PROCEEDINGS OF SCIENCE

Progress in kaon physics on the lattice

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CKM-unitarity, direct and indirect CP-violation and the $\Delta I = 1/2$ rule in full lattice QCD are the focus of this talk. To this end I will discuss and compare recent lattice results for leptonic, semi-leptonic and non-leptonic decays of the kaon and neutral kaon mixing and I will motivate current best estimates

$$f_K/f_\pi = 1.198(10)\,,\ f_+^{K\pi}(0) = 0.964(5) \ \text{ and } \ \hat{B}_K = 0.720(39)\ .$$

Moreover new theoretical advances that will improve the quality of these computations in the future will be discussed.

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1. Introduction

This talk reviews recent results and developments in the phenomenology of the kaon from the lattice focussing on simulations of full QCD (i.e. with a degenerate pair of dynamical u and d quarks ($N_f = 2$) and a dynamical s-quark ($N_f = 2+1$)). Many of the results presented here have been computed using different fermion discretisations. This has to be seen as a feature since we are entering a phase where in particular observables describing $SU(3)$-breaking effects in kaons can be computed on the lattice with sub-percent level precision. If used to constrain the Standard Model and also to constrain models that go beyond, consistency of the results from different discretisations will increase the reliability of the lattice output. To this end I want to draw the reader's attention to the plenary talks on recent simulations with domain wall fermions (DWF) [1], overlap fermions [2], twisted mass fermions (tmQCD) [3] and clover improved Wilson fermions [4] and also the two plenary talks by Creutz [5] and Kronfeld [6][1] discussing different perspectives on the consistency of the staggered fermion (KS) formulation.

Please be aware of the preliminary status of some of the results presented here and that I could not cover all recent results and developments of this field.

This talk is structured as follows: I will start with a discussion of the status of the first-row-unitarity of the CKM matrix, particularly focusing on the matrix element $|V_{us}|$ which can be determined either from leptonic kaon and pion decays or from semi-leptonic kaon decays. I will then discuss indirect CP-violation in neutral kaon mixing where the lattice can contribute in terms of the computation of the bag parameter $\hat{B}_K$. Finally, I will discuss direct CP-violation and the $\Delta I = 1/2$-rule in hadronic kaon decays, where the parameters $|\varepsilon'/\varepsilon|$ and ω can be computed on the lattice. While discussing these topics I will also present new theoretical and technical developments which will hopefully improve the quality and the precision of these calculations.

2. The determination of $|V_{us}|$

The three entries V_{ud}, V_{us} and V_{ub} make up the first row of the CKM-matrix [8, 9] and assuming its unitarity, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$. Any deviation from 1 on the r.h.s. would be a sign for physics beyond the Standard Model. $|V_{ud}|$ is very well known and neglecting $|V_{ub}|$ (since it is very small) the precision of $|V_{us}|$ is currently limiting the accuracy of the unitarity test.

2.1 $|V_{us}|$ from leptonic decays

In 2004 Marciano [10] first used the lattice determination of f_K/f_π to determine $|V_{us}|$ employing the relation

$$\frac{\Gamma(K\to\mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi\to\mu\bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K(1-\frac{m_\mu^2}{m_K^2})}{m_\pi(1-\frac{m_\mu^2}{m_\pi^2})} \times 0.9930(35)\,. \tag{2.1}$$

From the charged kaon and pion life-times and the leptonic partial widths in [11] one determines

$$\begin{aligned}\Gamma(K\to\mu\bar{\nu}_\mu(\gamma)) &= 2.528(2)\times 10^{-14}\mathrm{MeV}\,,\\ \Gamma(\pi\to\mu\bar{\nu}_\mu(\gamma)) &= 3.372(9)\times 10^{-14}\mathrm{MeV}\,,\end{aligned} \tag{2.2}$$

[1] See also Sharpe's plenary talk last year [7].

group		N_f	action	a/fm	a from	$Lm_\pi^\dagger$	m_π/MeV	f_K/f_π
QCDSF+ UKQCD	[14]	2	clover (NP)	$\gtrsim 0.06$	r_0	4.2	$\gtrsim 300$	1.21(3)
ETM	[15, 16]	2	max. tmQCD	0.09	f_π	3.2	$\gtrsim 290$	1.227(9)(24)
MILC	[17, 18, 19]	2+1	$KS^{\rm AsqTad}_{\rm MILC}$	$\gtrsim 0.06$	f_π	4	$\gtrsim 240$	$1.197(3)(^{+6}_{-13})$
HPQCD+ UKQCD	[20]	2+1	$KS^{\rm HISQ}_{\rm MILC}$	$\gtrsim 0.09$	Υ	3.8	$\gtrsim 250$	1.189(7)
NPLQCD	[21]	2+1	$KS_{\rm MILC}$/DWF	0.13	r_0	3.7	$\gtrsim 290$	$1.218(2)(^{+11}_{-24})$
RBC+ UKQCD	[22, 23]	2+1	DWF	0.11	Ω^-	4.6	$\gtrsim 330$	1.205(18)
PACS-CS	[4]	2+1	clover (NP)	0.09	ϕ	3	$\gtrsim 210$	1.219(26)

† lightest pion; NP: non-perturbatively improved; max. tmQCD: maximally twisted mass QCD

Table 1: Parameters of gauge configurations from which f_K/f_π has been determined. Errors on the results are either statistical and systematic or the combined error.

group	am_l-extrapolation	a	FVE
QCDSF+UKQCD	linear extrapolation	2 values of a	χPT
ETM	NLOχPT+NNLO (analytic terms) and also polynomial	-	χPT
MILC	rSχPT: first NLO + NNLO analytic terms on reduced set then include NNNLO-analyt. terms and include also larger values of am_l, 4 values of a		2 volumes rSχPT
HPQCD+UKQCD	NLOχPT+NNLO(analytic) a^2 (conventional+taste breaking)-terms 3 values of a		χPT
NPLQCD	NLOχPT+NNLO analytic terms	-	-
RBC+UKQCD	NLOχPT$^{SU(2)\times SU(2)}$	-	2 volumes
PACS-CS	NLOχPT	-	2 volumes, [24]

χPT: continuum chiral perturbation theory - rSχPT: rooted staggered chiral perturbation theory (including cut-off dependence)

Table 2: Summary of how the systematic effects due to the extrapolation in the quark mass, cut-off effects and finite volume effects have been treated in the determination of f_K/f_π.

which then yields

$$\frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f_K^2}{f_\pi^2} = 0.07602(23)_{\rm exp}(27)_{\rm RC}\,, \tag{2.3}$$

with errors from experiment and radiative corrections, respectively. Combining the very precise result $|V_{ud}| = 0.97377(11)_{\rm exp}(15)_{\rm nucl}(19)_{\rm RC}$[2] [13] from nuclear β-decay with the prediction for f_K/f_π from lattice computations one determines $|V_{us}|$. Table 1 summarises the basic parameters and results for the gauge field ensembles from which f_K/f_π has recently been determined. I begin with some comments on the treatment of the systematic error which is summarised in table 2:

Finite volume effects were studied by all collaborations except NPLQCD by comparing results from lattices with different physical volumes but otherwise constant physical parameters (MILC, RBC+UKQCD, PACS-CS) and/or using NLO chiral perturbation theory (QCDSF+UKQCD, ETM,

[2]A preliminary update for this number was given by Marciano at the Kaon 2007 International Conference: $|V_{ud}| = 0.97372(10)_{\rm exp}(15)_{\rm nucl}(19)_{\rm RC}$ [12]

MILC, HPQCD+UKQCD) [25, 26]. I should comment that the first method is clearly preferred and that values of $m_\pi L \approx 3$ very likely lead to finite volume effects that are not negligible. In addition to comparing results from two volumes for some simulation points PACS-CS estimated finite volume effects using the approach by Colangelo-Dürr-Haefeli [24]. It is based on Lüscher's idea to express finite volume effects of pion (kaon) masses in terms of the $\pi\pi(K)$ scattering phase shift [27] and predicts larger finite volume effects than NLO χPT. Since this approach depends only indirectly on chiral perturbation theory through the estimate of the scattering phase shift the predictions are expected to be more accurate.

chiral extrapolation: UKQCD+QCDSF don't see a curvature in their data for f_K/f_π and therefore linearly extrapolate to the physical point. Staggered fermions come in tastes and at finite lattice spacing the taste symmetry is broken and the corresponding pion spectrum is non-degenerate. The continuum chiral effective Lagrangian therefore does not correctly describe the spectrum of staggered lattice QCD. MILC, who are using AsqTad improved staggered valence and sea quarks, therefore employ $SU(3)\times SU(3)$ rooted staggered partially quenched chiral perturbation theory [28, 29] which simultaneously describes the approach to the chiral and the continuum limit. They first fit the NLO expression including NNLO analytic terms to a reduced data set including only lighter data points and subsequently extend the fit to the heavier data points by including NNNLO analytic terms [19]. HPQCD+UKQCD are simulating the partially quenched theory using HISQ [30] valence quarks on the MILC staggered sea [31, 19]. Arguing that for this setup the taste splitting is reduced sufficiently, they rely on NLO $SU(3)\times SU(3)$ continuum chiral perturbation theory with added NNLO analytical and cut-off terms. In their recent publication [20] HPQCD+UKQCD quote the smallest error for f_K/f_π among all the results presented here and it will be interesting to compare the details of their chiral and continuum extrapolation once they are accessible in an upcoming publication. ETM, MILC, HPQCD+UKQCD and NPLQCD also use $SU(3)\times SU(3)$ NLO continuum chiral perturbation theory adding NNLO analytical terms to the fit-ansatz in order to improve the fit quality; PACS-CS is using NLO only. RBC+UKQCD use a different strategy. Assuming that the strange quark mass is too heavy to be properly described by $SU(3)$ chiral perturbation theory and $m_s \gg m_{u,d}$ they use $SU(2)\times SU(2)$ chiral perturbation theory fits for the kaon sector [32], to describe the light quark mass dependence of the decay constants at fixed strange quark mass. The results obtained in this way are then interpolated to the physical point of the strange quark mass.

cut-off-effects: Apart from MILC and HPQCD+UKQCD who are doing a combined chiral and continuum extrapolation only QCDSF+UKQCD assess cut-off effects. The latter do not see any scaling violations in their data. Some of the remaining collaborations will supplement their current results with results on a finer lattice and/or rely on the crude estimates for cut-off effects to be of order $(a\Lambda_{\rm QCD})^2$ in the $O(a)$-improved theory.

2.2 $|V_{us}|$ from semi-leptonic decays

The Standard Model expectation for the $K\to\pi l\nu$ (K_{l3}) semi-leptonic decay rate is [33]

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} I\, S_{\rm EW}[1+2\Delta_{SU(2)}+2\Delta_{\rm EM}]|V_{us}|^2|f_+^{K\pi}(0)|^2\,. \qquad (2.4)$$

Here, $C_K^2 = 1/2(1)$ is the Clebsch-Gordan coefficient for the neutral (charged) kaon decay and I is the phase space integral which is typically determined from the shape of the experimentally measured form factor [33]. $S_{\rm EW}$ is the electro-weak short distance correction and $\Delta_{SU(2)}$ and $\Delta_{\rm EM}$ are $SU(2)$-isospin breaking and electromagnetic corrections, respectively. The non-perturbative contribution to the process is given in terms of the form factor $f_+^{K\pi}(0)$ defined through the QCD matrix element of the vector current $V_\mu = \bar{s}\gamma_\mu u$ between the kaon and the pion,

$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu\,, \tag{2.5}$$

where $q_\mu = (p_K - p_\pi)_\mu$ is the momentum transfer. Recently the FlaviaNet Kaon Decay Working Group determined the very accurate value $|V_{us} f_+^{K\pi}(0)| = 0.21673(46)$ [34] which can only be fully appreciated for a determination of $|V_{us}|$ if lattice computations of $f_+^{K\pi}(0)$ reach sub-percent precision.

In chiral perturbation theory $f_+^{K\pi}(0)$ is expanded as

$$f_+^{K\pi}(0) = 1 + f_2 + f_4 + \dots\,, \tag{2.6}$$

where f_2, which corresponds to the leading chiral correction, is fully determined in terms of the meson masses [35] and takes the value $f_2^{\rm phys} = -0.023$ at the physical point. Relevant for lattice computations are also the more recent evaluations of f_2 in partially quenched chiral perturbation theory and the evaluation of finite volume corrections [36, 37]. A first estimate of the higher order corrections was given in [33] and more recently estimates for f_4 were given in [38, 39, 40]. Thus, relying on our knowledge of f_2 in chiral perturbation theory, on the lattice one computes only the corrections,

$$\Delta f(m_K, m_\pi) = f_+(0, m_K, m_\pi) - (1 + f_2(m_K, m_\pi))\,. \tag{2.7}$$

Considering for a moment the first estimate $\Delta f = -0.016(8)$ for physical pion and kaon masses by Leutwyler and Roos in 1984 [33], it is clear that a precision of 30-40% for Δf is sufficient to reach sub-percent accuracy for $f_+^{K\pi}(0)$.

During the last three years a number of collaborations have computed this quantity using dynamical fermions. However, in terms of controlling systematic uncertainties the project by the RBC+UKQCD collaboration which use 2+1 flavours of dynamical domain wall fermions with pion masses down to 330 MeV is the current state of the art [41, 42, 43] (cf. the summary of recent simulations in table 3). At this conference also ETM [15, 44] and QCDSF [45] indicated that they will compute the K_{l3} form factor with twisted mass fermions and clover improved Wilson fermions, respectively. QCDSF gave a first estimate in their proceeding [45] and one has to wait for their final error analysis and the extension of their analysis to lighter pion masses and one coarser and one finer lattice spacing.

The technique for the high precision calculation of $f_+^{K\pi}(0)$ has been set out in [36] and consists of a 3-step procedure:

1) one first computes $f_0^{K\pi}(q^2_{\rm max}) = f_+^{K\pi}(q^2_{\rm max}) + q^2_{\rm max}/(m_K^2 - m_\pi^2)\, f_-(q^2_{\rm max})$, where $q^2_{\rm max} = (m_K - m_\pi)^2$, from

$$\frac{\langle \pi|V_0|K\rangle\langle K|V_0|\pi\rangle}{\langle \pi|V_0|\pi\rangle\langle K|V_0|K\rangle} = \frac{(m_K+m_\pi)^2}{4 m_K m_\pi}\left(f_0(q^2_{\rm max}, m_K, m_\pi)\right)^2\,. \tag{2.8}$$

The l.h.s. is obtained from ratios of suitable Euclidean three-point functions at large values of the Euclidean time. Since both mesons are at rest and due to cancellations of correlations and also the cancellation of the vector current renormalisation constants in the ratio in (2.8), $f_0^{K\pi}(q^2_{\rm max})$ can be determined with very high precision.

group		N_f	action	a/fm	L/fm	m_π/MeV	$f_+^{K\pi}(0)$
JLQCD	[46]	2	clover (NP)	0.09	1.8	$\gtrsim 550$	0.967(6)
RBC	[47]	2	DWF	0.12	2.5	$\gtrsim 490$	0.968(9)(6)
QCDSF	[45]	2	impr. Wilson	0.08	1.9	592	0.9647*
Fermilab, HPQCD, MILC	[48]†	2+1	Wilson d-quark impr. stag. u & s	‡	‡	‡	0.962(6)(9)
RBC+UKQCD	[41, 42, 43]	2+1	DWF	0.11	1.8,2.8	$\gtrsim 330$	0.964(5)

† computed $f_0^{K\pi}(q^2_{\rm max})$ and then used slope of form factor from experiment for extrapolation to $q^2=0$ ("exploratory study");
* Error analysis not yet finished
‡ The proceeding in which the result was published does not contain this information

Table 3: Lattice computations of the K_{l3} form factor using dynamical fermions. Errors on the results are either statistical and systematic or the combined error.

		q^2-fits				chiral fits		
		lin	quad	pole	z	const	lin	quad
JLQCD	[46]		●				●	
RBC	[47]	○	○	●		○	●	○
Fermilab FNAL,MILC	[48]			●†			●	
RBC/UKQCD	[41]	○	○	●	○	○	●	○

● = adopted; ○ = tested;
† = pole mass from experiment → extrapolate to $f_0(0)$ from $f_0(q^2_{\rm max})$

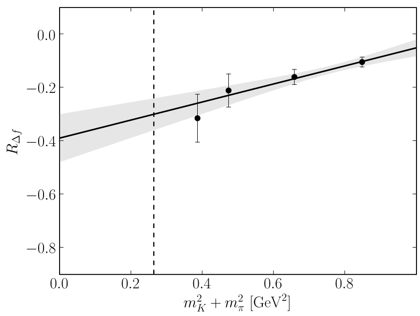


Figure 1: Left: Summary of how systematic in chiral extrapolation and q^2-interpolatoin was assessed; right: chiral extrapolation of $R_{\Delta f}$ (r.h.s. plot taken from [43]).

2) One then determines $f_0^{K\pi}(q^2)$ at various other values of $q^2 < q^2_{\rm max}$ from similar ratios of 3-point functions by inducing Fourier momenta ($|\vec{p}_{K,\pi}| = 2\pi/L, \sqrt{2}\,2\pi/L$) into the initial and/or final state. In order to interpolate the form factor to $q^2 = 0$ the collaborations have used first or second order polynomial ansätze in q^2, a pole dominance ansatz $f_{0,\rm pole}(q^2) = \frac{f_0(0)}{1-q^2/M^2}$ or the z-fit (polynomial with improved convergence [49]) as summarised in the table in figure 1.
3) In the last step the data for $f_+^{K\pi}(0) = f_0^{K\pi}(0)$ has to be extrapolated to the physical value of the quark masses and it is common practice to use the ratio

$$R_{\Delta f} \equiv \frac{\Delta f}{(m_K^2 - m_\pi^2)^2} = \frac{f_0^{K\pi}(0) - (1 + f_2(m_K, m_\pi))}{(m_K^2 - m_\pi^2)^2}\,, \tag{2.9}$$

with a polynomial ansatz constant, linear or quadratic in $(m_K^2 + m_\pi^2)$. Again the table in figure 1 summarises which ansatz has been studied by the various collaborations. An example for the extrapolation with a linear fit is shown in in the plot in figure 1 which is taken from James Zanotti's talk [43].

Both the q^2-interpolation and the chiral extrapolation are based on phenomenological fit-ansätze and are thus sources of systematic uncertainties which can be estimated in terms of the spread of results between the different ansätze. The RBC+UKQCD-collaboration [43] has observed a reduction in the final error when combining the q^2-interpolation and the chiral extrapolation into one global fit. In the next section we will briefly introduce a new technique [50] that may entirely remove the uncertainty due to the q^2-interpolation. Before let me make some comments:

- It would be interesting to have the expression in chiral perturbation theory of the K_{l3} form factor to order p^6 in [39] in terms of the quark masses. This would on the one hand allow to understand the slight tension between χPT and the lattice results which I will comment on later. On the other hand lattice results could then also be used to constrain or even determine the low energy constants appearing in these expressions.
- Taking the preliminary result by RBC+UKQCD [41] as the current bench-mark, the contributions to the overall error on Δf from the lattice decomposes in the following way[3]:

$$\begin{array}{llllll} \Delta f = -0.0161 & (45)^{\text{stat.}} & (15)^{\chi} & (16)^{q^2} & (8)^{a} \\ & 30\% & 9\% & 10\% & 4\%\,, \end{array} \tag{2.10}$$

  for the statistical error, error due to the chiral extrapolation, error due to the q^2-interpolation and error due to the finite lattice-cut-off, respectively. In the following we will briefly discuss two recent developments which should allow to further reduce the error due to the q^2-interpolation on the one hand and the statistical error on the other hand.

2.3 K_{l3} with partially twisted boundary conditions

Using twisted boundary conditions for the valence quark fields, i.e. $q(x+L\hat{k}) = e^{i\theta_k}q(x)$ for $k = 1,2,3$ and leaving the sea quark's boundary conditions untouched (*partially twisted boundary conditions*) [51, 52, 53], one can tune the momenta of hadrons in dynamical simulations in a finite box continously. For example the dispersion relation for a charged pion then follows

$$E_{\pi^\pm} = \sqrt{m^2_{\pi^\pm} + (\vec{p}_{\rm lat} - \frac{1}{L}(\vec{\theta}_u - \vec{\theta}_d))^2}\,, \tag{2.11}$$

where $\vec{\theta}_u$ and $\vec{\theta}_d$ are the twisting angles of the valence up and down quarks, respectively, and $\vec{p}_{\rm lat}$ is the conventional Fourier momentum. This relation was confirmed numerically in [54]. Twisted boundary conditions were also tested in the quenched theory [55] where they have subsequently been been applied to study the q^2-dependence of the K_{l3} form factor using techniques similar to the 3-step procedure detailed in the previous section [56]. An approach which goes beyond that study was developed in [50] where the matrix element

$$\langle \pi(p_\pi)|V_4(0)|K(p_K)\rangle = f_+^{K\pi}(0)(E_K + E_\pi) + f_-^{K\pi}(0)(E_K - E_\pi)\,, \tag{2.12}$$

of the 4th component of the vector current is evaluated for two choices of the kinematics, namely

$$\begin{array}{l} 1)\ \langle \pi(0)|V_4|K(\vec{\theta}_K)\rangle|_{q^2=0}:\ |\vec{\theta}_K| = L\sqrt{(\frac{m_K^2+m_\pi^2}{2m_\pi})^2 - m_K^2}\ \text{ and }\ \vec{\theta}_\pi = \vec{0}\,, \\ 2)\ \langle \pi(\vec{\theta}_\pi)|V_4|K(0)\rangle|_{q^2=0}:\ |\vec{\theta}_\pi| = L\sqrt{(\frac{m_K^2+m_\pi^2}{2m_K})^2 - m_\pi^2}\ \text{ and }\ \vec{\theta}_K = \vec{0}\,. \end{array} \tag{2.13}$$

The form factor $f_+^{K\pi}(0)$ in eq. (2.12) can directly be extracted from a linear combination of the matrix element in the kinematical situations 1) and 2) in (2.13), thus entirely avoiding the q^2-interpolation and difficult fits to ratios of correlation functions involving the spatial components

[3] In order to disentangle the various contributions to the systematic error I here quote the results from the individual q^2-interpolation and chiral extrapolation in [43].

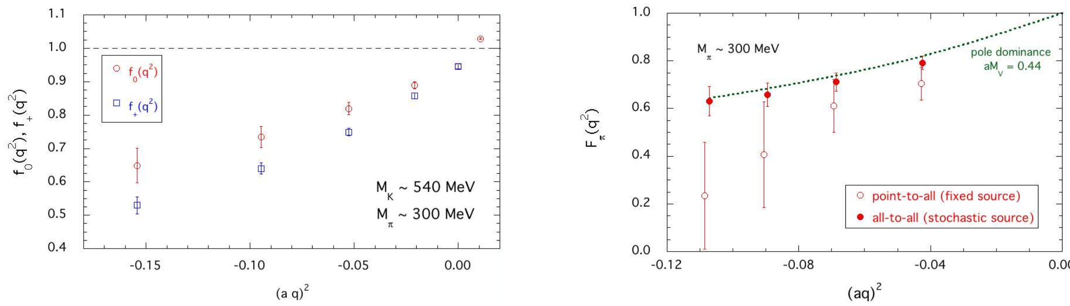


Figure 2: Left: $f_0^{K\pi}(q^2)$ with all-to-all propagators; right: the pion form factor $f_\pi(q^2)$ constructed from point-to-all and all-to-all propagators, respectively [44].

of the vector current, which are necessary in the 3-step procedure lined out above. The statistical errors of the results from the new approach are comparable to the errors using the conventional approach.

2.4 K_{l3}-decays using propagators from stochastic sources

Instead of constructing the three point functions relevant to the determination of the K_{l3}-form factor from point-to-all propagators, the ETM collaboration [44] uses all-to-all propagators generated with the one-end-trick [57, 58], thus gaining a volume averaging of the propagator source. The pion and kaon momenta in these simulations are induced using partially twisted boundary conditions [51, 52, 53] in combination with the 3-step procedure lined out in section 2.2. Preliminary results for this approach were shown in Simula's talk at this conference (cf. l.h.s. of figure 2). Since no data for comparison with results from point-source propagators were available for $f_+^{K\pi}(q^2)$ and $f_0^{K\pi}(q^2)$, one gains some feeling for the improvement by looking at Simula's plot of the pion vector form factor $f^{\pi\pi}(q^2)$ on the r.h.s. of figure 2.

2.5 Summary for $|V_{us}|$

Figure 3 shows a comparison of recent results for $|V_{us}|$ from both leptonic and semi-leptonic kaon decays. The grey band in the plot gives the result for $|V_{us}|$ which one gets using Leutwyler and Roos' result $f_+^{K\pi}(0) = 0.961(8)$ [33] and $|V_{us} f_+^{K\pi}(0)| = 0.21673(46)$ by the FlaviaNet Kaon Decay Working Group [34]. The lattice results for K_{l3} currently support Leutwyler and Roos' prediction. Both chiral perturbation theory for $f_+^{K\pi}(0)$ [39, 59, 40] and the lattice predictions for f_K/f_π tend towards smaller values for $|V_{us}|$, thus creating a slight tension. We also see that results for the CKM-matrix element from lattice calculations of leptonic kaon decays are becoming competitive with results from calculations of semi-leptonic decays.

RBC+UKQCD currently have the best control of systematic effects in the calculation of $f_+^{K\pi}(0)$ [1, 41, 43, 60] and I take their result as the current best estimate,

$$f_+^{K\pi}(0) \; = \; 0.964(5) \; \longrightarrow |V_{us}| = 0.2247(12)\,. \qquad (2.14)$$

Due to the preliminary status of some of the central values and error bars for f_K/f_π in table 1 it is difficult to compute a world average with a reliably estimated error. As the central value for the current best estimate I suggest the weighted average over all results in table 1 assuming Gaussian

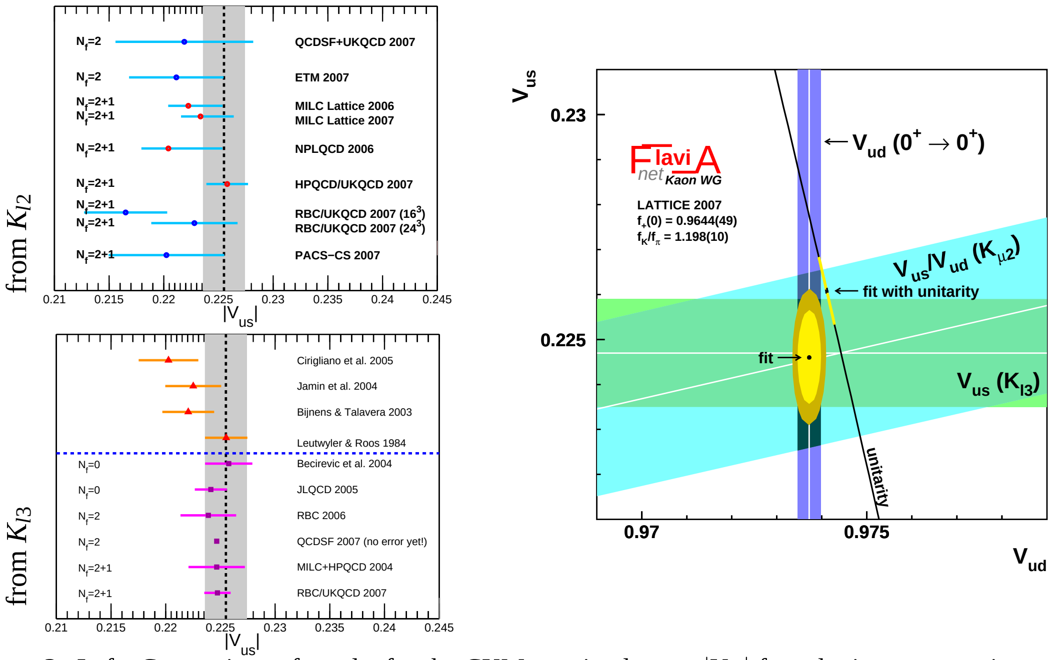


Figure 3: Left: Comparison of results for the CKM-matrix element $|V_{us}|$ from lattice computations of the leptonic and semi-leptonic kaon decays. In the lower plot also results from chiral perturbation theory are shown. Right: Compilation of the estimates (2.14) and (2.15) into a combined fit by the FlaviaNet Kaon Decay WG (like in [34]) to assess whether the first row of the CKM matrix fulfils the unitarity constraint.

and un-correlated errors[4]. MILC [19] has carried out the most extensive simulation with a very detailed study of the chiral extrapolation, the finite-volume and the cut-off effects all within the frame work of rooted staggered chiral perturbation theory and I therefore attach to the averaged central value their combined statistical and systematic error,

$$f_K/f_\pi = 1.198(10) \longrightarrow |V_{us}| = 0.2241(24)\,, \tag{2.15}$$

which reveals a slight tension with the experimental value 1.223(12) [11]. Note that this average is dominated by the MILC and HPQCD+UKQCD results with staggered fermions which have very small errors. The average without the results based on staggered fermions would instead be 1.211(10).

The FlaviaNet Kaon Decay Working Group was so kind to provide a version of their unitarity fit [34, 61] assuming the above best estimates which is shown in the r.h.s. plot in figure 3[5]. The blue vertical line represents Marciano's update for $|V_{ud}|$ from nuclear β-decays [12]. The horizontal band is the result for $|V_{us}|$ from RBC+UKQCD's lattice calculation of the semi-leptonic form factor [1, 41, 60] and the slightly tilted horizontal band represents (2.15). The solid black line represents CKM first-row unitarity (neglecting $|V_{ub}|$). This analysis which is partly based on preliminary results indicates a tension between the lattice results and CKM-unitarity. With $|V_{ud}| = 0.97372(26)$ and $|V_{us}| = 0.2246(11)$ one gets $|V_{ud}|^2 + |V_{us}|^2 - 1 = 0.0014(7)$. It will be extremely exciting to

[4]In the case of assymmetric error bars I have shifted the central value of the result and corrected the error correspondingly to be symmetric.

[5]Many thanks to M. Antonelli, F. Mescia and M. Moulson for generating this plot [62].

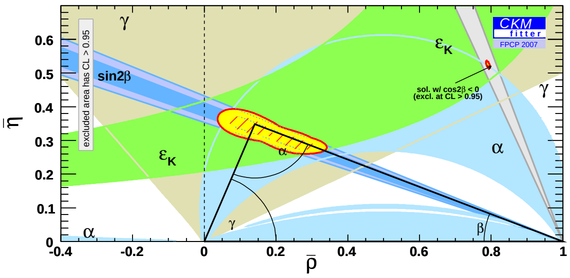


Figure 4: Status of the CKM-triangle by the CKMFitter group [66]

follow the development of this situation as the various collaborations finalize their analysis and in particular theoretical errors are further reduced.

3. *CP*-violation in kaon systems

The physical state K_L consists predominantly of the *CP*-odd K_2 and an admixture of the *CP*-even K_1. The decay of the K_2 into a *CP*-even two-pion state is called direct *CP*-violation and has been established experimentally in 1999 by NA48 and KTeV [63, 64]. Direct *CP*-violation which occurs when the K_1 decays into the *CP*-even two-pion state has been established in 1964 by Cronin and Fitch [65]. Direct and indirect *CP*-violation have been studied on the lattice for many years and in the following I will discuss recent advances and results.

3.1 Indirect *CP*-violation - neutral kaon mixing

The experimental measurement of $|\varepsilon_K| = |A(K_L \to (\pi\pi)_{I=0})/A(K_S \to (\pi\pi)_{I=0})|$ together with a prediction of the kaon bag parameter

$$\hat{B}_K = C(\mu)\frac{\langle \overline{K^0}|Q^{\Delta S=2}(\mu)|K^0\rangle}{\frac{8}{3} f_K^2 m_K^2}\,, \tag{3.1}$$

define a hyperbola in the plane of the Wolfenstein parameters $\hat{\eta}$ and $\hat{\rho}$ which constrains the apex of the unitarity triangle shown in figure 4. Here $Q^{\Delta S=2}(\mu)$ equals the difference of the parity even and parity odd four quark operators O_{VV+AA} and O_{VA+AV}, respectively. Before presenting recent results for $\hat{B}_K$ I will now briefly discuss and compare how the major fermion discretisations fare in the computation of the matrix element in eqn. (3.1).

Wilson fermions: As a result of the explicit breaking of chiral symmetry due to the Wilson term the B_K operator mixes with four other operators O_i [67],

$$O^{\Delta S=2}(\mu) = Z_{\rm VV+AA}(\mu, g_0^2)\left(O^{\rm latt}_{\rm VV+AA} + \sum_{i=1}^{4}\Delta_i(g_0^2)O_i^{\rm latt}\right)\,, \tag{3.2}$$

thus complicating the renormalisation process. It was realised in [68] that the parity even operator O_{VV+AA} is related to the parity odd operator O_{AV+VA} by an axial Ward identity and that the latter operator renormalises multiplicatively also for Wilson fermions. While the relevant matrix element of the parity even operator can be determined from three-point functions, the relevant matrix element for the parity odd operator has to be determined from four point functions. In some sense one trades the uncertainties due to the mixing in the case of the parity even operator for noisier signals of the four point functions in the case of the parity odd operator. Both approaches have been studied with dynamical fermions in [69, 70].

Twisted mass QCD: The the parity even operator in QCD is related to the parity odd operator in tmQCD by an axial rotation [71], $\langle K^0|O^{(0)}_{\rm VV+AA}|\overline{K^0}\rangle_{\rm QCD} = -i\langle K^0|O^{(\alpha)}_{\rm VA+AV}|\overline{K^0}\rangle_{\rm tmQCD}$. B_K can now be computed from 3-pt. functions of $O_{\rm VA+AV}$ which is expected to give a better signal than the 4-pt. function which has to be computed in the case of standard Wilson fermions. A benchmark computation using this approach in quenched tmQCD has recently been carried out by the ALPHA collaboration [72, 73]. In their simulation they used degenerate light and strange quark masses, thus neglecting possible $SU(3)$-breaking effects. The first case that was studied in that paper was with a Wilson s-quark and two twisted light quarks at twisting angle $\alpha = \pi/2$. Although non-degenerate s and d quarks are in principle feasible in this approach, here the masses of the s and the d quark were tuned to be degenerate, which is non-trivial since they have been discretised differently. The second case that ALPHA studied was $\alpha = \pi/4$ with twisted s and d quarks in which case the quarks are automatically degenerate. The simulations were carried out for a number of different lattice spacings, thus allowing for a very detailed study of the approach to the continuum limit. The authors used the non-perturbatively computed (Schrödinger functional) renormalisation constant for O_{VA+AV} [74]. One further outcome of this study is that the splitting of the meson spectrum due to the explicit breaking of the flavour symmetry with twisted mass fermions reduces as the continuum is approached where it is expected to vanish.

Staggered fermions: Van de Water and Sharpe [75] studied the transformation properties of the staggered B_K-operator that couples to external kaons of taste P which correspond to the lattice Goldstone kaon. The lattice representation of that operator mixes with many other operators of all tastes. In current simulations [76, 77] only the operators with the same taste as the lattice Goldstone kaon are actually implemented. The corresponding mixing coefficients are computed in perturbation theory. The mixing with other tastes at order α and higher orders in the strong coupling constant as well as lattice artefacts entering at order a^2 are described by staggered chiral perturbation theory:

$$O_K^{\rm stagg,cont} = \underbrace{O_K^{\rm stagg}[{\rm taste\,P}] + \frac{\alpha}{4\pi}[{\rm taste\,P}]}_{\rm simulation} + \frac{\alpha}{4\pi}[{\rm wrong\,tastes}] + \underbrace{\alpha^2[{\rm all\,tastes}]}_{\rm unknown\ 2-loop} + \underbrace{a^2[{\rm all\,tastes}]}_{\rm discretisation}\,. \quad (3.3)$$

The perturbative coefficients of the terms in $\alpha/(4\pi)$ are known. Since the higher order perturbative coefficients are not known, they are counted conservatively as α^2 rather than $(\alpha/(4\pi))^2$. The above expression has 37 free parameters which can be constrained e.g. by measuring the taste-splitting or by first determining a sub-set of parameters at a single lattice spacing. It is also known that HYP-smearing [78] reduces the mixing and taste breaking and also improves the convergence of perturbation theory and may thus yield a more favourable power counting [79, 80]. It has to be mentioned that non-perturbative renormalisation is in principle possible. However, all current simulations of B_K with staggered fermions rely on perturbative renormalization.

Domain wall fermions and overlap fermions: With chiral fermion formulations the parity-even operator O_{VV+AA} renormalises multiplicatively. For domain wall fermions residual mixing with wrong chirality operators was discussed in detail in [81]. It is suppressed by $(am_{\rm res})^2$ and therefore negligible (e.g. $am_{\rm res} \approx 10^{-3}$ for the current RBC+UKQCD domain wall fermion data set

		N_f		a	$m_\pi L$	m_π/MeV†	renorm.	degenerate	non-degenerate	χPT	$\hat{B}_K$
HPQCD+ UKQCD	[76]	2+1	KS^{HYP}_{MILC}	0.125 quenched scaling	4.5	360	PT	•		-	0.83(18)
Bae, Kim, Lee, Sharpe	[77]	2+1	KS^{HYP}_{MILC}	0.125	4.5	360	PT	•	•	NLO	
RBC+ UKQCD	[82] [83, 84]	2+1	DWF	0.11 quenched scaling	4.6	330	NPR	•	•	NLO	0.720(39)
JLQCD	[85]	2	Overlap fix. top.	0.12	2.7	290	NPR	•	•	NLO	0.723(12)‡

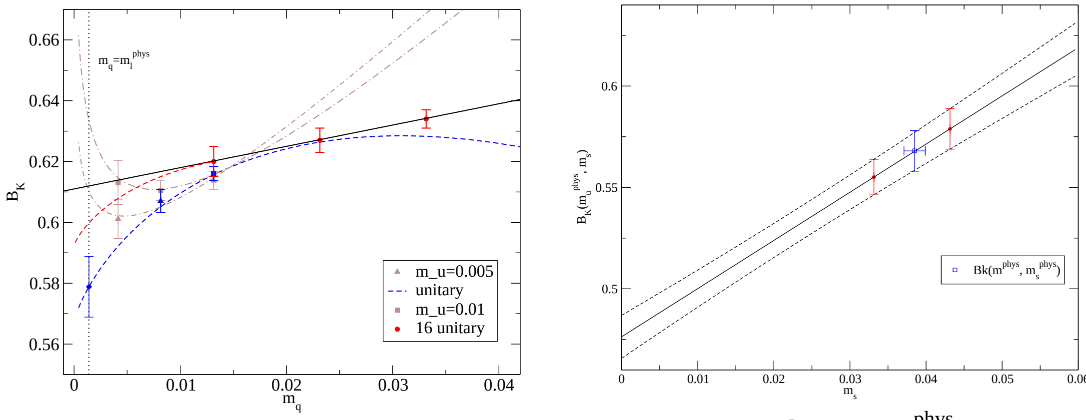
† for lightest pion; ‡ statistical error only

Table 4: Summary of current B_K-calculations.

[1]). The chiral symmetry of these actions provides automatic $O(a)$-improvement and continuum chiral perturbation theory can be used.

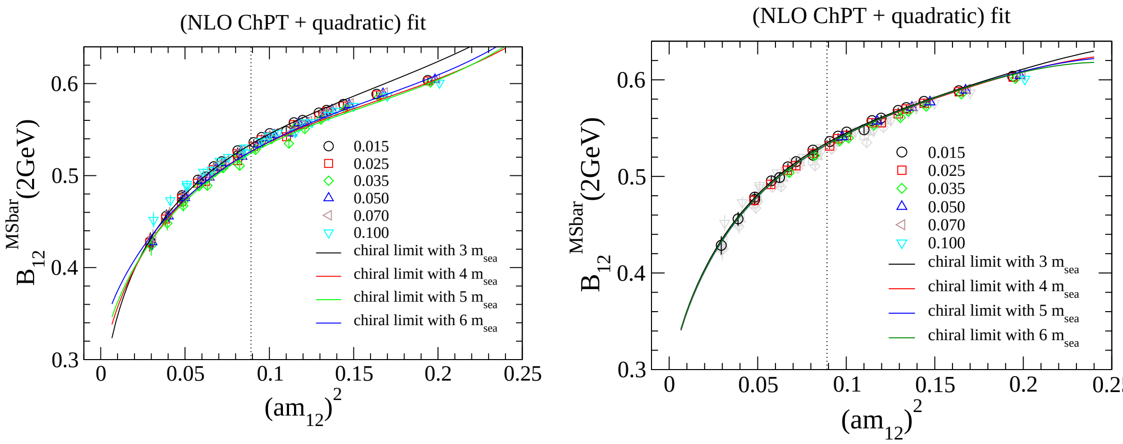
Recent simulations: Current efforts for the calculation of B_K on the lattice with dynamical fermions are summarised in table 4. HPQCD+UKQCD [76] on the one hand and Bae, Kim, Lee and Sharpe [77] on the other hand are using 2+1 flavour staggered quarks (MILC configurations [31]) at the same simulation parameters. Both collaborations use HYP smeared [78] valence quarks, thus simulating a partially quenched theory. While HPQCD+UKQCD is simulating for degenerate s- and d-quarks only, Bae and collaborators also investigate $SU(3)$-breaking effects and fit their data to continuum partially quenched chiral perturbation theory [75]. It remains to be studied in detail whether taste breaking effects for HYP-valence quarks are sufficiently suppressed that one can apply continuum chiral perturbation theory instead of staggered chiral perturbation theory in order to reliably describe and extrapolate the data. In order to estimate cut-off effects, HPQCD+UKQCD compare to simulations at various lattice spacings but otherwise similar simulation parameters within the quenched approximation. The underlying assumption is that cut-off effects in the quenched and unquenched theory behave similarly. RBC+UKQCD [82, 83, 1] and JLQCD [85] use chiral fermions with 2+1 flavours of domain wall quarks and 2 flavours of overlap quarks, respectively. RBC+UKQCD estimate the cut-off effects from the experience with the quenched case [81, 86] and both collaborations consider non-degenerate s- and d-quarks in the partially quenched framework and have renormalised the B_K operator non-perturbatively in the RI-MOM scheme [87]. JLQCD's overlap quarks are simulated at fixed topological charge and the corresponding finite volume effects are estimated to be at the percent level; this estimate is obtained by comparing results from different charge sectors at constant volume and quark mass. JLQCD has not yet finalised the error analysis for their value of B_K and it will be interesting to see the the full impact of finite volume effects ($m_\pi L \approx 2.7$) on the final error budget. Since the results of HPQCD+UKQCD and Bae, Kim, Lee and Sharpe have been nicely discussed in last year's plenary talk [79] I concentrate here on the ones by RBC+UKQCD and JLQCD.

RBC+UKQCD simulated on two volumes ($L = 1.8$fm and $L = 2.7$fm) and as can be gathered from the l.h.s. plot in figure 5, no significant finite volume effects were seen. The plot shows a number of partially quenched data points for configurations with sea quark masses that correspond to approximately 330 MeV and 420 MeV pions. The data was described and extrapolated using NLO $SU(2)\times SU(2)$ chiral perturbation theory [32] at fixed values of the strange quark mass which was then subsequently interpolated to the physical point (r.h.s. plot in figure 5). JLQCD's results



interpolation to $m_s^{\rm phys}$

Figure 5: RBC+UKQCD results for B_K [83, 82].



full data set reduced data set

Figure 6: JLQCD results for B_K [85].

are illustrated in figure 6. They used NLO $SU(3)\times SU(3)$ partially quenched chiral perturbation theory [75] to extrapolate their data to the physical point. The l.h.s. plot in figure 6 shows a significant dependence of the fit-results on the number of data points that were included into the fit. After applying the cut $m_l \leq m_s/2$ their fits however turned out to be stable and as the r.h.s. plot of figure 6 suggests that the fits all agree after inclusion of an additional NNLO analytic term into their fit-ansatz.

A summary of all recent lattice computations of B_K with dynamical fermions is given in figure 7 and I quote the numerical values of the most advanced simulations in table 4. The available results for the computation of the B_K-operator O_{VA+AV} with Wilson fermions either directly or via the axial Ward identity method have rather large errors. The reason is the lack of $O(a)$-improvement of the B_K-operator, rather heavy light quark masses and perturbative renormalization in these simulations. Also the HPQCD+UKQCD result from staggered fermions has a large error bar which contains a

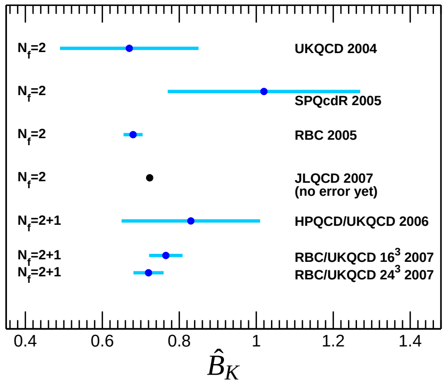


Figure 7: Summary of recent lattice simulations with dynamical fermions; for the JLQCD-result only the central value is shown since the error analysis is not yet finished.

large contribution from the perturbative treatment of the operator mixing. The implementation of non-perturbative renormalisation for the staggered B_K-operators should be considered in the future. The results with the smallest error bars have all been computed using chiral fermions thanks to the multiplicative renormalisation of O_{AA+VV} which was realized non-perturbatively in both simulations. JLQCD currently only quotes their central value for B_K since the error analysis is not yet finalized. The current best estimates for $\hat{B}_K$ from lattice QCD is the one by RBC+UKQCD [83],

$$\hat{B}_K = 0.720(39)\,. \tag{3.4}$$

This result is compatible with the result of the previous $N_f = 2$-simulations with domain wall fermions by RBC [88] and also with the central value of the new simulation by JLQCD [85].

New developments: Aubin, Laiho and Van de Water [89] developed the partially quenched chiral perturbation theory for domain wall valence fermions combined with AsqTad staggered sea quarks. In this mixed action ansatz [90] the symmetry properties of the domain wall valence quarks protects the B_K-operator from mixing with operators of non-trivial taste structure. Compared to continuum partially quenched chiral perturbation theory [75] there are only two additional parameters. With the large set of staggered sea quark configurations by the MILC collaboration [19] a computation of B_K using this approach seems feasible and very preliminary results have been presented at this conference [91].

The scale evolution of the B_K operator is usually carried out in perturbation theory. In order to remove the uncertainty due to perturbation theory the ALPHA collaboration has determined the scale evolution of the parity-odd operator O_{VA+AV} non-perturbatively in the continuum limit of the $N_f = 0,2$ Schrödinger functional scheme [92, 93]. If O_{VA+AV} is also renormalised in the Schrödinger functional scheme, the discretisation independent result for the scale evolution can be used to compute the running of B_K as obtained with Wilson fermions using the axial Ward identity method and the running of B_K as obtained using twisted mass QCD or Ginsparg-Wilson-type fermions.

Assuming that O_{VA+AV} in the continuum limit of twisted mass QCD has been non-perturbatively renormalised at the scale μ, it was suggested in [94] that the renormalisation constant for the corresponding operator determined using Ginsparg-Wilson-type fermions, $Z^{GW}(\mu,g_0)$, at non-vanishing lattice spacing could be defined via

$$O_{VA+AV}(\mu, m_{PS})|_{\rm c.l.\,of\,tmQCD} = Z^{\rm GW}(\mu, g_0) O^{\rm GW}_{VA+AV}(g_0, m_{PS}) + O(a^2)\,, \tag{3.5}$$

once m_{PS} on the l.h.s. and r.h.s. have been tuned to the same value. The identity holds due to the automatic $O(a)$ improvement of chiral fermions.

3.2 *The Holy Grail* - direct *CP*-violation

CP-violation is described in terms of the isospin amplitudes

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2 e^{i\delta_2} \;\text{ and }\; A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - \sqrt{\frac{1}{3}} A_2 e^{i\delta_2} \tag{3.6}$$

from which one constructs the parameters $\varepsilon' = \frac{\omega}{\sqrt{2}} e^{i\phi} \left(\frac{{\rm Im}A_2}{{\rm Re}A_2} - \frac{{\rm Im}A_0}{{\rm Re}A_0} \right)$ and $\omega = \frac{{\rm Re}A_2}{{\rm Re}A_0}$ describing direct *CP*-violation and the $\Delta I = 1/2$-rule, respectively. The isospin amplitudes are defined in terms of the matrix element

$$\langle \pi\pi(I)| - i\mathscr{H}|K^0\rangle = A_I e^{i\delta_I}\,. \tag{3.7}$$

where the relevant effective Hamiltonians [95, 96] are

$$\begin{aligned} &\text{for } u,d,c,s \;\; \mathscr{H}_c^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{\sigma=\pm} \left\{ k_1^\sigma(\mu) \mathscr{O}_1^\sigma(\mu) + k_2^\sigma(\mu) \mathscr{O}_2^\sigma(\mu) \right\}, \\ &\text{for } u,d,s \quad\; \mathscr{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)\,, \end{aligned} \tag{3.8}$$

for the four- and three flavour case, respectively, where $\mathscr{O}^\pm_{1,2}$ and $O_{1,2}$ are current-current operators, $O_{3,4,5,6}$ are QCD penguin operators and $O_{7,8,9,10}$ are EW penguin operators. $\mathscr{H}_c^{\Delta S=1}$ does not contain penguin diagrams. Giusti et al. [97] are studying this Hamiltonian in order to qualitatively assess the role of the charm quark in the $\Delta I = 1/2$ rule [97] and their programme has been nicely reported in Hernandez's plenary talk at Lattice 2006 [98]. Large scale dynamical lattice simulations with chiral fermions with the aim to compute the physical values of $|\varepsilon'/\varepsilon|$ and ω use $\mathscr{H}^{\Delta S=1}$ where the physical charm quark that is too heavy for current lattice simulations has been integrated out. Two-pion final states are notoriously difficult to handle on the lattice [99]. Instead of computing $\langle \pi\pi(I)| - i\mathscr{H}|K^0\rangle$ directly one uses chiral perturbation theory at LO and NLO in order to relate the matrix elements of interest to $K \to$ vacuum, $K \to \pi$ and $K \to \pi\pi$ matrix elements at unphysical kinematics which can be computed on the lattice more easily [100, 101, 102, 103]. In two impressive works by the CP-PACS [104] and the RBC collaboration [105] the approach has been demonstrated to work. The use of the quenched approximation and the rather large values of the light quark masses in these calculations of course have to be overcome. To this end Bae, Kim and Lee carried out exploratory studies of the calculation of $O^{(3/2)}_{7,8}$ using $N_f = 2+1$ flavours of staggered quarks [106, 107]. Very recently the RBC-collaboration has started to repeat their calculation but this time with $N_f = 2+1$ flavours of dynamical domain wall fermions. Their lattice spacing is $a^{-1} = 1.73$GeV and the spatial size of the lattice is $L = 2.7$fm. The collaboration is planning to simulate for 2 sea quark masses corresponding to pion masses of $m_\pi \approx 330$MeV and 410MeV, combined with 6 partially quenched valence quark simulation points and a fixed strange quark mass. All operators will be renormalised non-perturbatively (RI/MOM scheme [87]). One

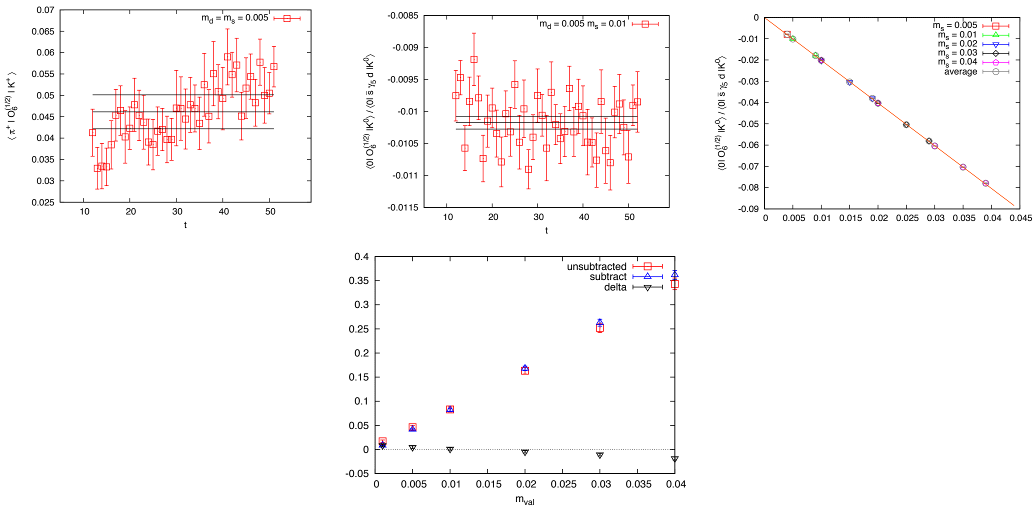


Figure 8: Power subtractions in the computation of $\Delta S = 1$ matrix elements: First row: Signal for $\langle \pi^+ | O_i^{(1/2)} | K^+ \rangle$ and $\frac{\langle 0|O_i|K^0\rangle}{\langle 0|(\bar{s}\gamma_5 d)|K^0\rangle}$ and the mass dependence of the latter. Second row: The power subtraction as in equation (3.10).

of the many technical difficulties that arise in the project is the appearance of power divergencies which have to be subtracted like e.g.

$$\langle \pi^+ | O_i^{(1/2)} | K^+ \rangle + \eta_{1,i}(m_s - m_d)\langle 0|(\bar{s}d)|K^+\rangle \,, \tag{3.9}$$

where the subtraction coefficient $\eta_{1,i}$ is determined from the slope of the quark mass dependence of the ratio of matrix elements

$$\frac{\langle 0|O_i|K^0\rangle}{\langle 0|(\bar{s}\gamma_5 d)|K^0\rangle} = \eta_{0,i} + \eta_{1,i}(m_s - m_d)\,. \tag{3.10}$$

The results for the involved matrix elements and the mass dependence of the ratio are illustrated in the plots in the first line of figure 8. That the subtraction (3.10) works very nicely in practice is illustrated in the second line of figure 8. The residual chiral symmetry breaking of domain wall fermions leads to an ambiguity in the normalisation of the subtraction procedure. This is however irrelevant here since only the slope of the subtracted matrix element with respect to the quark masses is of interest. The example given here is representative for the technical challenges encountered in this project which the RBC-collaboration is now carrying over to the NLO-level of chiral perturbation theory.

For completeness I also want to mention the study in [108] where the $K \to \pi\pi$ matrix element for $O_{7,8}^{(3/2)}$ was evaluated directly for unphysical kinematics and also another exploratory study of $K \to \pi\pi$ at NLO using $N_f = 2$ flavours of domain wall fermions in [109].

Another technical difficulty in the calculation of $K \to \pi\pi$ processes in the $\Delta I = 1/2$-channel is the computation of quark-disconnected diagrams that appear as a result of Wick contractions. Kim and Sachrajda [110] propose to compute the process $K\pi^- \to \pi^-$ instead of $K \to \pi^+\pi^-$. It turns out that the time-like disconnectedness in the latter process is replaced by a space-like disconnectedness in the former which is technically much easier to handle. The relation between the two processes has been worked out in NLO chiral perturbation theory.

4. Summary

In this talk I have reviewed recent progress in kaon physics. I discussed the determination of $|V_{us}|$ from lattice predictions for f_K/f_π and for $f_+^{K\pi}(0)$. The computation of the latter with a precision of 0.5% in full QCD is now feasible. The use of all-to-all propagators in the calculation of the relevant meson correlation functions and a new approach using partially twisted boundary conditions will very likely further reduce the error in the near future - new calculations based on these techniques are under way. The determination of $|V_{us}|$ from f_K/f_π will become an independent competitor for the determination from $f_+^{K\pi}(0)$. I expect the current precision of the various results (the best being 0.6% in the case of HPQCD/UKQCD) to be updated in the near future as collaborations continue extending their sets of gauge field configurations towards lighter masses and also towards smaller lattice spacings. This will then also allow to further improve the quality of the combined analysis of $|V_{us}|$ from f_K/f_π and from $f_+^{K\pi}(0)$ in view of CKM-unitarity which will be exciting to monitor.

For B_K the results from the various collaborations agree within errors. The magnitude of the errors however varies strongly and the results from chiral fermion formulations currently look most promising with an error of about 7% in full QCD. Collaborations using staggered quarks in their calculation of B_K should implement non-perturbative renormalisation in order to reduce the currently rather large error bars.

For $\Delta S = 1$ the RBC collaboration is in the middle of a large scale project with light domain wall fermions in large volume and it will be interesting to see whether they can reproduce the measured value of $|\varepsilon'/\varepsilon|$ and shed light on the $\Delta I = 1/2$-rule - the simulations are however technically extremely demanding.

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