

Braneworld Cosmology

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A brief review of the field of braneworld cosmology, from its inception with the large extra dimension scenario, to aspects of cosmology in warped extra dimensions, including the RS-I and RS-II models, braneworld inflation, the Goldberger-Wise mechanism, mirage cosmology, the radion-induced phase transition in RS-I, possible gravity wave signals, and the DGP model.

From Strings to LHC

January 2-10 2007

International Centre Dona Paula, Goa India

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I was asked by the organizers of “Strings to LHC” to provide a brief overview of brane cosmology. This is a large subject, and my scope will be necessarily limited to some of the key ideas. In particular, I will not discuss the many interesting developments with higher-codimension branes, rather focusing on the simplest case of codimension-one. Furthermore, this is not a review of string cosmology, nor of brane-antibrane inflation. Braneworld cosmology is a string-inspired framework, where the “branes” are merely delta-function sources in the stress-energy tensor, which generally do not have the other properties of D-branes, such as charge and internal dynamics. I will give a historically ordered account starting with large extra dimensions, then turning to warped geometries and the DGP model.

1. Flat extra dimensions

The modern era of brane cosmology began with ref. [1] in the context of large extra dimensions, made possible by the hypothesis that the standard model of particle physics is localized on a D-brane. Since only gravity was presumed to propagate in the extra dimensions, only short-distance (sub-millimeter) probes of the gravitational force could constrain the size of the extra dimensions. Experimentally it was known that gravity did not deviate from its known four-dimensional form down to distances just below a millimeter, hence allowing for extra dimensions of that magnitude. Additionally, with two extra dimensions, the fundamental gravity scale M_d would be lowered to a value consistent with the TeV scale, using the relation

$$M_p^2 \sim M_d^{d-2} R^{d-4} \quad (1.1)$$

in d spacetime dimensions, with $d - 4$ having size R . Cosmology becomes highly constrained in such a scenario because the phase space for Kaluza-Klein gravitons \tilde{g} is so large that they would be overproduced if the temperature of the universe was ever greater than some maximum value $T_* \sim 1$ MeV. The gravitons are populated by interactions like $\gamma\gamma \rightarrow \tilde{g}\tilde{g}$. Even though each interaction vertex is suppressed by $1/M_p$ in the 4D effective theory, the phase space is so dense that the total cross section is similar to having couplings that are only suppressed by $1/M_6$, where M_6 at the 1 – 10 TeV scale. The KK gravitons are constrained because they can overclose the universe, or distort the diffuse γ ray spectrum by their decays at late times.

One of the key papers motivating interest in brane cosmology was ref. [2] (BDL). Their work was inspired by string theory, the Hořava-Witten model [3], in which the 11th dimension of heterotic M-theory is compactified on an interval $y \in [0, 1]$, which can be considered as a circle parametrized by $y \in [-1, 1]$ with points identified under a Z_2 orbifold symmetry $y \rightarrow -y$. The ends of the interval are the orbifold fixed points. BDL considered a 5D line element of the form

$$ds^2 = -n^2(y)dt^2 + a^2(t,y)d\vec{x}^2 + b_0^2 dy^2 \quad (1.2)$$

with branes at $y = 0$ and $y = 1$, and b_0 assumed to be constant.

The branes provide delta-function sources in the Einstein equations, which take the simple form

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{n^2}{b_0^2} \left(\frac{a''}{a} + \left(\frac{a'}{a}\right)^2\right) + \frac{n^2}{3M_5^3 b_0} \sum_i \rho_i \delta(y - y_i), \quad (1.3)$$

$$\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{a}}{a} = \frac{n^2}{b_0^2} \left\{ 2\frac{a''}{a} + \frac{n''}{n} + \frac{a'}{a} \left(\frac{a'}{a} + 2\frac{n'}{n} \right) \right\} - \frac{n^2}{M_5^3 b_0} \sum_i p_i \delta(y - y_i) \quad (1.4)$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a} = \frac{n^2 a'}{b_0^2 a} \left(\frac{a'}{a} + \frac{n'}{n} \right) \quad (1.5)$$

$$\frac{n'}{n} \frac{\dot{a}}{a} + \frac{a'}{a} \frac{\dot{b}}{b} - \frac{\dot{a}'}{a} = 0 \quad (1.6)$$

For constant equation of state $p/\rho = w$ on both branes, the solution is also simple,

$$a = t^q \left(1 - \frac{b_0 \rho_0}{6M_5^3} |y| \right) \quad (1.7)$$

$$n = 1 + \frac{b_0 \rho_0}{2M_5^3} \left(w + \frac{2}{3} \right) |y| \quad (1.8)$$

where $q^{-1} = 3(1+w)$. The time-dependence of the scale factor differs from that in standard cosmology, where $q^{-1} = 3(1+w)/2$. A simple way of seeing why this happens is to consider the Einstein equation (1.3). By integrating it over the region $y \in [-\varepsilon, \varepsilon]$, we find that

$$\int_{-\varepsilon}^{\varepsilon} \frac{a''}{a} dy = \frac{\Delta a'}{a} = 2\frac{a'}{a} = -\frac{b_0 \rho}{3M_5^2} \quad (1.9)$$

where we used $a'(-\varepsilon) = -a'(\varepsilon)$ due to the Z_2 orbifold symmetry. Similarly, integrating (1.4) in this region gives

$$\frac{n'}{n} = \frac{b_0}{3M_5^2} (3p - \rho) \quad (1.10)$$

Using these values in (1.5) and choosing $n = 1$ at $y = 0$ gives

$$H^2 + \frac{\ddot{a}}{a} = \frac{1}{36M_5^6} \rho (\rho - 3p) \quad (1.11)$$

Because of energy conservation, $\dot{\rho} = 3H(\rho + p)$, which follows from (1.6), eq. (1.11) is consistent with the first-order equation

$$H^2 = \left(\frac{\rho}{6M_5^3} \right)^2 \quad (1.12)$$

This modified form for the Friedmann equation can be solved for $a(t)$ given the usual scaling $\rho \sim a^{-3(1+w)}$, to show that $a(t) \sim t^q$ as in (1.7).

Needless to say, the dependence $H \sim \rho$ instead of the standard relation $H = \sqrt{8\pi G \rho/3}$ is ruled out by the successful prediction of big bang nucleosynthesis (BBN) for the light element abundances. BBN is quite sensitive to changes in the expansion rate of the universe.

2. Warped extra dimensions

To overcome the problem with BDL's Friedmann equation, a simple modification was proposed in [4] and [5]: add a negative bulk cosmological constant, Λ_b , which further modifies the Friedmann equation to the form

$$H^2 = \left(\frac{\rho}{6M_5^3} \right)^2 + \frac{\Lambda_b}{6M_5^3} \quad (2.1)$$

This greatly improves the situation since we can let

$$\rho = \tau + \rho_m \quad (2.2)$$

where τ represents the tension of the brane at $y = 0$, while ρ_m stands for ordinary matter or radiation on the brane. By tuning $\tau \rightarrow \tau_0$ in such a way that the constant contribution to H vanishes,

$$\left(\frac{\tau_0}{6M_5^3}\right)^2 + \frac{\Lambda_b}{6M_5^3} = 0 \quad (2.3)$$

which is the usual tuning of the cosmological constant to zero, we are left with the modified Friedmann equation

$$H^2 = \frac{\tau}{18M_5^3} \rho_m \left(1 + \frac{\rho_m}{2\tau_0}\right) \quad (2.4)$$

where we should identify the coefficient as

$$\frac{\tau_0}{18M_5^3} = \frac{8\pi}{3} G \quad (2.5)$$

This is interesting because it reproduces the standard 4D result at low energies $\rho_m \ll \tau_0$, but predicts the BDL-like modification $H \sim \rho$ in the high-energy regime, $\rho_m \gg \tau_0$.

Moreover, the background solution in the presence of $\Lambda_b < 0$ becomes the warped solution of Randall and Sundrum (RS) [6] in the limit $\rho_m \rightarrow 0$,

$$a(y) = n(y) = e^{-bk|y|} \quad (2.6)$$

thus providing a link between brane cosmology and the warped compactification scenario which was proposed for completely noncosmological reasons—namely to solve the hierarchy problem. (If τ is not tuned to the static value τ_0 , one finds warped de Sitter solutions $a(t, y) = e^{Ht} (\cosh bk|y| - (\tau/\tau_0) \sinh bk|y|)$ [7].)

However this was not a complete solution to the problems which plagued the BDL model. In both BDL and in RS it was necessary to have a negative tension brane at $y = 1$, with equal and opposite tension to the $y = 0$ brane. This is not fatal since within string theory there exist physically consistent negative-tension objects, orientifold planes. The real problem was that the matter densities on the two branes had to also be tuned, such that

$$\rho_1 = -e^{2kb_0} \rho_0 \quad (2.7)$$

where the subscript indicates the position of the brane. String theory does not provide any sources of *matter* with negative energy density. Furthermore, it was not clear why it should be necessary to tune the matter densities on the two branes.

In retrospect, it should have been obvious that something else was seriously lacking in the model of [2] since it failed to reproduce the predictions of general relativity (GR) even if the extra dimension became arbitrarily small—notice that the strange Friedmann equation (1.12) is completely insensitive to the value of b_0 , which is the size of the extra dimension. The problem was shown to be due to artificially assuming that b was static, even though there was no mechanism

in the theory to stabilize it at any particular value [8]. This criticism applied equally to the warped models of [4, 5].

In fact, b_0 should be treated as a massless scalar field $b(x^\mu)$, the *radion*, which leads to a scalar-tensor theory of gravity rather than GR. Although it is possible to find solutions where this massless scalar is constant by tuning the brane matter densities as in (2.7), fifth force constraints oblige us to give the radion a mass, and stabilize it at some particular value. Ref. [8] showed that once this is done, the standard Friedmann equation is recovered in either the warped or unwarped models, up to corrections which are suppressed by the radion mass, rather than the brane tension.

It is interesting to see how the argument which led to eq. (1.12) is changed once we add a potential energy $V(b)$ for the radion (and possibly a bulk cosmological constant Λ_b). The 5D Einstein equations become

$$3H^2 - 3\frac{n^2}{b_0^2} \left(\frac{a''}{a} + \left(\frac{a'}{a} \right)^2 \right) = \frac{n^2}{M_5^3} \left(\sum_i \rho_i \delta(b_0(y - y_i)) + \Lambda_b + V(b_0) \right) \quad (2.8)$$

$$H^2 + 2\frac{\ddot{a}}{a} - \frac{n^2}{b_0^2} \left\{ 2\frac{a''}{a} + \frac{n''}{n} + \frac{a'}{a} \left(\frac{a'}{a} + 2\frac{n'}{n} \right) \right\} = -\frac{n^2}{M_5^3} \left(\sum_i p_i \delta(b_0(y - y_i)) + \Lambda_b + V(b_0) \right) \quad (2.9)$$

$$-3 \left(H^2 + \frac{\ddot{a}}{a} \right) + 3\frac{n^2}{b_0^2} \frac{a'}{a} \left(\frac{a'}{a} + \frac{n'}{n} \right) = -\frac{n^2}{M_5^3} \left(\Lambda_b + V(b_0) + b_0 V'(b_0) \right) \quad (2.10)$$

where $V(b_0)$ is the radion potential, and we are still assuming that b_0 is constant in space and time for simplicity. The junction conditions (1.9-1.10) are unchanged by the radion potential, but the 5-5 Einstein equation (2.10) no longer determines $H^2 + \frac{\ddot{a}}{a}$ in terms of brane sources because of the extra $V(b_0)$ contribution. Instead this equation determines the shift in the radion due to the expansion of the universe, $b \rightarrow b_0 + \delta b$. For the Randall-Sundrum model, ref. [8] finds

$$\frac{\delta b}{b} \sim \frac{(\rho - 3p)}{m_b^2 a^2 M_p^2} \quad (2.11)$$

where m_b is the radion mass, and a is the warp factor; therefore $aM_p \equiv \Lambda$ is the TeV (or infrared) scale.

It is no longer simple to compute the modifications to the Friedmann equation once the radion is stabilized. Moreover the result depends upon whether the expansion of the universe is measured at the Planck brane ($y = 0$) or the TeV brane ($y = 1$). At the TeV brane, the leading correction was found to be [9]

$$H^2|_{y=1} = \frac{8\pi G}{3} \left(\rho + \frac{c\rho^2}{m_b^2 \Lambda^2} (1 - 3w)(1 + w) + O(\rho^2) \right) \quad (2.12)$$

where $c = O(1)$, ρ is the warped energy density on the TeV brane, and $w = p/\rho$. Curiously the ρ^2 correction vanishes when $w = 1/3$ (radiation) or $w = -1$ (inflation).

3. Randall-Sundrum II cosmology

Despite its shortcomings (because it ignored the need for radion stabilization), the modified Friedmann equation (2.4) turned out to have a deeper significance. This is in connection with

the Randall-Sundrum II model [10], in which the TeV brane is displaced to $y \rightarrow \infty$ (equivalently $b_0 \rightarrow \infty$) and thereby removed from the theory. There is no physical radion in this theory (it can be gauged away by a coordinate transformation), hence no need for stabilization. Eq. (2.4) is still the correct Friedmann equation, but now it is fully justified, unlike in the RSI model. Big bang nucleosynthesis puts a weak constraint on the brane tension; $\tau > (\text{MeV})^4$ to insure that H has the standard form during BBN. Terrestrial measurements of the gravitational force give a much stronger constraint since the exchange of KK gravitons modifies the Newtonian gravitational potential to the form

$$V = -G \frac{m_1 m_2}{r} \left(1 + \frac{1}{r^2 k^2} \right) \quad (3.1)$$

with $k = M_5^3/M_p^2$. Interpreting the current limit from the Eöt-Wash experiment [11] to imply that $kr > 1$ at the scale 0.04 mm, we find the bound

$$k > 5 \text{ meV} \quad (3.2)$$

hence

$$M_5 = k^{1/3} M_p^{2/3} > 3 \times 10^5 \text{ TeV} \quad (3.3)$$

and the brane tension is

$$\tau = 24kM_5^3 > (7 \text{ TeV})^4, \quad (3.4)$$

far above the energy scale of BBN.

3.1 Inflation with nonstandard Friedmann equation

An obvious application of the modified Friedmann equation is to inflation at scales greater than $\tau^{1/4}$, where $H \sim \rho$. Ref. [12] examined chaotic inflation in this context, noting that one can get inflation to work with steeper potentials than usual, since H is larger than its conventional value and Hubble damping is more effective. It was shown that the slow-roll parameters get modified relative to their conventional values (denoted by ϵ_0 and η_0),

$$\epsilon = \epsilon_0 \left(\frac{1 + V/\tau}{(1 + V/2\tau)^2} \right) \sim \frac{4\tau}{V} \epsilon_0 \quad (3.5)$$

$$\eta = \eta_0 \frac{1}{1 + V/2\tau} \sim \frac{2\tau}{V} \eta_0 \quad (3.6)$$

where V is the potential and the final estimates are in the limit $V \gg \tau$. One can show that inflation occurs when $\dot{\phi}^2 < \frac{2}{3}V$ ($p < -\frac{2}{3}\rho$) instead of the usual condition $\dot{\phi}^2 < V$ ($p < -\frac{1}{3}\rho$) in this regime. Furthermore the number of e-foldings as a function of the inflaton field is given by

$$N \cong -\frac{1}{M_p^2} \int d\phi \frac{V}{V'} \left(1 + \frac{V}{2\tau} \right) \quad (3.7)$$

and the scalar power amplitude is enhanced relative to its usual value,

$$A_s^2 = A_0^2 \left(1 + \frac{V}{2\tau} \right)^2 \quad (3.8)$$

The spectral index formula keeps its usual form in the new slow-roll parameters, but their relation to the number of e-foldings in general differs from the standard result (shown below in parentheses) in the high-energy regime [13]:

$$n_s - 1 \cong -6\epsilon + 2\eta \cong \begin{cases} -\frac{5}{2N} \text{ (c.f. } -\frac{2}{N}), & V = m^2\phi^2 \\ -\frac{3}{N} \text{ (c.f. } -\frac{3}{N}), & V = \lambda\phi^4 \end{cases} \quad (3.9)$$

Coincidentally, the prediction for ϕ^4 chaotic inflation does not change, although [13] shows that in the intermediate region where $V \sim \tau$ this is no longer the case, and the tension between $\lambda\phi^4$ theory and the CMB data is made somewhat worse. For the ϕ^2 model, the increase in $|n_s - 1|$ in the high V regime degrades the agreement with the data (viewed as likelihood contours in the plane of the tensor-to-scalar ratio r versus $n_s - 1$) relative to the standard $V \ll \tau$ regime: it pushes it outside the $1\text{-}\sigma$ contour.

A novel feature is that inflation can occur with subPlanckian field values, $\phi < M_p$, which is not the case for conventional chaotic inflation, and is often regarded as a drawback. However, one still needs $\phi > M_5$; for $V = m^2\phi^2$, inflation occurs for $\phi \sim 300M_5$, and the COBE normalization requires $m = 5 \times 10^{-5}M_5$.

Braneworld inflation was also considered using the potential

$$V = V_0 e^{-\alpha\phi/M_p} \quad (3.10)$$

in [14], which relies upon gravitational particle production to get reheating since there is no minimum around which ϕ can oscillate to give ordinary reheating. In standard cosmology, one needs to have $\alpha^2 < 2$ in order to get inflation from (3.10), but this is no longer true in the high-energy regime. Ref. [14] finds that with the COBE normalization the inflationary scale, brane tension and 5D Planck mass are fixed in terms of α as

$$V_{\text{inf}} \sim \left(\frac{10^{15} \text{ GeV}}{\alpha} \right)^4, \quad \tau \sim \left(\frac{10^{15} \text{ GeV}}{\alpha^{3/2}} \right)^4, \quad M_5 \sim \frac{3 \times 10^{16} \text{ GeV}}{\alpha} \quad (3.11)$$

The number of e-foldings as a function of field value is given by

$$N = \frac{1}{2\tau\alpha^2} (V(\phi) - V_{\text{end}}) \quad (3.12)$$

in the high-energy phase. This shows that α can be taken much larger than the standard range if τ is appropriately adjusted downward, and so the potential can be much steeper than usual while still supporting slow-roll inflation. The eventual reheating temperature is found to be $T \sim 10^3/\alpha$ GeV. Independently of α , the predicted relation between spectral index and number of e-foldings is

$$n_s - 1 = -\frac{4}{N+1} \quad (3.13)$$

leading to $n_s \cong 0.92$ as well as the tensor-to-scalar ratio $r = 0.03$. This value of n_s is somewhat too low relative to the currently favored WMAP3 value $n_s = 0.95$.

4. Randall-Sundrum I cosmology

We now return to the more rich and complicated case of two branes in a compact warped geometry. Notice that eq. (2.11) predicts that the radion gets destabilized at temperature

$$T \sim \sqrt{m_b \Lambda} \sim \varepsilon \times \text{TeV} \quad (4.1)$$

where $\varepsilon \sim m_b/\Lambda$ is a small parameter which we shall specify below. To understand the dynamics of this phase transition between compact and noncompact extra dimensions, we must discuss the mechanism of stabilization. The most popular such mechanism was proposed by Goldberger and Wise (GW) [15].

4.1 Goldberger-Wise mechanism

The GW mechanism uses a bulk scalar field Φ with Lagrangian

$$\mathcal{L} = (\partial\Phi)^2 - m^2\Phi^2 - V_0\delta(y) - V_1\delta(y-1) \quad (4.2)$$

where the brane potentials $V_{0,1}$ are minimized at $\Phi = v_0$ and $\Phi = v_1$, respectively. There are two competing effects in this model. The mass term

$$b \int dy e^{-4kby} m^2 \Phi^2 \quad (4.3)$$

is minimized when b is small, while the gradient energy

$$\frac{1}{b} \int dy e^{-4kby} \Phi'^2 \quad (4.4)$$

is minimized at large b . The brane potentials insure that the trivial solution $\Phi = 0$ is energetically disfavored; instead (assuming the coefficients of $V_{0,1}$ are sufficiently large) Φ must vary between the boundary values v_0 and v_1 . The result of this competition is that b gets stabilized at some intermediate value,

$$b = \frac{1}{k\varepsilon} \ln \left(\frac{v_0}{v_1} \right) \quad (4.5)$$

where $\varepsilon \cong m^2/4k^2 \ll 1$. The canonically normalized radion field can be written approximately as

$$\phi = f e^{-kb(x)}, \quad f = \sqrt{6}M_p \quad (4.6)$$

and its VEV using (4.5) is related to the warp factor by

$$\frac{1}{f} \langle \phi \rangle = e^{-kb} = \eta^{1/\varepsilon} \equiv \left(\frac{v_1}{v_0} \right)^{1/\varepsilon} \quad (4.7)$$

Notice that the large hierarchy is naturally generated by raising a moderately small number η to a moderately large power $1/\varepsilon$. The radion potential has the approximate form

$$V(\phi) = \frac{k v_0^4}{f^4} \phi^4 \left[\left(\left(\frac{\phi}{f} \right)^\varepsilon - \eta \right)^2 - \frac{\varepsilon}{4} \eta^2 \right] \quad (4.8)$$

This looks like a double-well potential, fig. 1, with a slightly deeper minimum at $\phi = \langle \phi \rangle$ than at $\phi = 0$. (This depth can be changed by adjusting the TeV brane tension.) The local minimum at $\phi = 0$ corresponds to the decompactified RS II theory, where $b \rightarrow \infty$. This effective potential predicts that the radion mass scales like $\varepsilon^{3/4}\Lambda$ [16, 17], but a more careful analysis shows that actually $m_b \sim \varepsilon\Lambda$ [18].

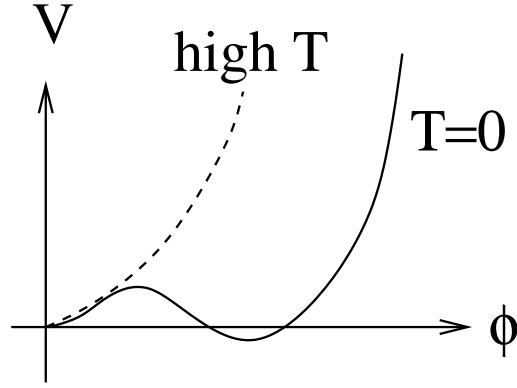


Figure 1: The Goldberger-Wise radion potential at zero temperature (solid curve) and at high temperature (dashed curve).

4.2 Decompactification phase transition

At high temperatures, the GW potential naturally gets a thermal correction which lifts the nontrivial minimum and leads to a transition between the compactified and decompactified phases of the theory (fig. 1). This was first studied in [17], and has been more recently considered in [19]-[21]. A simple way of understanding the destabilization at high temperature is through the coupling of the radion to matter. It couples conformally, *i.e.* to the trace of the stress-energy tensor,

$$\mathcal{L}_{\text{int}} \sim \frac{\phi}{\Lambda} T_{\mu}^{\mu} = \frac{\phi}{\text{TeV}} (\rho - 3p) \quad (4.9)$$

This predicts no shift in the radion during a radiation-dominated era; however at high T there is also a correction to the radion mass, $T^2\phi^2$, and its effect is parametrically the same as that of the linear coupling if we simply estimate $\rho - 3p \sim T^4$. If V_0 is the zero-temperature potential, we can estimate the shift in the radion by taking

$$\frac{dV}{d\phi} = \frac{dV_0}{d\phi} - \frac{T_{\mu}^{\mu}}{\Lambda} = 0 \quad (4.10)$$

Letting $\phi = \phi_0 + \delta\phi$, this implies

$$\frac{d^2V_0}{d\phi^2} \delta\phi = m_b^2 \delta\phi \sim \frac{T^4}{\Lambda} \quad (4.11)$$

Using the fact that $\phi_0 = f e^{-kb_0} \sim \Lambda$, we see that

$$\frac{\delta\phi}{\phi_0} \sim \frac{T^4}{\Lambda^2 m_b^2} \quad (4.12)$$

which becomes of order unity at the critical temperature given by eq. (4.1).

More insight into the high-temperature limit of the theory can be gained by considering the 5D solution to the Einstein equations at finite temperature: this is the AdS-Schwarzschild solution

where there is a black hole in the bulk, located at the AdS horizon $r = 0$ [22] (which corresponds to $y = \infty$ in the RS coordinate system),

$$ds^2 = - \left(\frac{r^2}{\ell^2} - \frac{\mu}{r^2} \right) dt^2 + r^2 d\vec{x}^2 + \left(\frac{r^2}{\ell^2} - \frac{\mu}{r^2} \right)^{-1} dr^2 \quad (4.13)$$

The temperature of the system is identified with the Hawking temperature of the black hole, whose mass is μ , and $\ell = 1/k$ is the AdS curvature length scale. In this coordinate system the branes are moving through the bulk, toward larger values of r , with the Planck brane at $r = R(t)$ say, and the TeV brane at some smaller value of r . There is a horizon at

$$r = \mu^{1/4} \ell \quad (4.14)$$

behind which the TeV brane is hidden before the compactification phase transition.

The brane motion can be understood by considering the induced metric on the brane at $r = R(t)$:

$$ds^2 = - \left(\frac{R^2}{\ell^2} - \frac{\mu}{R^2} - \dot{R}^2 \left(\frac{R^2}{\ell^2} - \frac{\mu}{R^2} \right)^{-1} \right) dt^2 - R^2 d\vec{x}^2 \equiv -d\tau^2 + R^2(\tau) d\vec{x}^2 \quad (4.15)$$

where τ is the proper time for an observer on the brane. It is clear from the latter form that $R(\tau)$ plays the role of the scale factor for the cosmological expansion measured by the brane observer. The dynamics of $R(\tau)$ are determined by the junction condition at the brane, leading to the Friedmann equation

$$H^2 = \frac{\mu}{R^4} \quad (4.16)$$

which has the interpretation of μ/R^4 being the energy density of ‘‘dark radiation,’’ the Hawking radiation emitted by the black hole, consisting of thermal KK gravitons in the bulk. Notice that this Friedmann equation has the normal form, $H^2 \sim \rho$, rather than ρ^2 , since radiation redshifts like $1/R^4$. In the AdS/CFT correspondence, these are the thermalized CFT degrees of freedom. This picture in which the geometry is static but the branes are moving has been called ‘‘mirage cosmology,’’ [23] since the expansion of the universe appears to be an illusion on the part of the brane observers, due to their motion through the bulk.

Of course, the same physics can also be seen in the RS coordinates, where the geometry is explicitly time dependent and the branes are stationary [24]. The metric functions are

$$a^2(t, y) = a_0^2(t) e^{-2kby} + \frac{c}{a_0^2(t)} \sinh^2 kby; \quad n(t, y) = \frac{a_0}{a} \left(e^{-2kby} - \frac{c}{a_0^4(t)} \sinh^2 kby \right) \quad (4.17)$$

and the horizon is at

$$\begin{aligned} \frac{e^{-2kby}}{\sinh^2 kby} &\cong 4e^{-4kby} = \frac{c}{a_0^4} \\ \implies y &= \frac{1}{4kb} \ln \frac{4c}{a_0^4} \end{aligned} \quad (4.18)$$

which recedes toward the AdS horizon as the universe expands. In these coordinates c/a_0^4 is the dark radiation density, so $c \sim \mu$ in terms of the coordinates in (4.13). At some critical temperature

the TeV brane is unclocked by the horizon, signaling the compactification phase transition [25]. In the prior epoch when the TeV brane is hidden below the horizon, it is as good as nonexistent, since it is inaccessible to observers outside the horizon. For them, the TeV brane might as well be at $r = 0$, which is the decompactified phase of the theory.

The purely geometrical interpretation given above does not include the effects of radion stabilization. From the shape of the radion potential shown in fig. 1, we see that there is a first-order phase transition of a peculiar kind: within the high- T 5D single-brane universe, bubbles nucleate, inside of which the universe is 4D and the TeV brane has appeared. References [17]-[20] show that the phase transition does not necessarily complete (as in old inflation) for some ranges of parameters of the GW stabilization mechanism. Ref. [17] found that the bubble nucleation rate $\Gamma \sim e^{-S}$ is suppressed by the tunneling action

$$S \sim \frac{\Lambda^2 M_p^2 e^{-2/\sqrt{\epsilon}}}{\epsilon v_1 \sqrt{k} T^2} \sim \frac{e^{-2/\sqrt{\epsilon}} \Lambda^3}{T^2 m_b} \quad (4.19)$$

at temperature T , so the transition is slower if the radion mass m_b is small or if ϵ is large. Ref. [19] obtains quantitatively different but qualitatively similar results.

An interesting recent development is the prediction of gravity waves produced during the phase transition, which can be large enough to be observable at LISA if the phase transition is strongly enough first order [20]. The gravity waves are produced by bubble collisions and by turbulence. However they are only observably large for model parameters where perturbation theory is starting to break down, so it would be interesting to undertake a more careful calculation of the effect.

5. The DGP model

Dvali, Gabadadze and Porrati (DGP) found a remarkably simple way in which branes can modify gravity at large distances [26], using the action

$$S = 2M_p^2 \int d^4x \sqrt{-g} R^{(4)} + 2M_5^3 \int d^4x dy \sqrt{-G} R^{(5)} \quad (5.1)$$

where g and G are the 4D and 5D metric determinants, respectively. By including the 4D Einstein term on a 3-brane, gravity is quasilocalized there, up to the distance scale

$$r_c \sim \frac{M_p^2}{M_5^3} \quad (5.2)$$

At distances $r < r_c$, gravity looks 4D, while for $r > r_c$ it looks 5D. The cosmology of this model was first studied in [27, 28], which found the modified Friedmann equation

$$\frac{\rho}{3M_p^2} = H^2 \pm \frac{H}{r_c} \quad (5.3)$$

having two branches of solutions,

$$H = \sqrt{\frac{\rho}{3M_p^2} + \frac{1}{4r_c^2}} \mp \frac{1}{2r_c} \quad (5.4)$$

The upper sign leads to conventional cosmology as $\rho \rightarrow 0$, while the lower one has the interesting property that $H \rightarrow 1/r_c$ even as the matter density on the brane vanishes. This is called the *self-accelerating* solution, and gives rise to the possibility of explaining the accelerated expansion of the universe without an explicit cosmological constant term. To match the current Hubble rate, a small 5D gravity scale is required:

$$M_5 = (M_p^2 H_0)^{1/3} \sim 10 \text{ MeV} \quad (5.5)$$

In principle this kind of acceleration is experimentally distinguishable from that due to dark energy, even if it has an arbitrary (but constant) equation of state $w = p/\rho$. The luminosity distance $d_L(z)$ as a function of redshift z ,

$$d_L = (1+z) \int_0^z dx \frac{H_0}{H(x)} \quad (5.6)$$

has a different shape than in conventional dark energy models, so if the expansion history can be probed accurately as a function of z , as with type I supernovae, one can test the DGP model relative to fluid models.

Interestingly, the DGP model also predicts nonstandard gravity at much shorter scales, potentially including the solar system. This comes about because of an extra scalar graviton polarization π which changes the strength of the Newtonian gravitational potential in a distance-dependent way. One can think of π as a “brane-bending” mode, associated with transverse fluctuations of the brane. It modifies the strength of gravity at distances $r > r_* = (r_g r_c^2)^{1/3}$, where $r_g = 2M/M_p^2$ is the gravitational radius of a source of mass M , for example the sun if we are the testing solar system. Thus

$$r_* = \left(\frac{2MM_p^2}{M_5^3} \right)^{1/3} \quad (5.7)$$

Naively, the extra polarization π would spoil GR down to zero distance, but nonlinear terms in the Einstein equations become important when $r < r_*$ and these suppress the scalar contribution to the gravitational potential [29]. The different distance regimes and corresponding gravitational potentials are illustrated in fig. 2.

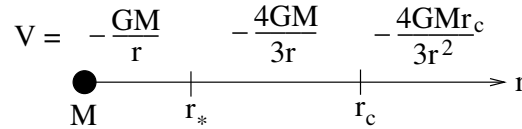


Figure 2: Distance scales and corresponding behavior of gravitational potential in the DGP model.

Of course even in the 4D regime, there are small corrections to the Newtonian potential, and these lead to anomalous precession of planetary orbits, and that of the moon. The precession rate turns out to be independent of the masses of the source or the orbiting body (see [30]),

$$\frac{d\Delta\phi}{dt} = \mp \frac{3}{8r_0} = \mp 5 \mu\text{as/year} \quad (5.8)$$

which might be probed by future lunar laser ranging experiments.

There is yet another mass/distance scale associated with the π mode,

$$\Lambda = \frac{M_5^2}{M_p} \equiv r_\Lambda^{-1} \sim (10^3 \text{ km})^{-1} \quad (5.9)$$

For $r < r_\Lambda$, the self coupling $(\partial\pi)^2 \square\pi/\Lambda^3$ of the effective Lagrangian for the π becomes strong. If generic operators of the form $(\partial\pi)^{2N}/\Lambda^{4N-4}$ were also present, as would be true for an ordinary Goldstone boson, this strong coupling would cause the theory to lose predictivity at any $r < r_\Lambda$. Remarkably, such operators are not present in the DGP model, due to the unconventional shift symmetry $\partial_\mu\pi \rightarrow \partial_\mu\pi + c_\mu$, which is a remnant of the 5D Lorentz symmetry. Hence predictivity is not spoiled in the DGP model. However, precisely the absence of the $(\partial\pi)^4/\Lambda^4$ term leads to a violation of causality and locality, as shown by ref. [31]. This indicates that the DGP model has no ultraviolet completion which respects analyticity of the S-matrix, hence making it appear quite unlikely that it can be embedded within string theory.

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