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Gauge-gravity duality – Super Yang Mills Quantum Mechanics

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We describe the conjectured holographic duality between Yang-Mills quantum mechanics and type IIa string theory. This duality allows us to use lattice Monte Carlo simulations to probe the physics of the gravitational theory - for example, at low energies it provides a computation of black hole entropy in terms of a sum over microstates of the dual gauge theory. Numerical results are presented of the 4 supercharge theory at finite temperature

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1. Introduction

The possible equivalence of string and gauge theory has long history going back to birth of string theory as a possible theory of the strong interactions and the recognition that Feynman diagrams of perturbative large N gauge theory could be naturally classified into a sum over surfaces of given genus.

However, there has been renewed interest in the subject in recent years stemming from Maldecena's discovery that the type II string theory in anti-deSitter space could be described in terms of a particular supersymmetric gauge theory in four dimensions [1]. This theory has sixteen real supersymmetries and gauge group $SU(N)$. This so-called AdSCFT correspondence has been tested most extensively in the limit $N \to \infty$ and for low energies in which case the string theory reduces to classical supergravity. However it is widely conjectured to be true away from these limits. Perhaps surprisingly the gauge theory in question looks very different from QCD – it is a superconformal field theory with neither asymptotic freedom nor confinement.

Since then many other such *dualities* have been postulated corresponding to varying the dimension of the gauge theory, adding additional matter fields and breaking all or part of the supersymmetry and conformal invariance. The dual gauge systems possess the common feature of residing on the boundary of the space on which the string theory is defined and are hence often referred to as giving a *holographic* representation of the gravitational system. One such class of dualities arise naturally from the original AdSCFT construction and consist of a mapping between type II string theory containing N Dp-branes and $(p+1)$ dimensional supersymmetric gauge theories with gauge group $SU(N)$ [2]. In this talk we will concentrate on perhaps the simplest of these systems which describes the dynamics of $D0$-branes in string theory in terms of super Yang Mills quantum mechanics [3].

We will be interested in this mapping at finite temperature corresponding to a background string geometry in which the time coordinate is Euclidean and periodic. At low energies the resultant supergravity equations contain a black hole with charge N and metric

$$ds^2 = \alpha'[-h(U)dt^2 + h^{-1}(U)dU^2 + \frac{c^{\frac{1}{2}}\sqrt{\lambda}}{U^{\frac{3}{2}}} d\Omega_8^2 \tag{1.1}$$

where ($\lambda = N g_s \alpha'^{-3/2}$) and the function $h(U)$ is given by

$$h(U) = \frac{U^{\frac{7}{2}}}{c^{\frac{1}{2}}\sqrt{\lambda}} \left(1 - \left(\frac{U_0}{U}\right)^7\right) \tag{1.2}$$

with $\alpha' = l^2_{\rm string}$ is the (inverse) string tension and g_s the string coupling. Such a black hole has a temperature and entropy given by

$$\frac{T}{\lambda^{\frac{1}{3}}} \sim \left(\frac{U_0}{\lambda^{\frac{1}{3}}}\right)^{\frac{5}{2}} \qquad S \sim N^2 \left(\frac{U_0}{\lambda^{\frac{1}{3}}}\right)^{\frac{9}{2}} \tag{1.3}$$

The holographic conjecture states that the dual Yang-Mills model has N colors and is to be computed at the *same* temperature with λ identified as usual 't Hooft coupling $\lambda = g^2_{YM} N$. The statement of duality implies that the free energies of both gauge theory and black hole are equal. Duality thus offers a way of understanding black hole entropy as arising from a counting of microstates in the dual gauge theory.

The analysis that leads to the duality conjecture is only valid in which the string theory reduces to supergravity. This requires $Ng_s >> 1$ and $\alpha' \to 0$. Thus the dual super Yang Mills quantum mechanics is *strongly coupled*. This is rather a generic feature of these gauge-gravity dualities – typically the regime in which the string theory is tractable corresponds to a strongly coupled large N gauge theory. Furthermore, in the quantum mechanics case the dynamics of the gauge theory depends only on the dimensionless coupling $\beta = \frac{\lambda^{\frac{1}{3}}}{T}$ which then implies it is the low temperature behavior of the gauge theory that provides a description of the dual black hole thermodynamics.

It is not known what happens as we increase the temperature of the gauge theory. Familiarity with thermal gauge theories leads us to speculate that the system can undergo a deconfining phase transition. Indeed, as we shall show just such a transition occurs in the quenched theory. In the gravitational language raising the temperature corresponds to increasing α' and with it the strength of classical string theory corrections. It is not known for sure what happens in this case. It is possible that the duality breaks down and no correspondence exists between the gauge and gravity models. Or more exotic possibilities could exist with the existence of a thermal phase transition in the gauge theory signaling some sort of phase transition in the gravitational system - for example a transition from black holes to a hot gas of strings and branes. One of the goals of our work will be to map out this gauge-gravity correspondence and to understand the phase diagram of the string theory by conducting numerical simulations of the gauge theory.

In this talk we will report on numerical simulations of a related model with four supersymmetries. General arguments exist [4] that this model may lie in a similar universality class to its sixteen supercharge cousin. This theory has also been studied recently in [5].

2. Action and supersymmetries

The $\mathscr{Q} = 16$ super Yang-Mills QM is obtained by dimensional reduction of $\mathscr{N} = 1$ SYM in $D = 10$ dimensions.

$$S = \frac{N}{\lambda}\int^R d\tau \left((D_\tau X_i)^2 - [X_i, X_j]^2 + 2i\Psi^\alpha D_\tau \Psi_\alpha + 2\Psi^\alpha (\Gamma_i)^\beta_\alpha [X_i, \Psi_\beta]\right) \tag{2.1}$$

where Ψ^α is a 16-component spinor and $\Gamma_i = \gamma_0\gamma_i$ where $\gamma_i, i = 0\ldots 10$ are the $D = 10$ Dirac matrices in Majorana representation. The bosonic sector of the model consists of 9 scalars X_i and a gauge field A. All fields take values in the adjoint representation of the gauge group. The supersymmetries are given by

$$\delta A = -2i\Psi_\alpha \varepsilon^\alpha \tag{2.2}$$

$$\delta X_i = -2\varepsilon^\alpha (\Gamma_i)^\beta_\alpha \Psi_\beta \tag{2.3}$$

$$\delta \Psi^\alpha = \frac{1}{2}\left((\Gamma_i)^\alpha_\beta i D_\tau X_i + \frac{1}{2}([\Gamma_i, \Gamma_j])^\alpha_\beta [X_i, X_j]\right)\varepsilon^\beta \tag{2.4}$$

After integration over the fermions one encounters a Pfaffian which on generic scalar field backgrounds is complex. This renders simulation of the model problematic. Because of this we have initially focused on a related four supercharge model corresponding to the dimensional reduction of $\mathscr{N}=1$ super Yang Mills in four dimensions. This model has received extensive attention in the literature as it may be discretized on a lattice while preserving part of the supersymmetry algebra exactly[1][6]. The $\mathscr{Q}=4$ model has an action similar to that shown above with the restrictions that there are now just 3 scalars and the fermions are 4 component Majorana fields with Yukawa couplings related to the corresponding four dimensional Dirac matrices in Majorana representation.

3. Lattice Theory

The transition to a lattice theory is potentially delicate. Naive discretizations generically break supersymmetry at the classical level leading to the appearance of supersymmetry breaking counterterms in the quantum effective action. Some of these may be relevant which leads to a fine tuning problem as their associated couplings must then be fine tuned to recover supersymmetry in the continuum limit.

However, super Yang-Mills quantum mechanics is, of course, super-renormalizable, which implies that the continuum theory contains only a finite number of superficially U.V divergent Feynman graphs and they occur for small numbers of loops. It is only these terms that can generate new supersymmetry violating interactions on the lattice. This is the content of a powerful theorem on lattice Feynman diagrams due to Reisz [7] but it is easy to understand – the lattice propagators resemble continuum propagators for light modes and only depart strongly for modes near the cut-off. In a theory with no cut-off sensitivity these U.V modes are irrelevant and the lattice diagram converges for vanishing lattice spacing to its continuum counterpart.

In our quantum mechanics case there is only one potentially U.V sensitive diagram – a one loop fermion tadpole correction to the scalar field. However the gauge index structure of this term ensures it must vanish even in the lattice theory. Thus we conclude that any discretization of the continuum theory prohibiting doublers will flow to the correct supersymmetric continuum limit *without fine tuning*.

Hence we have employed the following lattice action:

$$S=\kappa\sum_{\tau}\left(\sum_{i=1}^{3}\left(D^{+}X_i\right)^2-\sum_{i<j}[X_i,X_j]^2+\Psi^T(1)D^{+}\overline{\Psi}^{(2)}+\Psi^T(2)D^{-}\Psi^{(1)}+\Psi^T(\hat{\Gamma}_i[X_i,\Psi]\right) \quad (3.1)$$

Here $\Psi^{(1)},\Psi^{(2)}$ label the upper and lower components of the fermion field. The matrices $\hat{\Gamma}_i$ are essentially the Γ_i up to an redefinition of the Euclidean time direction in order that the lattice kinetic term takes the explicitly antisymmetric form given above [2]

The lattice covariant derivative is just

$$D^{+}f=U_1(t)f(t+\hat{t})U_1^{\dagger}(t)-f(t) \quad (3.2)$$

[1] A property it shares with the $\mathscr{Q}=16$ model

[2] This trick happens automatically in the twisted formulations described in [6]. In the $\mathscr{Q}=4$ case described here it is possible to rewrite the Pfaffian as the determinant of matrix operator of half the size and discretizations of this operator can then choose a kinetic term proportional to the unit matrix. We have also used this equivalent formulation in our numerical work

and the lattice coupling is given by $\kappa = \frac{NM^3}{\beta^3}$ where we measure the inverse temperature β in units of the 't Hooft coupling λ and M is the number of lattice points. The continuum limit is simply $M \to \infty$ at fixed β. We also look for limiting behavior for large N.

4. Simulation details

It is not hard to show that the Pfaffian arising after integration over the fermions is real and positive semi-definite allowing it to be replaced by the term $\left(M^\dagger M\right)^{\frac{1}{4}}$ where M is the fermion operator described in the previous section and we use antiperiodic thermal boundary conditions for the fermions. This latter weight is generated by an auxiliary integration over pseudofermion fields $F, F^\dagger$ with action

$$S_{\rm PF} = F^\dagger \left(M^\dagger M\right)^{-\frac{1}{4}} F \tag{4.1}$$

Following the usual RHMC algorithm the latter is approximated by a partial fraction expansion of the form

$$\frac{1}{x^{\frac{1}{4}}} \sim \alpha_0 + \sum_{i=1}^{Q} \frac{\alpha_i}{x+\beta_i} \tag{4.2}$$

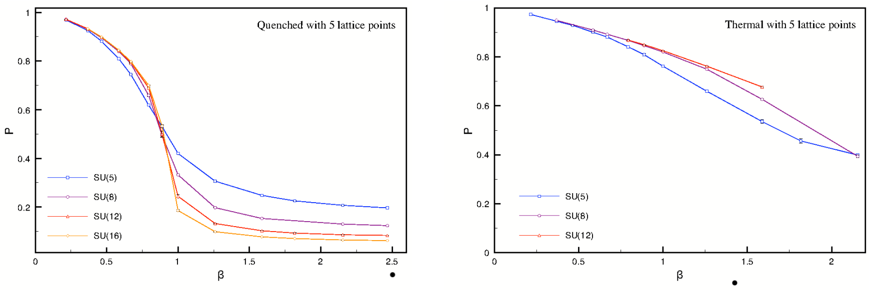
where the α_i, β_i are computed using the remez algorithm [8]. The resultant action is simulated using standard HMC techniques together with a multimass CG-solver to solve the Q linear systems in a time independent of Q [9].

We have simulated lattices with $5--12$ points at values of the inverse temperature β ranging from $0.01--3.0$. Typically we have amassed between $10^3 - 10^4$ $\tau = 1$ HMC trajectories for systems with a number of colors in the range $N = 5--16$.

Our primary observables are the absolute value of the trace of the Polyakov loop P, its associated susceptibility $\chi_P = \frac{1}{N}\left(< P^> - < P >^2\right)$ and the mean energy of the system E. A simple scaling argument shows that the latter is simply related to the expectation value of the bosonic action S_B. We have also examined the mean extent of the scalar fields given by $W = \frac{1}{\beta^{\frac{3}{4}}} \int d\mu |\mu| P(\mu)$. We have simulated both the full supersymmetric theory and a quenched theory in which the effects of dynamical fermions are removed.

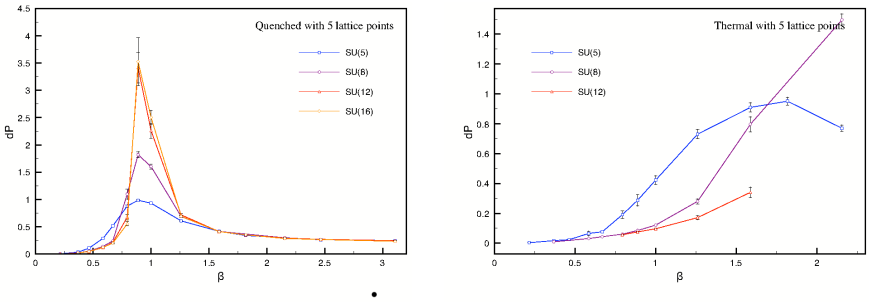
5. Results

Our results for the Polyakov line P in both dynamical and quenched ensembles are shown in figure 1 We see good evidence for 't Hooft scaling in both cases. In the quenched theory we see evidence of a rapid crossover between a low temperature confined phase and a high temperature deconfined phase. This crossover seems to strengthen with increasing number of colors N. However no such crossover is seen in the supersymmetric case and the system seems to exist in a single deconfined phase. This conclusion is strengthened by looking at the susceptibility χ_P in figure 2 which shows a a sharp peak around $\beta_c \sim 0.9$ which grows with N in the quenched theory but no such peak in the supersymmetric theory - a very broad peak is seen at small N and for large β but this seems to move rapidly to larger β with increasing N and we conclude it will not survive the large N limit. Further evidence of this dramatic difference between quenched and supersymmetric theories is seen in the energy E shown in figure 3. In the case of the quenched theory the linear



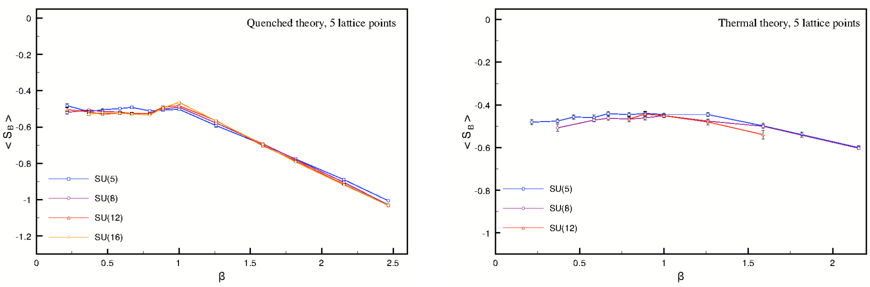
Quenched Dynamical

Figure 1: Polyakov line – quenched and supersymmetric cases



Quenched Dynamical

Figure 2: Polyakov susceptibility – quenched and supersymmetric cases



Quenched Dynamical

Figure 3: Mean energy – quenched and supersymmetric cases

slope visible in $\beta < E >$ for large β yields the non-zero vacuum energy of the model while the horizontal regime below β_c is consistent with classical equipartition and the appearance of $O(N^2)$ weakly coupled degrees of freedom. The dynamical system is quite different – we see good 't Hooft scaling but only weak β dependence – the large β behavior is qualitatively similar to the quasi free behavior seen at high temperature. Again the simplest interpretation is that the deconfined phase persists for all finite temperatures.

6. Conclusions

Holographic dualities offer the possibility of studying string and quantum gravity theories using the tools of (supersymmetric) gauge theory. Typically the gauge theories that arise in these contexts are strongly coupled which means that lattice methods are useful. Furthermore, low dimensional examples exist which allow for naive discretization methods which nevertheless regain supersymmetry with little or no fine tuning. These models can be simulated relatively easily using modern algorithms even in the 't Hooft large N limit.

In this talk we have presented results for a toy model with just $\mathscr{Q} = 4$ supercharges. Many features of this model are thought to be common to the target $\mathscr{Q} = 16$ model but the toy model is easier to simulate primarily because it possesses a real positive definite Pfaffian after integration over the fermions.

We have contrasted the supersymmetric simulations with quenched simulations. In both cases good large N scaling is seen. In the quenched case the system exists in two phases in the limit $N \to \infty$ – a low temperature confined phase with non-zero vacuum energy and a high temperature deconfined phase with $O(N^2)$ quasi free degrees of freedom. In the supersymmetric case the deconfined phase appears to extend to low temperatures. This is in agreement with conjectures about the $\mathscr{Q} = 16$ supercharge theory [4]. The latter theory has recently been studied using momentum space methods in [10] with interesting results. Perhaps most encouraging is the observation that the phase fluctuations of the Pfaffian were rather small over the temperature regime studied. We are currently investigating the $\mathscr{Q} = 16$ supercharge theory and hope to report results soon [11].

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