

## Recent lattice results on finite temperature and density QCD, part II

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We discuss recent progress in studies of QCD thermodynamics with almost physical light quark masses and a physical value of the strange quark mass. We summarize results on the transition temperature in QCD and analyze the relation between deconfinement and chiral symmetry restoration.

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## 1. Introduction

The QCD vacuum has a complicated structure that manifests itself most prominently in the confinement of quarks and gluons and the spontaneous breaking of chiral symmetry. These non-perturbative properties of QCD are known to be temperature dependent and eventually will disappear at high temperature; at least in the limit of vanishing quark masses we expect that QCD undergoes a phase transition from a hadronic phase to a new phase of deconfined quarks and gluons in which chiral symmetry is restored. It has been speculated that there could be two separate transitions in QCD at which quarks and gluons deconfine and chiral symmetry gets restored [1, 2], a scenario that has indeed been observed in lattice calculations of gauge theories with fermions in the adjointed representation [3]. In QCD with quarks being in the fundamental representation, however, it seems that at least for vanishing quark chemical potential there is a unique transition in the chiral limit at which quarks and gluons deconfine and chiral symmetry gets restored<sup>1</sup>.

While in massless QCD the chiral condensate is a unique order parameter for chiral symmetry restoration, there is no counterpart for deconfinement. The Polyakov loop, which is an order parameter for deconfinement in the limit of infinitely heavy quarks [5], is non-zero at all values of the temperature whenever quarks have finite masses. Nonetheless, the deconfining properties of the QCD transition are clearly reflected in the behavior of bulk thermodynamic observables, e.g. in the rapid rise of the energy or entropy density as well as in the sudden increase in fluctuations of light and strange quark numbers. The sudden change in the latter reflects the liberation of many light degrees of freedom, quarks and gluons, which dominate the properties of the thermal medium at high temperature. In the chiral limit these sudden changes go along with singularities in bulk thermodynamic observables, the specific heat as well as quartic fluctuations of the light quark number diverge or develop a cusp. At the same temperature the chiral order parameter and its derivative with respect to the quark mass, the chiral susceptibility, show singular behavior. In this limit it is obvious that the singular behavior in observables related to deconfinement and chiral symmetry restoration, respectively, are closely related. To what extent this close relation persists also for non-zero values of the quark masses then becomes a quantitative question that should be answered through numerical calculations in lattice QCD.

This became of particular interest in view of a recent calculation [6] that suggested that there might be a large difference in the transition temperature related to deconfinement observables on the one hand and observables sensitive to chiral symmetry restoration on the other hand. It has been suggested that in the continuum limit this difference can be as large as 25 MeV. However, calculations performed with  $\mathcal{O}(a^2)$  improved staggered fermions [7, 8] so far did not show such a large difference.

In this write-up we will discuss some results from lattice calculations concerning the interplay of deconfinement and chiral symmetry restoration. As these results have been presented in July/August of 2007 in a very similar format at the '4th International Workshop on Critical Point and Onset of Deconfinement' and at the 'XXV International Symposium on Lattice Field Theory' the write-up of these talks has been splitted into two parts. In the first part [9] we discussed recent results on the QCD equation of state. In this second part we will concentrate on results that can

<sup>1</sup>Large  $N_c$  arguments suggest that the situation could be more complicated for non-zero quark chemical potential [4].

give insight into properties of the QCD transition itself. We will focus here on a presentation of results obtained with  $\mathcal{O}(a^2)$  improved staggered fermion formulations. Results obtained with the 1-link, stout smeared staggered fermion action [6] have been presented at both meetings separately [10].

## 2. The transition temperature

Before entering the discussion on deconfinement and chiral symmetry restoration, let us briefly summarize the current status of calculations of the QCD transition temperature using various discretization schemes. The goal here is, of course, to determine the transition temperature in the continuum limit of lattice regularized QCD with its physical spectrum of two light and a heavier strange quark mass. While the heavier quarks, e.g. the charm quarks, may influence thermodynamics at high temperature [11, 12], they are not expected to affect the transition temperature. In fact, even dynamical strange quark degrees of freedom seem to have little influence on the value of the transition temperature. Differences in the transition temperatures of 2, (2+1)-flavor and 3-flavor QCD still seem to be well within the current statistical and systematic uncertainty. This is in agreement with the observed weak dependence of the transition temperature on the light quark mass or, equivalently, on the light pseudo-scalar meson mass,

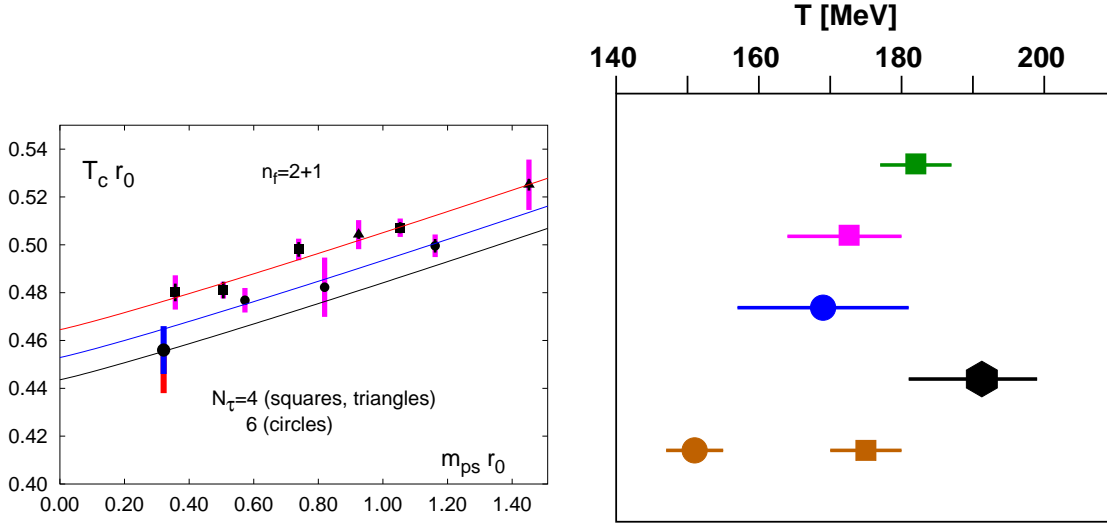
$$r_0 T_c(m_{PS}) - r_0 T_c(0) \simeq A (r_0 m_{PS})^d, \quad (2.1)$$

with  $d \simeq 1$ ,  $A \lesssim 0.05$ . To be specific we have used here the distance  $r_0$  extracted from the static quark potential (see part I [9]) to set the scale for  $T_c$ . The weak quark mass dependence of  $T_c$  is consistently found in calculations with  $\mathcal{O}(a^2)$  improved staggered fermions [7, 16, 17] as well as with Wilson fermions [13, 14, 15]. This has been taken as an indication for the importance of a large number of rather heavy resonances for building up the critical conditions, e.g. a sufficiently large energy density, needed to deconfine the partonic degrees of freedom in QCD. As these heavy resonances are only weakly dependent on the quark mass values the light chiral sector of QCD may play a subdominant role for the quantitative value of the transition temperature, while it does, of course, control the universal properties of thermodynamic observables in the chiral limit. In 2-flavor QCD for instance<sup>2</sup>, the scaling exponent  $d$  appearing in Eq. 2.1, will be related to critical exponents ( $\beta$ ,  $\delta$ ) of 3-dimensional,  $O(4)$  symmetric spin models,  $d = 2/\beta\delta = 1.08$ .

In Fig. 1(left) we show results on the quark mass dependence of the transition temperature obtained in calculations with the p4fat3 staggered fermion action on lattices with two different values of the cut-off,  $aT = 1/4$  and  $1/6$  [7]. As expected, in addition to the obvious quark mass dependence of  $T_c$  also a dependence on the cut-off,  $a$ , is clearly visible. Asymptotically the cut-off dependence is expected to be proportional to  $a^2$ , *i.e.* we expect to find

$$r_0 T_c(m_{PS}, N_\tau) - r_0 T_c(0, \infty) \simeq A (r_0 m_{PS})^d + B/N_\tau^2. \quad (2.2)$$

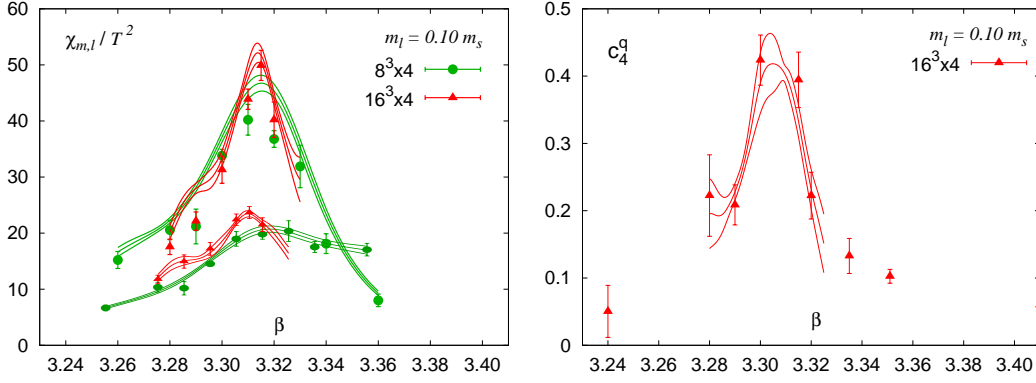
<sup>2</sup>The discussion carries over to the light quark sector of (2+1)-flavor QCD. However, in this case one has to keep in mind that a second order transition point might be reached already at a non-zero value of the light quark mass. This effectively will change the universality class of the transition from  $O(4)$  to  $Z(2)$  and modifies the singular structure of the free energy.



**Figure 1:** Left: Quark mass and cut-off dependence of the transition temperature calculated with the p4fat3 staggered fermion action on lattices with temporal extent  $N_\tau = 4$  and 6 [7]. Right: Transition temperatures determined in several recent studies of QCD thermodynamics. From top to bottom the first two data points show results obtained in simulations of 2-flavor QCD using clover improved Wilson fermions on lattices with temporal extent  $N_\tau = 8, 10$  and  $12$  [13, 14] and  $N_\tau = 4$  and  $6$  [15], respectively. The remaining data points have been obtained in simulations of QCD with 2 light quark masses and a physical strange quark mass. They are based on calculations with staggered fermions using the asqtad action on  $N_\tau = 4, 6$  and  $8$  lattices [16], the p4fat3 action on  $N_\tau = 4, 6$  [7] and 1-link, stout smeared action on  $N_\tau = 4, 6, 8$  and  $10$  lattices [6]. Circles indicate that the determination of the transition temperature is based on observables sensitive to chiral symmetry restoration, i.e. the chiral condensate and susceptibilities deduced from it. Squares indicate that observables sensitive to deconfinement have been used to determine the transition temperature, e.g. the Polyakov loop, its susceptibility and/or light and strange quark number susceptibilities. The diamond indicates that both sets of observables have been analyzed. With the exception of results presented in [15] all calculations aimed at an extrapolation to the continuum limit ( $N_\tau \rightarrow \infty$ ) for physical values of the quark masses. All results have been rescaled to a common physical scale using  $r_0 = 0.469$  fm [20].

This ansatz generally is used to extrapolate to the continuum limit and to extract the transition temperature,  $T_c \equiv T_c(0, \infty)$ . Of course, when using Eq. 2.2 for an extrapolation to the continuum limit one has to make sure that the asymptotic scaling regime has been reached. In Ref. [7] the extrapolation is only based on two different values of the lattice cut-off,  $aT = 1/4$  and  $1/6$ , which may not be close enough to the continuum limit. This has been taken into account in the analysis performed in Ref. [7] by estimating a systematic error for the possible scaling violations. This lead to an estimate of the transition temperature  $T_c = 192(7)(4)$  MeV with the second error denoting an estimate for the systematic uncertainty in the extrapolation. An earlier analysis performed with the asqtad action on lattices with temporal extent  $N_\tau = 4, 6$  and  $8$  but smaller spatial volume,  $N_\sigma/N_\tau = 2$ , lead to the estimate  $T_c = 169(12)(4)$  MeV [16]. Both calculations currently get improved in a systematic comparison of simulations performed with the p4fat3 and asqtad action on lattices of size  $32^3 \times 8$  [8].

In calculations with the p4fat3 action Polyakov loop and chiral susceptibilities have been examined. The transition temperature has been determined by locating peaks in these susceptibilities



**Figure 2:** Disconnected part of the light quark chiral susceptibility and the Polyakov loop susceptibility (left) [7] and the quartic fluctuations of the light quark number (right) [23] calculated on lattices with temporal extent  $N_\tau = 4$  in simulations with the p4fat3 action.

(see Fig.2(left)). This lead to consistent results on larger volumes, although systematic deviations have been observed for smaller spatial volumes, e.g. for  $N_\sigma/N_\tau = 2$ . Calculations performed with the 1-link, stout smeared action [6] on lattices with temporal extent ranging from  $N_\tau = 4$  up to  $N_\tau = 10$ , on the other hand, suggest that there exist differences in the location of the peak positions in chiral susceptibilities and inflection points determined for observables like the Polyakov loop and strange quark number susceptibility. These differences seem to become increasingly significant with increasing  $N_\tau$ . In this analysis a transition temperature related to chiral properties is determined to be  $T_c = 151(3)(3)$  MeV while observables related to deconfinement suggest a transition temperature  $T_c \simeq 175$  MeV. For further discussion of these calculations see also [10].

In the Wilson formulation a discussion of chiral symmetry restoration becomes more involved than in the staggered case. For this reason only observables related to deconfinement, *i.e.* the Polyakov loop and its susceptibility, have generally been analyzed in studies with Wilson fermions. Compared to calculations of the QCD transition temperature performed with staggered fermions in  $(2+1)$ -flavor [6, 7, 16] and 3-flavor [19] QCD, calculations with clover-improved Wilson fermions [13, 14, 15] performed in 2-flavor QCD typically use rather large quark masses,  $m_{PS}r_0 \gtrsim 1$ . This makes an extrapolation to physical masses difficult. Nonetheless, a straightforward application of the scaling ansatz, Eq. 2.2, for extrapolations of the transition temperature down to the physical values of the pion mass,  $m_{PS}r_0 \simeq 0.32$ , yields results that are in reasonably good agreement with values determined within staggered fermion formulations. The current status of calculations of transition temperatures with Wilson and staggered fermions is summarized in Fig. 1(right).

### 3. Deconfinement and chiral symmetry restoration

The relation between deconfinement and chiral symmetry restoration in QCD has been discussed since a long time [1, 2]. Although both phenomena seem to be related to physics on different length scales lattice calculations seem to suggest that both phenomena happen at approximately the same temperature even at finite, non-zero values of the quark masses when none of the symmetries related to confinement ( $Z(3)$  center symmetry) or chiral symmetry breaking ( $SU_L(n_f) \times SU_R(n_f)$ ) are realized exactly. In calculations with almost physical light quark masses and a physical value

of the strange quark mass this has recently been confirmed on lattices with temporal extent  $N_\tau = 4$  and 6 in a detailed analysis performed with  $\mathcal{O}(a^2)$  improved staggered fermions [7]. The gradual restoration of chiral symmetry with increasing temperature is signaled by changes in the light and strange quark chiral condensates and the corresponding susceptibilities,

$$\langle \bar{\psi}\psi \rangle_q = \frac{T}{V} \frac{\partial \ln Z}{\partial m_q}, \quad \chi_{m,q} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial m_q^2}, \quad (3.1)$$

where  $q = l, s$  for the light and strange quark sector, respectively. The onset of deconfinement, on the other hand, can be examined through an analysis of the Polyakov loop  $L$  and its susceptibility,  $\chi_L$ ,

$$L = \left\langle \frac{1}{3N_\sigma^3} \sum_{\vec{n}} \text{Tr} \prod_{n_0=1}^{N_\tau} U_{(n_0, \vec{n}), \hat{0}} \right\rangle, \quad \chi_L = N_\sigma^3 (\langle L^2 \rangle - \langle L \rangle^2), \quad (3.2)$$

where  $U_{(n_0, \vec{n}), \hat{0}}$  denote the gauge field variables defined on temporal links of a lattice of size  $N_\sigma^3 N_\tau$ . Some results for the light quark chiral susceptibility,  $\chi_{m,l}$ , and the Polyakov loop susceptibility,  $\chi_L$ , calculated in a simulation with a physical value of the strange quark mass and a light quark mass that corresponds to a light pseudo-scalar mass of about 220 MeV ( $m_l/m_s = 0.1$ ) on lattices of size  $16^3 4$  are shown in Fig. 2(left).

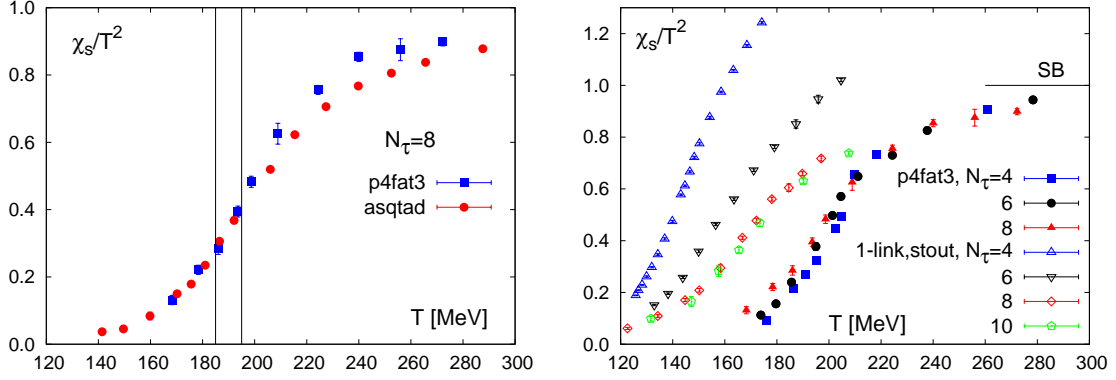
As can be deduced from this figure the Polyakov loop susceptibility does not provide a strong signal for deconfinement in calculations with light dynamical quarks; the Polyakov loop itself is non-zero at all temperatures and rises smoothly through the transition region. This results only in a shallow peak in  $\chi_L$ , which nonetheless is in good agreement with the peak position in the light quark chiral susceptibility,  $\chi_{m,l}$ .

In part I [9] we have presented results on bulk thermodynamic observables, e.g. the energy density ( $\epsilon/T^4$ ), as well as light and strange quark number susceptibilities ( $\chi_{l,s}/T^2$ ). At least on lattices with temporal extent  $N_\tau = 4$  and 6 they both rise rapidly in about the same temperature range. In both cases this is due to the deconfining nature of the QCD transition; as indicated in the introduction both quantities are sensitive to the liberation of many light quark and gluon degrees of freedom. In the chiral limit the sudden rise of, e.g.  $\epsilon/T^4$  and  $\chi_{l,s}/T^2$  seems to be closely related to the singular behavior of the QCD partition function that arises from the restoration of chiral symmetry. To be specific let us discuss here the chiral limit of 2-flavor QCD (see footnote 2). The singular part of the free energy,  $f_s$ , is controlled by a reduced 'temperature'  $t$  that is a function of temperature as well as the quark chemical potential  $\mu_q$  [22]. The latter adds quadratically to the reduced temperature in order to respect charge symmetry at  $\mu_q = 0$ ,

$$f_s(T, \mu_q) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{2-\alpha}, \quad \text{with } t = \left| \frac{T - T_c}{T_c} \right| + c \left( \frac{\mu_q}{T_c} \right)^2, \quad (3.3)$$

where  $b$  is an arbitrary scale parameter and  $\alpha$  denotes a critical exponent. As a consequence the specific heat as well as the quartic fluctuations of the light quark number,  $c_4^q \sim (\langle N_q^4 \rangle - 3\langle N_q^2 \rangle^2)$ , show singular behavior at the critical point  $t = 0$  that is controlled by the same critical exponent  $\alpha$ ,

$$C_V \sim \frac{\partial^2 \ln Z}{\partial T^2} \sim t^{-\alpha}, \quad c_4^q \sim \frac{\partial^4 \ln Z}{\partial \mu_q^4} \sim t^{-\alpha}, \quad \text{for } \mu_q = 0. \quad (3.4)$$

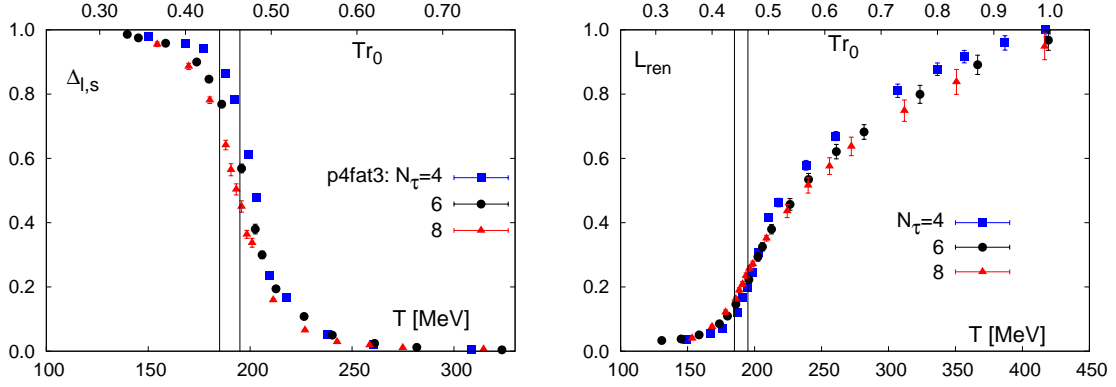


**Figure 3:** Preliminary results of the hotQCD collaboration [8] for the strange quark number susceptibility calculated on lattices of size  $32^3 8$  using two different  $\mathcal{O}(a^2)$  improved staggered fermion actions, asqtad and p4fat3 (left). The vertical lines indicate a band of temperatures,  $185\text{MeV} \leq T \leq 195\text{MeV}$ , which characterizes the transition region in the  $N_\tau = 8$  calculations [8] (see also Figs. 4 and 5). The right hand figure shows a comparison of calculations performed with the p4fat3 action on lattices of temporal extent  $N_\tau = 4, 6$  [23] and  $8$  [8] and with the 1-link, stout smeared action for  $N_\tau = 4, 6, 8$  and  $10$  [6]. Note that different conventions have been used to define the temperature scale (see text)

Unlike the Polyakov loop susceptibility the quartic fluctuations of the quark number thus provide a strong signal for deconfinement. This is shown in Fig. 2(right). In fact, a comparison of  $c_4^q$  calculated here in  $(2+1)$ -flavor QCD with light quark masses that correspond to a light pseudo-scalar (pion) mass of about  $220\text{ MeV}$  [23] and earlier calculations in 2-flavor QCD with 10 times heavier quarks corresponding to a pion mass of about  $770\text{ MeV}$  [25] show that the quartic fluctuations rise strongly with decreasing quark mass.

In Fig. 5 of part I [9] we have shown results for the strange quark number susceptibility, *i.e.* the fluctuations of strangeness number  $\chi_s \sim \langle N_s^2 \rangle$ , calculated with the p4fat3 and asqtad actions on lattices of temporal extent  $N_\tau = 4, 6$  [16, 23] and  $8$  [8] in  $(2+1)$ -flavor QCD and a light to strange quark mass ratio  $m_l/m_s = 0.1$ . The  $N_\tau = 8$  results are preliminary results obtained by the hotQCD collaboration. They are discussed in more detail in [8]. These calculations indicate a quite good agreement between results obtained with the two different  $\mathcal{O}(a^2)$  improved discretization schemes, although in particular at temperatures above the crossover region some differences show up. This is more clearly seen in Fig. 3(left) where we compare the preliminary results obtained within both discretization schemes on  $N_\tau = 8$  lattices. These differences may be due to small differences in the choice of quark masses that define the constant line of physics along which the calculations have been performed and may partly be also due to differences in the discretization errors for both actions which may be about<sup>3</sup> 6% for  $N_\tau = 8$ . These differences as well as the cut-off dependence of results obtained on  $N_\tau = 4, 6$  and  $8$  lattice with the asqtad and p4fat3 actions are, however, small when

<sup>3</sup>In the infinite temperature limit deviations from the continuum value,  $\chi_{free,m=0}^{SB}$ , can be calculated analytically. For massless free staggered fermions on lattices with temporal extent  $N_\tau = 8$  this yields  $\chi_{free,m=0}/\chi_{free,m=0}^{SB} = 0.92$  (asqtad),  $0.98$  (p4fat3),  $1.47$  (1-link,stout). Like in the case of the pressure and other bulk thermodynamic observables, cut-off effects in the quark number susceptibility are  $\mathcal{O}(a^2)$  improved for the asqtad and p4fat3 action and only start with  $1/N_\tau^4$  corrections in the infinite temperature limit.



**Figure 4:** The difference of light and strange quark chiral condensates normalized to its zero temperature value as defined in Eq. 3.5 (left) and the renormalized Polyakov loop expectation value (right). Shown are results from simulations on  $N_\tau = 4$  and 6 lattice obtained with the p4fat3 [21] action as well as preliminary results for  $N_\tau = 8$  obtained by the hotQCD Collaboration [8]. The upper axis shows the temperature in units of the distance  $r_0$  extracted from the heavy quark potential. The lower temperature scale in units of MeV has been obtained from this using  $r_0 = 0.469$  fm [20]. The vertical lines indicate a band of temperatures,  $185\text{MeV} \leq T \leq 195\text{MeV}$ , which characterizes the transition region in the  $N_\tau = 8$  calculations.

compared to results obtained with the 1-link, stout smeared staggered fermion action [6] as shown in Fig. 3(right). The differences between the asqtad and p4fat3 calculations on the one hand and the 1-link, stout smeared calculations on the other hand arise from two sources. For small values of  $N_\tau$ , the quark number susceptibilities calculated with 1-link staggered fermion actions overshoot the continuum Stefan-Boltzmann result at high temperatures and reflect the strong cut-off dependence of thermodynamic observables calculated with this action. This is well-known to happen in the infinite temperature, ideal gas limit and influences the behavior of thermodynamic observables in the high temperature phase of QCD (see footnote 3 and also Fig. 2 in [9]). On the other hand the differences also arise from the different choice for the zero temperature observable used to set the temperature scale. While the temperature scale in the asqtad and p4fat3 calculations has been obtained from the static quark potential (the distance  $r_0$ ), the kaon decay constant has been used in calculations with the 1-link, stout smeared action. Of course, this should not make a difference after proper continuum extrapolations have been carried out. At finite values of the cut-off, however, one should make an effort to disentangle cut-off effects in thermodynamic observables from cut-off effects that only arise from a strong lattice spacing dependence in a zero temperature observable that is used to define a temperature scale. In this respect, the scale parameter  $r_0$  extracted from the heavy quark potential is a safe quantity which is easy to determine; it has been studied in detail and its weak cut-off dependence is well controlled [21, 24].

Let us now turn our attention to observables sensitive to chiral symmetry restoration which, of course, is signaled by changes in the chiral condensate (Eq. 3.1). This also is reflected in pronounced peaks in the light quark chiral susceptibility as shown in Fig. 2. As the chiral condensate receives additive as well as multiplicative renormalization, one should look at appropriate combinations that eliminate the renormalization effects. An appropriate choice is to subtract a fraction of the strange quark condensate from the light quark condensate and normalize the finite temperature



difference with the corresponding zero temperature difference,

$$\Delta_{l,s}(T) = \frac{\langle \bar{\psi}\psi \rangle_{l,T} - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_{s,T}}{\langle \bar{\psi}\psi \rangle_{l,0} - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_{s,0}} . \quad (3.5)$$

We note, that the strange quark contribution to this quantity will drop out in the chiral limit;  $\Delta_l(T) \equiv \lim_{m_l \rightarrow 0} \Delta_{l,s}(T)$  thus will become the standard order parameter for chiral symmetry restoration. This normalized difference of condensates obtained in calculations with the p4fat3 action on lattices with temporal extent  $N_\tau = 4, 6$  [21] and 8 [8] is shown in Fig. 4(left).

In the right hand part of this figure we show results for the renormalized Polyakov loop obtained in the same set of calculations. As can be seen the most rapid change in both quantities occurs in the same temperature range also on lattices with temporal extent  $N_\tau = 8$  [8]. A cut-off dependence, which shifts the transition region to smaller temperatures, is visible in both observables. It, however, seems to be small and correlated in both observables.

The rapid change in the chiral condensate reflected in Fig. 4 by the drop in  $\Delta_{l,s}(T)$ , of course, is correlated to a peak in the light quark chiral susceptibility,  $\chi_{m,l}$ , introduced in Eq. 3.1. This susceptibility actually is composed of two contributions, usually referred to as the connected and disconnected part,

$$\chi_{\text{tot}} \equiv \chi_{m,l} = \chi_{\text{disc}} + \chi_{\text{con}} \quad (3.6)$$

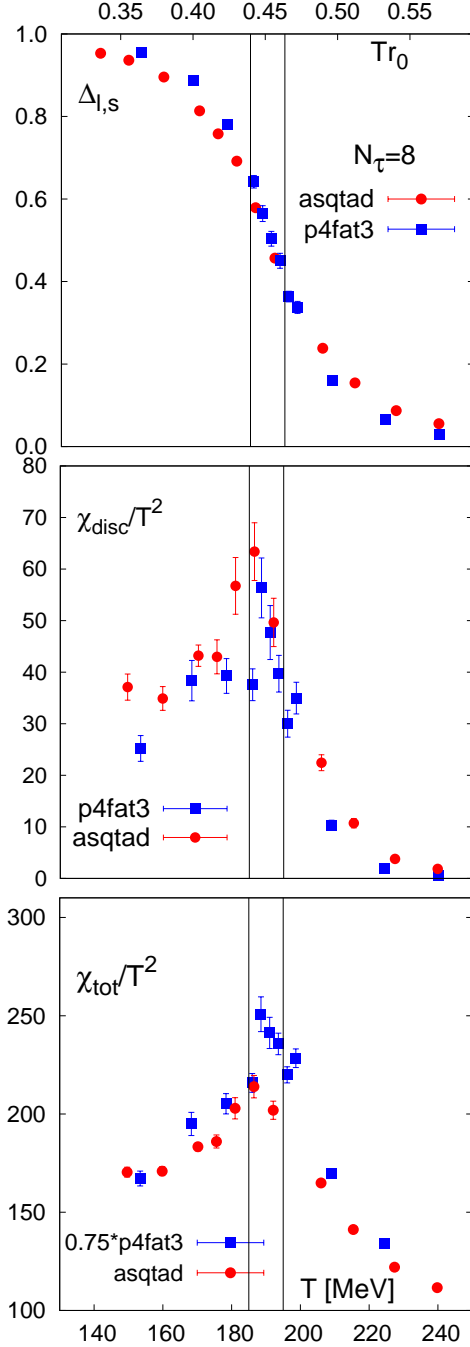
with

$$\begin{aligned} \chi_{\text{disc}} &= \frac{1}{4N_\sigma^3 N_\tau} \left\{ \langle (\text{Tr} D_l^{-1})^2 \rangle - \langle \text{Tr} D_l^{-1} \rangle^2 \right\} , \\ \chi_{\text{con}} &= -\frac{1}{2} \sum_x \langle D_l^{-1}(x,0) D_l^{-1}(0,x) \rangle . \end{aligned} \quad (3.7)$$

Here  $D_l$  denotes the staggered fermion matrix for the light quarks. In Fig. 5 we show results for the disconnected part of the light quark chiral susceptibility and the combined total susceptibility obtained in calculations performed on lattices of size  $32^3 8$  with the asqtad and p4fat3 actions. This is compared to the subtracted, normalized chiral condensate,  $\Delta_{l,s}(T)$ , calculated with both actions on the same size lattices. As can be seen the peak in  $\chi_{\text{disc}}/T^2$  as well as  $\chi_{\text{tot}}/T^2$  obtained from calculations within both discretization schemes is in good agreement and corresponds well to the region of most rapid change in  $\Delta_{l,s}(T)$ .

#### 4. Conclusions

Studies of the thermodynamics of (2+1)-flavor QCD with a physical value of the strange quark mass and almost physical values of the light quark masses have been performed at vanishing chemical potential with two versions of  $\mathcal{O}(a^2)$  improved staggered fermions, the asqtad and p4fat3 actions. Already on lattices with temporal extent  $N_\tau = 6$  they yield a consistent description of bulk thermodynamics, e.g. of the temperature dependence of energy density and pressure. This also holds true for the structure of the transition region and is confirmed through calculations closer to the continuum limit performed on lattices with temporal extent  $N_\tau = 8$ . These calculations yield



**Figure 5:** Subtracted finite temperature chiral condensates normalized by the corresponding zero temperature quantity evaluated at the same value of the cut-off (top), the disconnected part of the light quark chiral susceptibility (middle) and the total light quark chiral susceptibility (bottom). All figures show preliminary results of the hotQCD Collaboration obtained with two different  $\mathcal{O}(a^2)$  improved staggered fermion actions on lattices of size  $32^3 \times 8$  [8].

preliminary results for the transition temperature that may differ by a few MeV, depending on the observable used to identify the transition at non-zero quark mass values. In particular, on the  $N_\tau = 8$  lattice no large differences in the determination of the transition temperature arises from observables related to deconfinement and chiral symmetry restoration respectively. The preliminary results of the hotQCD collaboration indicate that the crossover region for both deconfinement and chiral symmetry restoration lie in the range  $T = (185-195)$  MeV for  $N_\tau = 8$  and  $m_l/m_s = 0.1$ .

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