Gamma_{a_1..a_d} = \Gamma_{[a_1 \cdots a_d]} \quad . \tag{1.18}$$

This kappa symmetry has the following important consequences:

1) The symmetry is achieved only if certain constraints on the antisymmetric tensor field strength $F_{MNP..Q}(\mathrm{Z})$ and the supertorsion are satisfied. In particular the Bianchi identity $dF = 0$ then requires the Γ matrix identity

$$\left(d\bar{\theta}\Gamma_a d\theta\right)\left(d\bar{\theta}\Gamma^{ab_1 \dots b_{d-2}} d\theta\right) = 0 \tag{1.19}$$

for a commuting spinor dθ. This is satisfied only for certain values of d and D. Specifically, for $d \geq 2$

$$\begin{aligned} d &= 2: \quad D = 3, 4, 6, 10 \\ d &= 3: \quad D = 4, 5, 7, 11 \\ d &= 4: \quad D = 6, 8 \\ d &= 5: \quad D = 9 \\ d &= 6: \quad D = 10 \quad . \end{aligned} \tag{1.20}$$

Note that we recover as a special case the well-known result that Green-Schwarz superstrings exist *classically* only for $D = 3, 4, 6$, and 10. Note also $d_{max} = 6$ and $D_{max} = 11$. The upper limit of $D = 11$ is already known in supergravity but there it is necessary to make extra assumptions concerning the absence of consistent higher spin interactions. In supermembrane theory, it follows automatically.

2) The matrix Γ of (1.18) is traceless and satisfies

$$\Gamma^2 = 1 \tag{1.21}$$

when the equations of motion are satisfied and hence the matrices $(1 \pm \Gamma)/2$ act as projection operators. The transformation rule (1.16) therefore permits us to gauge away one half of the fermion degrees of freedom. As described below, this gives rise to a matching of physical boson and fermion degrees of freedom on the worldvolume.

3) In the case of the eleven-dimensional supermembrane, it has been shown that the constraints on the background fields ${E_M}^A$ and B_{MNP} are nothing but the equations of motion of eleven-dimensional supergravity.

1.4 The brane scan

The matching of physical bose and fermi degrees of freedom on the *worldvolume* may, at first sight, seem puzzling since we began with only spacetime supersymmetry. The explanation is as follows. As the p-brane moves through spacetime, its trajectory is described by the functions $X^M(\xi)$ where X^M are the spacetime coordinates ($M = 0, 1, \ldots, D-1$) and ξ^i are the worldvolume coordinates ($i = 0, 1, \ldots, d-1$). It is often convenient to make the so-called *static gauge choice* by making the $D = d + (D-d)$ split

$$X^M(\xi) = (X^\mu(\xi), Y^m(\xi)), \tag{1.22}$$

where $\mu = 0, 1, \ldots, d-1$ and $m = d, \ldots, D-1$, and then setting

$$X^\mu(\xi) = \xi^\mu. \tag{1.23}$$

Thus the only physical worldvolume degrees of freedom are given by the $(D-d)$ $Y^m(\xi)$. So the number of on-shell bosonic degrees of freedom is

$$N_B = D - d. \tag{1.24}$$

To describe the super p-brane we augment the D bosonic coordinates $X^M(\xi)$ with anticommuting fermionic coordinates $\theta^\alpha(\xi)$. Depending on D, this spinor could be Dirac, Weyl, Majorana or Majorana-Weyl. The fermionic kappa symmetry means that half of the spinor degrees of freedom are redundant and may be eliminated by a physical gauge choice. The net result is that the theory exhibits a *d-dimensional worldvolume supersymmetry* where the number of fermionic generators is exactly half of the generators in the original spacetime supersymmetry. This partial breaking of supersymmetry is a key idea. Let M be the number of real components of the minimal spinor and N the number of supersymmetries in

D spacetime dimensions and let m and n be the corresponding quantities in d worldvolume dimensions. Let us first consider $d > 2$. Since kappa symmetry always halves the number of fermionic degrees of freedom and going on-shell halves it again, the number of on-shell fermionic degrees of freedom is

$$N_F = \frac{1}{2}mn = \frac{1}{4}MN. \tag{1.25}$$

Worldvolume supersymmetry demands $N_B = N_F$ and hence

$$D - d = \frac{1}{2}mn = \frac{1}{4}MN. \tag{1.26}$$

A list of dimensions, number of real dimensions of the minimal spinor and possible supersymmetries is given in Table 2, from which we see that there are only 8 solutions of (1.26) all with $N = 1$, as shown in Table 1. We note in particular that $D_{\max} = 11$ since $M \geq 64$ for $D \geq 12$ and hence (1.26) cannot be satisfied. Similarly $d_{\max} = 6$ since $m \geq 16$ for $d \geq 7$. The case $d = 2$ is special because of the ability to treat left and right moving modes independently. If we require the sum of both left and right moving bosons and fermions to be equal, then we again find the condition (1.26). This provides four more solutions all with $N = 2$, corresponding to Type II superstrings in $D = 3, 4, 6$ and 10 (or 8 solutions in all if we treat Type IIA and Type IIB separately). Both the gauge-fixed Type IIA and Type IIB superstrings will display $(8, 8)$ supersymmetry on the worldsheet. If we require only left (or right) matching, then (1.26) is replaced by

$$D - 2 = n = \frac{1}{2}MN, \tag{1.27}$$

which allows another 4 solutions in $D = 3, 4, 6$ and 10, all with $N = 1$. The gauge-fixed theory will display $(8, 0)$ worldsheet supersymmetry. The heterotic string falls into this category. The results are indicated by the points laled S in Table 1. Point particles with $d = 1$ are sometimes omitted from the brane scan but in Table 1 we have included them.

An equivalent way to arrive at the above conclusions is to list all scalar supermultiplets and to interpret the dimension of the target space, D, by

$$D - d = \text{ number of scalars.} \tag{1.28}$$

In particular, we can understand $d_{\max} = 6$ from this point of view since this is the upper limit for scalar supermultiplets. In summary, according to the above classification, Type II p-branes do not exist for $p > 1$. We shall return to this issue, however, in section (1.6).

There are four types of solution with $8 + 8$, $4 + 4$, $2 + 2$ or $1 + 1$ degrees of freedom respectively. Since the numbers 1, 2, 4 and 8 are also the dimension of the four division algebras, these four types of solution are referred to as real, complex, quaternion and octonion respectively. The connection with the division algebras can in fact be made more precise.

1.5 Type IIA superstring in D=10 from supermembrane in D=11

We begin with the bosonic sector of the $d=3$ worldvolume of the $D=11$ supermembrane given in (1.29).

$$S_3 = T_3 \int d^3\xi \left[-\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\partial_i x^M \partial_j x^N g_{MN}(x) + \frac{1}{2}\sqrt{-\gamma} + \frac{1}{3!}\epsilon^{ijk}\partial_i x^M \partial_j x^N \partial_k x^P A_{MNP}(x) \right] , \tag{1.29}$$

where T_3 is the membrane tension, ξ^i $(i=0,1,2,)$ are the worldvolume coordinates, γ^{ij} is the worldvolume metric and $x^M(\xi)$ are the spacetime coordinates $(M=0,1,\ldots,10)$. Kappa symmetry then demands that the background metric G_{MN} and background 3-form potential A_{MNP} obey the classical field equations of $D=11$ supergravity.

To see how a double worldvolume/spacetime compactification of the $D=11$ supermembrane theory on S^1 leads to the Type IIA string in $D=10$, let us denote all $(d=3, D=11)$ quantities by a hat and all $(d=2, D=10)$ quantities without. We then make a ten-one split of the spacetime coordinates

$$\hat{x}^{\hat{M}} = (x^M, y) \qquad M = 0,1,\ldots,9 \tag{1.30}$$

and a two-one split of the worldvolume coordinates

$$\hat{\xi}^{\hat{i}} = (\xi^i, \rho) \qquad i = 1,2 \tag{1.31}$$

in order to make the partial gauge choice

$$\rho = y \ , \tag{1.32}$$

which identifies the eleventh dimension of spacetime with the third dimension of the worldvolume. The dimensional reduction is then effected by taking the background fields $\hat{g}_{\hat{M}\hat{N}}$ and $\hat{A}_{\hat{M}\hat{N}\hat{P}}$ to be independent of y. The string backgrounds of dilaton ϕ, string σ-model metric g_{MN}, 1-form A_M, 2-form B_{MN} and 3-form A_{MNP} are given by

$$\begin{aligned} \hat{g}_{MN} &= e^{-2\phi/3} \begin{pmatrix} g_{MN} + e^{\Phi} A_M A_N & e^{2\phi} A_M \\ e^{2\phi} A_N & e^{\Phi} \end{pmatrix} \\ \hat{A}_{MNP} &= A_{MNP} \\ \hat{A}_{MNY} &= A_{MN} \ . \end{aligned} \tag{1.33}$$

The choice of dilaton prefactor, $e^{-2\phi/3}$, is dictated by the requirement that g_{MN} be the $D = 10$ string σ-model metric. (To obtain the $D = 10$ fivebrane σ-model metric, the prefactor is unity because the reduction is then spacetime only and not simultaneous worldvolume/spacetime. This explains the remarkable "coincidence" between $\hat{g}_{MN}$ and the $D = 10$ fivebrane σ-model metric.)

The action (1.29) now reduces to

$$S_2 = T_2 \int d^2\xi \Big[\quad - \quad \frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\partial_i X^M \partial_j X^N g_{MN}(X) \\ \quad - \quad \frac{1}{2!}\epsilon^{ij}\partial_i X^M \partial_j X^N B_{MN}(X) + \cdots \Big] \tag{1.34}$$

One may repeat the procedure in superspace to obtain

$$S_2 = T_2 \int d^2\xi \Big[-\frac{1}{2}\sqrt{-\gamma}\gamma^{ij} E_i{}^a E_j{}^b \eta_{ab} + \frac{1}{2!}\epsilon^{ij}\partial_i X^M \partial_j X^N B_{MN}(Z) \Big] \tag{1.35}$$

which is just the action of the Type IIA superstring.

1.6 Type II p-branes: the brane scan revisited

According to the classification described in section (1.4), no Type II p-branes with $p > 1$ can exist. Moreover, the only brane allowed in $D = 11$ is $p = 2$. These conclusions were based on the assumption that the only fields propagating on the worldvolume were scalars and spinors, so that, after gauge fixing, they fall only into *scalar* supermultiplets, denoted by S on the brane scan of Table 1. Indeed, for some time, these were the only kappa symmetric actions. Using soliton arguments, however, it was later realized that both Type IIA and Type IIB superfivebranes exist after all. Moreover, the Type IIB theory also admits a self-dual superthreebrane. The no-go theorem is circumvented because in addition to the superspace coordinates X^M and θ^α there are also higher spin fields on the worldvolume: vectors or antisymmetric tensors. This raised the question: are there other super p-branes and if so, for what p and D? An attempt to answer this question can be made by asking what new points on the brane scan are permitted by bose-fermi matching alone. Given that the gauge-fixed theories display worldvolume supersymmetry, and given that we now wish to include the possibility of vector and antisymmetric tensor fields, it is a relatively straightforward exercise to repeat the bose-fermi matching conditions of the section (1.4) for vector and antisymmetric tensor supermultiplets.

Let us begin with vector supermultiplets. Once again, we may proceed in one of two ways. First, given that a worldvolume vector has $(d - 2)$ degrees of freedom, the scalar multiplet condition (1.26) gets replaced by

$$D-2=\frac{1}{2}\,mn=\frac{1}{4}\,MN. \tag{1.36}$$

Alternatively, we may simply list all the vector supermultiplets and once again interpret D via (1.28). The results are shown by the points laled V in Table 1.

Next we turn to antisymmetric tensor multiplets. In $d=6$ there is a supermultiplet with a second rank tensor whose field strength is self-dual: $(B^{-}_{\mu\nu},\lambda^{I},\phi^{[IJ]})$, $I=1,\ldots,4$. This is has chiral $d=6$ supersymmetry. Since there are five scalars, we have $D=6+5=11$. There is thus a new point on the scan corresponding to the $D=11$ superfivebrane. One may decompose this $(n_{+},n_{-})=(2,0)$ supermultiplet under $(n_{+},n_{-})=(1,0)$ into a tensor multiplet with one scalar and a hypermultiplet with four scalars. Truncating to just the tensor multiplet gives the zero modes of a fivebrane in $D=6+1=7$. These two tensor multiplets are shown by the points laled T in Table 1. Several comments are now in order:

1) The number of scalars in a vector supermultiplet is such that, from (1.28), $D=3,4,6$ or 10 only.

2) Vector supermultiplets exist for all $d\leq 10$, as may be seen by dimensionally reducing the $(n=1,d=10)$ Maxwell supermultiplet. However, in $d=2$ vectors have no degrees of freedom and in $d=3$ vectors have only one degree of freedom and are dual to scalars. In this sense, therefore, these multiplets will already have been included as scalar multiplets in section (1.4). There is consequently some arbitrariness in whether we count these as new points on the scan. For example, by dualizing a vector into a scalar on the gauge-fixed $d=3$ worldvolume of the Type IIA supermembrane, one increases the number of worldvolume scalars, *i.e.* transverse dimensions, from 7 to 8 and hence obtains the corresponding worldvolume action of the $D=11$ supermembrane. Thus the $D=10$ Type IIA theory contains a hidden $D=11$ Lorentz invariance!

3) This dualizing of the scalar into a vector on the 3-dimensional worldvolume, which has the effect of lowering the spacetime dimension by one, is a special case of a more general phenomenon of dualizing scalars into antisymmetric tensors of rank $(d-2)$ on a d-dimensional worldvolumes. For example, one could argue that one should also include new points on the scan with $(d=6,D=9)$, $(d=5,D=8)$, $(d=4,D=7)$ and $(d=4,D=5)$ obtained by dualizing one of the four scalars in the hypermultiplets describing the known points at $(d=6,D=10)$, $(d=5,D=9)$, $(d=4,D=8)$ and $(d=4,D=6)$. The problem with this is knowing when to stop.

4) In listing vector multiplets, we have focussed only on the abelian theories obtained by

dimensionally reducing the Maxwell multiplet. One might ask what role, if any, is played by non-abelian Yang-Mills multiplets. See below.

5) We emphasize that the points laled V and T merely tell us what is allowed by bose/fermi matching. We must then try to establish which of these possibilities actually exists. When this scan was first written down in 1993 we knew of the following soliton solutions: First the Type IIA and Type IIB superfivebranes and the self-dual Type IIB superthreebrane (the first example of a supermembrane carrying Ramond-Ramond charges) all found in 1991, then the $D = 11$ superfivebrane found by Gueven in 1992, then the Type IIA p branes with all $p = 0, 1, 2, 3, 4, 5, 6$ found in 1993. The other points laled V were still something of a mystery. To see why these choices of p were singled out, we recall that Type II string theories differ from heterotic theories in one important respect: in addition to the usual Neveu-Schwarz charge associated with the 3-form field strength, they also carry Ramond-Ramond charges associated with 2-form and 4-form field strengths in the case of Type IIA and 3-forms and 5-forms in the case of Type IIB. Accordingly, the new solutions of the Type IIA string equations were found to describe *electric* super p-branes with $p = 0, 2$ and their *magnetic* duals with $p = 6, 4$ and new solutions of Type IIB string equations were found to describe *electric* super p-branes with $p = 1, 3$ and their *magnetic* duals with $p = 5, 3$. Interestingly enough, the Type IIB superthreebrane is *self-dual*, carrying a magnetic charge equal to its electric charge.

However, the whole subject of Type II supermembranes underwent a major sea change in 1995 when Polchinski realized that Type II super p-branes carrying Ramond-Ramond charges admit the interpretation of *Dirichlet-branes* that had been proposed earlier in 1989. These D-branes are surfaces of dimension p on which open strings can end. The Dirichlet p-brane is defined by Neumann boundary conditions in $(p+1)$ directions (the worldvolume itself) and Dirichlet boundary conditions in the remaining $(D-p-1)$ transverse directions. In $D = 10$, they exist for even $p = 0, 2, 4, 6, 8$ in the Type IIA theory and odd $p = -1, 1, 3, 5, 7, 9$ in the Type IIB theory, in complete correspondence with the points marked V on the brane scan of Table 1. The fact that these points preserve one half of the spacetime supersymmetry and are described by dimensionally reducing the $(n = 1, d = 10)$ Maxwell multiplet fits in perfectly with the D-brane picture. In hindsight there also exists a Type IIB supergravity interpretation of the (-1)-brane, which is an instanton, and its 7-brane dual. The 9-brane emerges from the fact that purely Neumann strings can end anywhere. Also in hindsight the 8-branes of the Type IIA theory also admit a soliton interpretation because there exists a version of $D = 10$ Type IIA supergravity with a gravitino mass term and cosmological constant proposed by Romans. But in D dimensions a cosmological term is equivalent to a rank D antisymmetric tensor field strength, and hence yields a $(D-2)$ -brane, which is a *domain wall*.

Moreover when N branes coincide, the individual $U(1)$s on each brane conspire to form a non-abelian $U(N)$ thus filling in the non-abelian gap in the supermultiplets.

1.7 Bibliography

Introductory treatments of supermembranes may be found in [1, 2, 3, 4, 5, 6, 7, 8, 9]

References

[1] M. J. Duff, *Supermembranes: The first fifteen weeks*, Class. Quant. Grav. **5**, 189 (1988).

[2] M. J. Duff, R. R. Khuri and J. X. Lu, *String solitons*, Phys. Rep. **259** (1995) 213, hep-th/9412184

[3] M. J. Duff, *Classical and quantum supermembranes*, Class. Quant. Grav. **6**, 1577 (1989).

[4] M. J. Duff, *Supermembranes*, Lectures given at the Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 96), Boulder, 1996, hep-th/9611203.

[5] K. S. Stelle, *An introduction to p-branes*, In *Seoul/Sokcho 1997, Dualities in gauge and string theories 39-104.

[6] P. K. Townsend, *Four lectures on M-theory*, hep-th/9612121.

[7] J. Polchinski, *String Theory*, Volumes I and II, Cambridge University Press, 1998.

[8] C. Johnson, *D-branes*, C.U.P. 2003.

[9] T. Ortin, *Gravity and Strings*, C. U. P. 2004