PROCEEDINGS OF SCIENCE

Meson masses at large $\mathrm{N_c}$

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We analyze the meson spectrum in $SU(N_c)$ lattice gauge theory. We use Wilson quarks to measure the pseudoscalar and vector masses in $SU(2)$, $SU(3)$, $SU(4)$ and $SU(6)$, and extrapolate to the large–N_c limit. We find that finite–N_c corrections are small at all values of the quark mass.

The XXV International Symposium on Lattice Field Theory
July 30-4 August 2007
Regensburg, Germany

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PoS(LATTICE 2007)050

1. Introduction

The number of colours N_c is an implicit parameter of QCD. In the large N_c limit the theory becomes simpler [1]; amongst other features, mesons and glueballs become exactly stable. The finite–N_c corrections in the pure gauge theory are $\mathscr{O}(1/N_c^2)$. This is also the case in quenched lattice field theory, since in that case fermion loops do not contribute. For full QCD the corrections are $\mathscr{O}(1/N_c)$. Clearly it would be very interesting to evaluate these corrections and establish how large they are in the case of QCD ($N_c = 3$).

The corrections can be estimated by carrying out quenched calculations at finite N_c. The leading N_c dependence will then give the $\mathscr{O}(1/N_c^2)$ quenched corrections, and an $N_c \to \infty$ extrapolation will give the large N_c limit. Finally this can be compared to unquenched $SU(3)$ results (or indeed directly to experiment) to estimate the $\mathscr{O}(1/N_c)$ unquenched corrections.

Calculation of quantities in the large–N_c limit is also important for attempts to extend the AdS/CFT correspondence [2] to the case of QCD. This approach, known as AdS/QCD, attempts to learn about QCD by studying a five–dimensional theory that is dual to the large–N_c limit of QCD [3, 4, 5, 6, 7, 8]. The form of this five–dimensional dual (if it exists) is unknown, and additional information on the QCD side would help to constrain it. Since the duality occurs in the large–N_c limit such information must also be calculated in that limit.

The masses of low–lying glueballs have been calculated on the lattice for N_c up to 8 [9]. The finite–N_c corrections were found to be small, reaching only about 10% in the case of $SU(2)$. The pion mass has been calculated for $N_c = 17$ to 23, with no N_c dependence observed [10]. Other meson masses have not to our knowledege been calculated at large N_c.

In this work we present calculations of the pion and rho masses up to $SU(6)$. We describe our methods in the next Section and our results in Section 3. We sum up in Section 4.

2. Methods

We use the lattice software package *Chroma* [11], which we adapt to work for arbitrary N_c. We use unimproved Wilson fermions, together with the Wilson plaquette action for the gauge fields.

To set the scale, we use the string tension calculations by Lucini *et al.* [12]. We choose the coupling $\beta = 2N_c/g^2 = 2N_c^2/\lambda$, where λ is the 't Hooft coupling, such that the string tension in lattice units, $a\sqrt{\sigma}$, is the same for each N_c. We use the value $a\sqrt{\sigma} = 0.2093$: for $SU(3)$ this corresponds to $\beta = 6.0175$. The values for other N_c are shown in Table 1. Adopting a value of 420 MeV for the string tension, the lattice spacing is then 0.099 fm in each case. The values of $a\sqrt{\sigma}$ used in the fits in [12] are very accurate [9], so the errors on our estimate of the lattice spacing are less than 1%.

The volume of our lattices is $16^3 \times 32$ in lattice units, corresponding to a spatial volume of 1.58^3 fm. Finite–volume effects are expected to decrease with N_c, and to be zero at infinite N_c as long as the box is larger than a critical length l_c [13]. This means that we should obtain the correct large–N_c limits for the masses we calculate, despite any finite–volume effects at small N_c.

We find that, as expected, the cost of updating a gauge configuration is approximately proportional to N_c^3, and the cost of inverting a propagator is approximately proportional to N_c^2 (the number of conjugate gradient steps required for the inversion is approximately independent of N_c). For the

N_c	β
2	2.4645
3	6.0175
4	11.028
6	25.452

Table 1: Value of the coupling β for each N_c.

relatively small values of N_c we use, the latter in fact dominates. Furthermore, we find that the correlators become less noisy at larger N_c (see below), so fewer are needed to calculate a mass to a given precision. This reduces the total cost to $\sim N_c$. A heuristic argument supporting this observed reduction in noise is the increase of the degrees of freedom of the statistical system $\propto N_c^2$ at fixed volume.

We calculate local–local and local–smeared correlators on our configurations. We smear using 0 to 100 iterations of Gaussian smearing, $\psi'(x) = \psi(x) + \kappa \sum_\mu (U_\mu(x)\psi(x+\mu) + U_\mu^\dagger(x-\mu)\psi(x-\mu))$, with smearing parameter $\kappa = 4$. We smear the gauge links using 10 iterations of APE smearing, $U'_\mu(x) = \alpha U_\mu(x) + \sum_\nu (U_\nu(x)U_\mu(x+\nu)U_\nu^\dagger(x+\mu) + U_\nu^\dagger(x-\nu)U_\mu(x-\nu)U_\nu(x+\mu-\nu))$, with smearing parameter $\alpha = 2.5$

3. Results

We search for the critical value of the hopping parameter, κ_c, by finding the value of κ for which the pion mass m_π vanishes. We show a plot $(am_\pi)^2$ as a function of $1/\kappa$ in Fig. 1. For suffciently small masses the dependence is linear, as expected, and we can extrapolate to zero pion mass. The values of κ_c we obtain are shown in Table 2. It is clear that they are rapidly converging to an $N_c = \infty$ value.

N_c	κ_c
2	0.15327(16)
3	0.156397(45)
4	0.158168(39)
6	0.159060(44)

Table 2: Critical hopping parameter κ_c for each N_c.

As mentioned above, we find that correlators become less noisy as N_c increases. We illustrate this in Fig. 2, where we compare pseudoscalar correlators on individual gauge configurations in $SU(3)$ and $SU(6)$. The pion masses are almost identical, $am_\pi = 0.283(5)$ and $0.286(3)$ respectively, but the scatter between individual configurations, a measure of the noise, is about twice as large in $SU(3)$, in agreement with the naive degrees of freedom argument above.

Since fluctuations decrease as N_c increases, and we use correlators of fluctations to extract masses, one might worry that we will be unable to calculate masses in this way in the large–N_c limit. Fortunately this is not the case. The correlators do indeed decrease as $1/N_c^2$, but the fluctations in

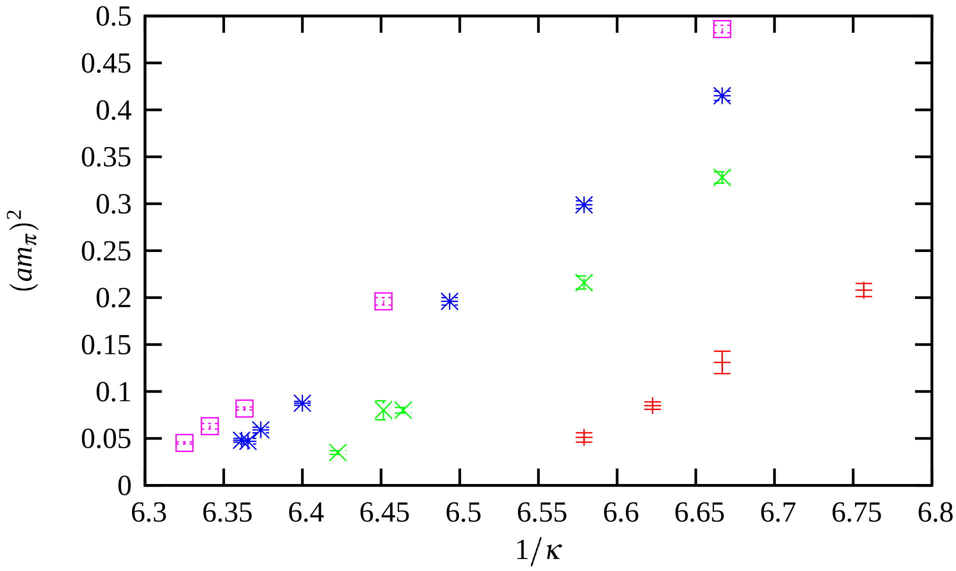

Figure 1: $(am_\pi)^2$ as a function of $1/\kappa$ for $SU(2)$ ($+$), $SU(3)$ ($\times$), $SU(4)$ ($*$) and $SU(6)$ ($\square$).

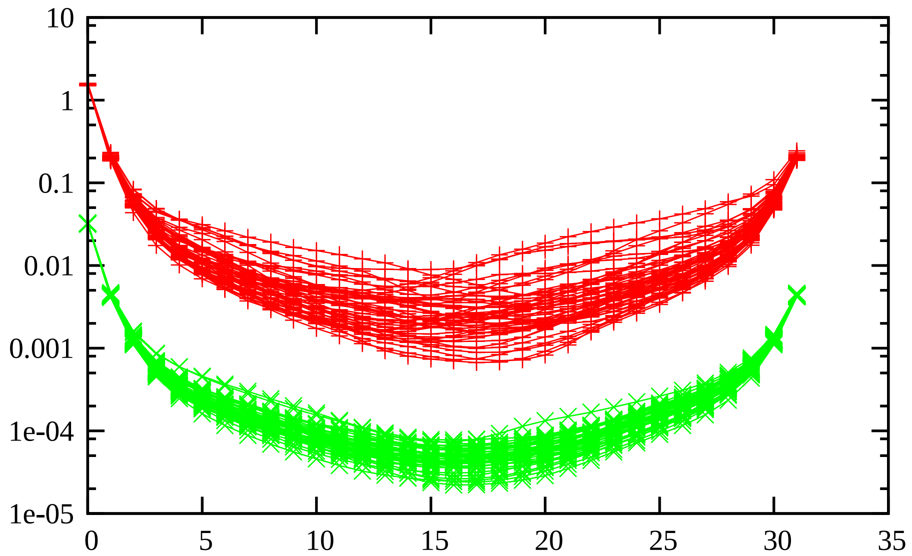

Figure 2: Point–point pseudoscalar correlators on individual gauge configurations in $SU(3)$ at $\kappa = 0.1547$ ($+$) and in $SU(6)$ at $\kappa = 0.15715$ ($\times$). $SU(6)$ results have been shifted vertically for clarity.

the correlators decrease at the same rate, so the signal–to–noise ratio remains constant. For a more detailed discussion of this issue see [9].

We have calculated the lowest Dirac eigenvalues at our lightest pion mass. We plot the distributions of eigenvalues we obtain in $SU(3)$ and $SU(6)$ in Fig. 3. These are at similar pion masses, $am_\pi = 0.188(6)$ and $0.212(3)$ respectively, but we see that the distribution of eigenvalues is much narrower in $SU(6)$, with no near–exceptional configurations. This suggests that it may be possible to use unimproved Wilson quarks at much lighter pion masses at large N_c than is possible for $SU(3)$.

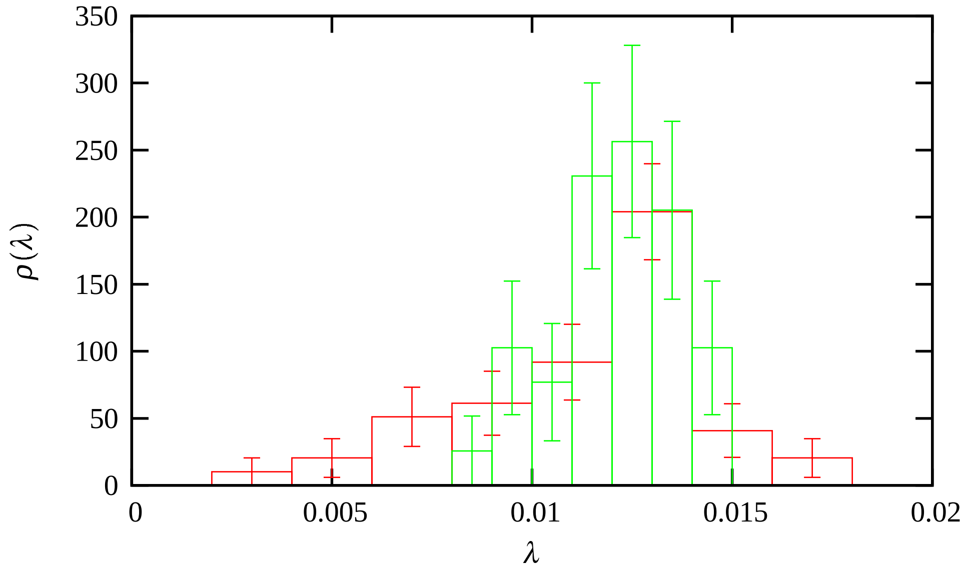

Figure 3: Eigenvalue density for the smallest eigenvalue of $\gamma_5 D$ in $SU(3)$ at $\kappa = 0.1557$ (red) and in $SU(6)$ at $\kappa = 0.1581$ (green).

We extract the ground state pseudoscalar and vector masses from correlators at a range of quark masses. The lowest pion mass is approximately $m_\pi = 0.2a^{-1} = 410$ MeV, corresponding to a quark mass somewhat lighter than the strange quark mass, while the heaviest is around $m_\pi = 0.6a^{-1} = 1220$ MeV. At our lighest masses $m_\pi/m_\rho \approx 0.5$. We show the extracted masses in Fig. 4. We do not have exact matching of the pion masses for different N_c in all cases, so we plot the rho masses against the squares of the pion masses — if there is no N_c–dependence these should all fall on a common curve. We see that for the entire range of quark masses, the points do indeed fall onto the same line, the only exception being $SU(2)$, for which m_ρ appears to be about 10% higher at the lowest quark masses. Finite volume effects are expected to be largest at small N_c and light quark masses, but we would expect these to increase m_π^2 by a larger amount than m_ρ, shifting the $SU(2)$ points below the line, the wrong direction to explain the observations. If the deviations are indeed real finite–N_c effects, they are of order 10%, similar to the deviations observed for glueballs in [9]. Assuming they are dominated by the $1/N_c^2$ corrections, the corresponding deviations for $SU(3)$ should be around 4%.

4. Conclusions

We have calculated the masses of the pion and rho mesons up to $SU(6)$, at a cost approximately proportional to N_c. We have shown that the N_c dependence of the masses is small, approximately 10% for $SU(2)$ at small pion masses, and that there is no observable dependence at higher masses. If these differences are dominated by the $1/N^2$ corrections, which seems very plausible, there will be a difference of about 4% between $SU(3)$ and $SU(\infty)$. Thus, at least for the masses of these mesons, quenched QCD is very close to the large–N_c limit.

The distribution of the lowest eigenvalue of $\gamma_5 D$ becomes much narrower as N_c increases. This suggests that it maybe be possible to use Wilson quarks at significantly lower quark masses without

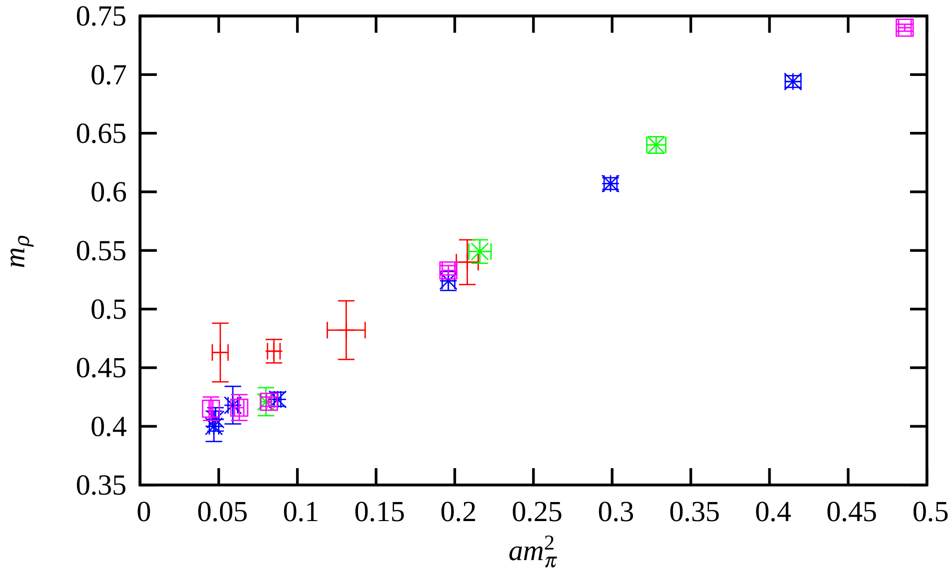

Figure 4: Rho mass against pion mass squared, for $SU(2)$ $(+)$, $SU(3)$ $(\times)$, $SU(4)$ $(*)$ and $SU(6)$ $(\Box)$.

running into exceptional configurations. We intend to explore this possibility in future work. We will also examine smaller volumes to check that the volume dependence does indeed decrease.

Apart from the ground state pseudoscalar and vector, we intend to also calculate the masses of the excited states. It would also be interesting to look at other quantum numbers, in particular the scalar mesons.

Finally we note that our calculations have been carried out at only one lattice spacing. Our results will thus be affected by lattice corrections. However, these lattice corrections will themselves have a large–N_c limit, which will affect our results for all N_c equally. Only an unlikely cancellation between the continuum and lattice $1/N_c^2$ corrections could cause us to see the very small N_c–dependence we observe. Thus, although our values for the meson masses have yet to be extrapolated to the continuum limit, our conclusion that the N_c–dependence is very small is unlikely to be affected. One should of course check this by repeating the calculations at a smaller lattice spacing, and we intend to do so in future.

Acknowledgments

This work is supported by the EC Hadron Physics I3 Contract RII3-CT-2004-506087 and by the GSI University Program Contract RSCHAE.

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