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Masses, Mass Shifts and Higgs

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The general relativistic notion of gravitational and inertial mass is discussed from the general viewpoint of the tidal forces implicit in the curvature and the Einstein field equations within ponderable matter. A simple yet rigorously general derivation is given for the Tolman gravitational mass viewpoint wherein the computation of gravitational mass requires both a rest energy contribution (the inertial mass) and a pressure contribution. The pressure contribution is extremely small under normal conditions which implies the equality of gravitational and inertial mass to a high degree of accuracy. However, the pressure contribution is substantial for conformal symmetric systems such as Maxwell radiation, whose constituent photons are massless. On the other hand the standard model of particle physics attributes the mass growth of many elementary particles to a conjectured Higgs field. If such is the case, there should exist a subtle connection between mass as it enters into general relativity and mass as it enters into Higgs symmetry breaking. This connection is here briefly explored.

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1. Introduction

In standard model particle physics, the masses of elementary particles are thought to be determined by the Higgs symmetry breaking mechanism. It is also thought that mass is the ultimate source of gravitational fields. This strongly suggests a connection between gravity and the Higgs mechanism which has hardly been explored. Our purpose is to take some small initial steps in this direction. In Sec.2, we discuss the connection between inertial mass and gravitational mass as they enter into general relativity. Surprisingly, inertial and gravitational masses are not quite equal. In Sec.3, we discuss the coupling between the gravitational field and the Higgs particle. Surprisingly, we find that the decay of the Higgs into two gravitons may well be the dominant decay mechanism over all other known Higgs decay modes. In Sec.4, we discuss a proximity effect wherein the mass of one particle grows because there is another heavy particle in its neighborhood. Surprisingly, we find that mass shifts allow for the detection of the Higgs field possibly below the threshold for producing the Higgs particle.

2. Inertial and Gravitational Mass

In modern field theories, including supersymmetry, the sources of mass are just as obscure as were the sources of standard model interactions before the introduction of the gauge symmetry principle. General relativity does give to us a principle for the mass of ponderable matter in which the inertial and gravitational masses are not quite equal. To see what is involved, let us consider a condensed matter flow with a four velocity v^μ and a *scalar* energy density ε. These macroscopic fields may be computed by solving an eignenvalue problem for the energy-pressure tensor $T_{\mu\nu}$. In detail

$$T_{\mu\lambda}v^\lambda = -\varepsilon v_\mu \quad \text{wherein} \quad v^\mu v_\mu = -c^2 \quad \text{and} \quad \varepsilon = \rho_i c^2. \tag{2.1}$$

We have identified the scalar *inertial* mass density ρ_i in a self evident manner as being equivalent to a scalar energy density ε.

The energy-pressure tensor then has the form

$$T_{\mu\lambda} = \varepsilon\frac{v^\mu v^\lambda}{c^2} + P_{\mu\lambda} \quad \text{wherein} \quad P_{\mu\lambda}v^\lambda = 0. \tag{2.2}$$

The pressure tensor $P_{\mu\nu}$ has three scalar eigenvalues corresponding to space-like eigenvectors;

$$P_{\mu\lambda}n_j^{\ \lambda} = P_j n_{j\mu} \quad \text{wherein} \quad n_i^{\ \mu} n_{j\mu} = \delta_{ij} \quad \text{and} \quad 3P = \sum_{j=1}^{3} P_j. \tag{2.3}$$

Since one third the sum of these eigenvalues determines the mean scalar pressure $P = P^\mu_\mu/3$, the trace of the total energy-pressure tensor reads

$$\Theta \equiv -T^\mu_\mu = \varepsilon - 3P. \tag{2.4}$$

To compute the scalar *gravitational* mass density ρ, one may begin with the tidal force tensor [1] [2] $\Phi_{\mu\lambda}$ which in general relativity is determined by the curvature $R_{\mu\sigma\lambda\beta}$ via

$$\Phi_{\mu\lambda} = R_{\mu\sigma\lambda\beta}v^\sigma v^\beta. \tag{2.5}$$

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Employing Eqs.2.1-2.5 along with the Einstein gravitational field equations

$$R_{\mu\lambda} = \frac{8\pi G}{c^4}\left(T_{\mu\lambda} - \frac{1}{2}g_{\mu\lambda}T^{\sigma}_{\ \sigma}\right) \tag{2.6}$$

it follows rigorously from general relativity that the trace of the tidal force tensor obeys[3]

$$\Phi^{\mu}_{\ \mu} = R_{\mu\lambda}v^{\mu}v^{\lambda} \quad \Rightarrow \quad \Phi^{\mu}_{\ \mu} = \frac{4\pi G}{c^2}(\varepsilon + 3P). \tag{2.7}$$

Since Eq.2.7 is the rigorous general relativistic version of the non-relativistic Newtonian field equation $\Phi^{j}_{\ j} \equiv \nabla^2\Phi = 4\pi G\rho$ in thinly disguised form, it follows that the scalar gravitational mass density ρ is given by

$$\rho = \frac{\varepsilon + 3P}{c^2} = \rho_i + \frac{3P}{c^2}\ . \tag{2.8}$$

Only in the non-relativistic formal limit $c \to \infty$ can the inertial and gravitational mass densities be identified; i.e. $\rho \approx \rho_i$ if $|3P| \ll \rho_i c^2$. General relativistic arguments require that $\varepsilon \geq 3P$ which implies an inequality between gravitational mass density ρ and inertial mass density ρ_i,

$$\rho \leq 2\rho_i. \tag{2.9}$$

(i) For a gas of non-interacting photons $\Theta = \varepsilon - 3P = 0$ yielding $\rho = 2\rho_i$ which must be taken into account to obtain the correct bending of light around the sun. (ii) For systems which are conformally invariant $\Theta = \varepsilon - 3P = 0$ so that gravitational masses are twice as large as inertial masses. (iii) The gluon Lagrangian exhibits conformal symmetry. If such symmetry remained unbroken, then the gravitational mass of a glue ball would be twice the inertial mass of a glue ball. If one is to maintain that glue is responsible for much of the mass in macroscopic systems, then conformal symmetry must be broken. (iv) In principle, one must prove conformal symmetry breaking within the QCD model by firstly assigning a mass $\tilde{m}$ to the gluon. Secondly, after proceeding to calculate the infinite volume limit $\Theta_{\text{Glue}}(\tilde{m}) = \lim_{V\to\infty}\left\langle -(T_{\text{Glue}})^{\mu}_{\ \mu}\right\rangle_V$, one must prove that as the assigned gluon mass goes to zero there is indeed a symmetry break; $\lim_{\tilde{m}\to 0}\Theta_{\text{Glue}}(\tilde{m}) \neq 0$. Such a proof, even for pure glue, does not yet exist. (v) Since the matter which we see around us satisfies to a high accuracy the equality of inertial and gravitational mass, $\rho \approx \rho_i$ must be a requirement on any dynamical scheme of conformal symmetry breaking. Let us see how this works for the case of the Higgs model.

3. Mass and Gravity in the Higgs Model

There is a Higgs potential energy density $U(\varphi)$ constructed, by close analogy with Ginzburg-Landau superconductivity theory, to yield a vacuum expectation value $v = \langle 0|\varphi|0\rangle \neq 0$. The hope that a Higgs particle will be observed is based on an expansion

$$\varphi = \frac{1}{\sqrt{2}}\,(v + \sigma) \quad \text{wherein} \quad |\sigma| \ll v. \tag{3.1}$$

It should be noted that the particle of the superconductivity Ginzburg-Landau model (the Cooper pair) is not usually treated in this manner. With the masses of elementary particles written as

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$m_a = (\hbar f_a/c)v$, the trace of the energy-pressure tensor

$$-\Theta = \sum_a m_a \left(\frac{\partial \mathscr{L}}{\partial m_a}\right), \tag{3.2}$$

may be employed to write the lowest order interaction between a Higgs particle and the rest of the particles; i.e.

$$\mathscr{L}_{int} = -\left(\frac{\sigma}{v}\right)\Theta + \cdots \ . \tag{3.3}$$

The particle nature of the Higgs field should be made manifest via the propagator,

$$i\,\langle 0\,|\sigma(x)\sigma(y)|\,0\rangle_+ \equiv \tilde{D}_H(x-y) = \int D_H(k^2 - i0^+)e^{ik\cdot(x-y)}\ \frac{d^4k}{(2\pi)^4}\ , \tag{3.4}$$

wherein the subscript "+" indicates time ordering. With the lowest order Higgs mass $M_H = (\hbar\kappa/c)$, one finds the propagator expression

$$D_H(k^2) = \frac{1}{k^2 + \kappa^2 - \Pi(k^2)}\ . \tag{3.5}$$

In virtue of Eqs.3.3-3.5, one finds the lowest order expression for the Higgs self-energy,

$$\tilde{\Pi}(x-y) = \left(\frac{1}{\hbar c v}\right)^2 i\,\langle 0\,|\Theta(x)\Theta(y)|\,0\rangle_+ = \int \Pi(k^2)e^{ik\cdot(x-y)}\ \frac{d^4k}{(2\pi)^4}\ , \tag{3.6}$$

which implies

$$\Pi(k^2) = \left(\frac{\sqrt{2}G_F}{\hbar c^5}\right) i \int \langle 0\,|\Theta(x)\Theta(0)|\,0\rangle_+ e^{-ik\cdot x}\ d^4x, \tag{3.7}$$

wherein G_F is the Fermi weak interaction coupling strength. The Higgs decay rate is thereby

$$\Gamma_H = \frac{c}{\kappa}\Im m\ \Pi(\kappa^2 - i0^+) = \left(\frac{\sqrt{2}G_F}{\hbar c^4 \kappa}\right)\Re e\left[\int \langle 0\,|\Theta(x)\Theta(0)|\,0\rangle_+ e^{-ik\cdot x}\ d^4x\right]_{k^2=-\kappa^2}\ . \tag{3.8}$$

Employing the partial stress tensor trace $-\Theta_a$ from Eq.3.2 allows for the computation of the Higgs decay rate into a particle-antiparticle pair $\Gamma_a \equiv \Gamma(H \to a + \bar{a})$. It is

$$\Theta_a = -m_a\left(\frac{\partial \mathscr{L}}{\partial m_a}\right)\ \Rightarrow\ \Gamma_a = \left(\frac{\sqrt{2}G_F}{\hbar c^4 \kappa}\right)\Re e\left[\int \langle 0\,|\Theta_a(x)\Theta_a(0)|\,0\rangle_+ e^{-ik\cdot x}\ d^4x\right]_{k^2=-\kappa^2}\ . \tag{3.9}$$

Conventional Higgs decay calculations have been based on Eq.3.9.

On the other hand, the Einstein Eq.2.7 yields a scalar curvature identity when operating on physical quantum states,

$$R(x)\,|\text{physical}\rangle = -\left(\frac{8\pi G}{c^4}\right) T^{\mu}_{\mu}(x)\,|\text{physical}\rangle = \left(\frac{8\pi G}{c^4}\right)\Theta(x)\,|\text{physical}\rangle\,, \tag{3.10}$$

so that

$$\langle 0\,|\Theta(x)\Theta(0)|\,0\rangle_+ = \left(\frac{c^4}{8\pi G}\right)^2 \langle 0\,|R(x)R(0)|\,0\rangle_+\,. \tag{3.11}$$

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Eqs.3.8 and 3.11 imply the possibility for a Higgs to decay into two gravitons at a rate

$$\Gamma(H \to g+g) = \left(\frac{\sqrt{2}G_F}{\hbar c^4 \kappa}\right)\left(\frac{c^4}{8\pi G}\right)^2 \Re e\left[\int \langle 0|R(x)R(0)|0\rangle_+ e^{-ik\cdot x}\, d^4x\right]_{k^2=-\kappa^2} . \quad (3.12)$$

Expanding the scalar curvature in terms of operators creating and destroying two gravitons and inserting Fermion ghost terms to preserve gravitational gauge invariance, yields the total rate[4][5]

$$\Gamma(H \to g+g) = \frac{\sqrt{2}}{16\pi}\left(\frac{G_F M_H^2}{\hbar c}\right)\left(\frac{M_H c^2}{\hbar}\right). \quad (3.13)$$

Note that the above decay rate $\Gamma(H \to g+g)$ is independent of the gravitational coupling strength G. Presuming the Higgs mass were in the range 120 Gigavolt $< M_H c^2/e <$ 160 Gigavolt, the two graviton decay would be dominant over all of the other studied decay channels of the Higgs. The gravitons would interact so weakly with ponderable matter that their existence could only be inferred from missing energies and momenta. This experimental situation would be closely analogous to case of known decays into neutrinos, say $Z \to \nu + \bar{\nu}$, wherein the neutrinos can only be detected via missing energies and momenta.

4. Higgs Induced Mass Interactions

The effective action describing the exchange of a Higgs which employs Θ as a source and sink follows from Eq.3.3. It is

$$S_{\text{H}-\text{exhcnage}} = \frac{\sqrt{2}G_F}{2c^5}\int\int \Theta(x)\tilde{D}_H(x-y)\Theta(y)d^4x d^4y. \quad (4.1)$$

For a classical particle moving on a path $\mathscr{P}$ determined by the space-time position X as a function of proper time τ, the source field for the Higgs

$$\Theta_{\mathscr{P}}(x) = Mc^2\int_{\mathscr{P}} \delta(x-X(\tau))(cd\tau). \quad (4.2)$$

The interaction between two such particles is thereby

$$S_{ab} = \sqrt{2}G_F M_a M_b c\int_{\mathscr{P}_a}\int_{\mathscr{P}_b} \tilde{D}_H(X_a(\tau_a)-X_b(\tau_b))d\tau_a d\tau_b \quad (4.3)$$

For two paths wherein the particles are fixed at spatial points $\mathbf{r}_a$ and $\mathbf{r}_b$ separated by $r = |\mathbf{r}_a - \mathbf{r}_b|$, the Higgs particle exchange potential is computed from $S_{ab} = -\int U_{ab}(r)dt$ wherein $t = (t_a+t_b)/2$. In detail, the Higgs exchange induces a potential between two masses M_a and M_a given by

$$U_{ab}(r) = -\left(\frac{\sqrt{2}G_F M_a M_b}{4\pi}\right)\frac{e^{-\kappa r}}{r} \quad \text{wherein} \quad \kappa = \frac{M_H c}{\hbar} . \quad (4.4)$$

Eq.4.4 may be compared with the Newtonian gravitational energy $U_{ab}^{\text{gravity}}(r) = -GM_a M_b/r$. For the Higgs case, the exchange potential is screened on the length scale κ^{-1}. However, in both the Higgs and graviton cases, the force is proportional to the product of the masses.

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If two particles are in the neighborhood of one another, then the Higgs field produced by one particle will renormalize the mass of the other particle. If a complex X decays into two stable particles, then these particles will not be in the neighborhood of each other for virtually all of their world line proper times. On the other hand, if X decays into a particle anti-particle pair $X \to a + \bar{a}$, each of which has a finite life-time $\Gamma_a^{-1} = \Gamma_{\bar{a}}^{-1}$ then over a finite fraction of their lifetimes, the pair of particles are in each other's neighborhood and thereby should exhibit mass shifts; i.e. the measured masses when the particles are produced separately may not quite be the same as when they are produced as a pair; i.e. there will be a Higgs induced mass shift [6, 7, 8, 9, 10]

$$\Delta M_a = \frac{\hbar\Gamma_a}{2\pi c^2} \ln\left(\frac{M_a c^2}{\hbar\Gamma_a}\right) \left[\left(\frac{\sqrt{2}G_F M_a^2}{\hbar c}\right) \frac{M_a^2}{M_{a\bar{a}}\sqrt{M_{a\bar{a}}^2 - 4M_a^2}}\right], \tag{4.5}$$

wherein $M_a = M_{\bar{a}}$ is the mass of of a single produced particle, $\Gamma_a = \Gamma_{\bar{a}}$ is the single particle inverse lifetime and $\sqrt{s} = M_{a\bar{a}}$ is the invariant mass of the pair.

To understand the basic physics of how life times and energy shifts are intimately related, as in Eq.4.5, consider how one calculates in quantum mechanics employing the Hamiltonian operator. This Hamiltonian view requires an explicit choice of reference frame. To compute perturbative transitions rates one often employs the "Fermi Golden Rule" spectral function,

$$\Gamma_i(\varepsilon) = \frac{2\pi}{\hbar} \sum_f |V_{fi}|^2 \delta(\varepsilon + E_f - E_i), \tag{4.6}$$

while the energy shifts are often computed via second order perturbation theory,

$$\Delta E_i = \sum_f \frac{|V_{fi}|^2}{E_i - E_f} . \tag{4.7}$$

Eqs.4.6 and 4.7 employing reasonable approximations yield

$$\Delta E_i = \frac{\hbar}{2\pi} \int \frac{\Gamma_i(\varepsilon) d\varepsilon}{\varepsilon} \approx \frac{\hbar}{2\pi} \Gamma_i(0) \ln\left(\frac{E_i}{\hbar\Gamma_i(0)}\right). \tag{4.8}$$

The relativistic version of Eq.4.8 reads,

$$c^2 \Delta M_a^{(0)} \approx \frac{\hbar\Gamma_a}{2\pi} \ln\left(\frac{M_a c^2}{\hbar\Gamma_a}\right), \tag{4.9}$$

indicating that any interaction giving a particle a lifetime also induces a finite mass shift. Consider the processes $X \to a + \bar{a}$ as shown in Figure1. The final state wave functions of the a and $\bar{a}$ must be renormalized to take into account lifetimes and mass shifts as in (for example) Eq.4.9. When the vertex is renormalized by Higgs exchange, there is a further modification of the mass in the form

$$\Delta M_a = \Delta M_a^{(0)} \mathscr{H}(s) \ \text{ wherein } \ \mathscr{H}(s) = \left[\left(\frac{\sqrt{2}G_F M_a^2}{\hbar c}\right) \frac{M_a^2}{\sqrt{s}\sqrt{s - 4M_a^2}}\right]. \tag{4.10}$$

Eqs.4.9 and 4.10 produce the central Eq.4.5 which allows, near threshold, for a new experimental method for detecting the Higgs mechanism for producing masses.

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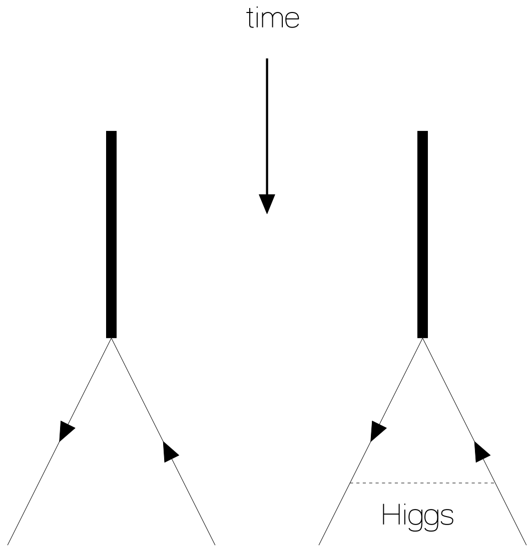

Figure 1: For a decay $X \to a + \bar{a}$, the widths of the final state mass distributions $\Gamma_a = \Gamma_{\bar{a}}$ give rise to lifetimes and mass shifts $\Delta M_a^{(0)} = \Delta M_{\bar{a}}^{(0)}$. When the vertex is further corrected by Higgs exchange, as shown above, there is an additional mass shift correction of the form $\Delta M = \Delta M^{(0)} \mathscr{H}$ wherein the Higgs field produced by the one particle changes the mass of the other particle within its proximity.

5. Conclusions

In the standard model of matter, one begins with an $SU_{\text{color}}(3) \times SU_{\text{left}}(2) \times U(1)$ field theory with conformal symmetry even for the quark and lepton sectors of the theory. The conformal symmetry is broken by a conjectured Higgs field which grows masses on some of the elementary particles, specifically on $(Z, W^{\pm}, e, \mu, \tau)$ in the electroweak interaction sector and on the quarks (u, d, c, s, t, b) in the strong interaction sector. For the model to hold true and also give the observed gravitational as well as inertial masses, one must hold the Higgs field largely responsible for growing macroscopic gravitational mass as well as inertial mass on the elementary constituent particles. The gravitational implications of the Higgs nechanicsm of growing inertial and gravitational masses on elementary particles have yet to be fully explored.

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