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$I = 2$ $\pi\pi$ scattering length with dynamical overlap fermion

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We report on a calculation of the $I = 2$ $\pi\pi$ scattering length using the overlap fermion for both sea and valence quarks. The calculation is performed on the gauge configurations generated by the JLQCD collaboration on a $16^3 \times 32$ lattice at a lattice spacing ~ 0.12 fm. We discuss the consistency with chiral perturbation theory.

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1. Introduction

The $I = 2$ $\pi\pi$ scattering provides a testing ground for the method of calculating the hadron interactions on the lattice, since it is the easiest process to compute among other more involved interactions, such as $I = 0$ or 1 $\pi\pi$ scatterings, πN, NN interactions *etc*. The method to extract the scattering length (or, in general, scattering phase shift) on the Euclidean lattice has been known already since middle eighties [1, 2] and some attempts were made to calculate the $I = 2$ $\pi\pi$ scattering length with or without the quenched approximation (see, for early attempts, [3, 4]), but realistic calculation with light dynamical quarks has been made possible only recently [5].

The main interest with the dynamical fermions is the consistency with the chiral perturbation theory (ChPT). Since the pion interactions occur mainly through derivative couplings in ChPT and their structure is completely constrained by chiral symmetry, the scattering length is entirely determined by the pion mass and decay constant at the leading order (small quark mass limit). At the next-to-leading order there exists a non-analytic term, so-called the chiral logarithm, with a definite numerical coefficient. Therefore, its calculation may give a stringent test for the lattice method as far as the chiral limit is sufficiently approached.

In practice, there is a complication due to the violated chiral symmetry (and/or flavor symmetry) when one uses conventional lattice fermion formulations, such as the Wilson-type fermions (or staggered fermions). For these fermions the beautiful relations derived from ChPT are not guaranteed to be satisfied unless the continuum limit is carefully taken first. Therefore, for the stringent test, the use of chiral lattice fermions is mandatory. In the recent work by the NPLQCD collaboration [5] the domain-wall fermion is used on the gauge configurations generated with staggered sea quarks. Although there is a plausible argument that the ChPT relations are not badly distorted with this choice, more rigorous approach using chiral fermions for both sea and valence quarks is desirable. This work is the first such attempt.

We calculate the $I = 2$ $\pi\pi$ scattering length using the overlap fermion for both sea and valence quarks. The calculation is done on a $16^3 \times 32$ lattice at $a \simeq 0.12$ fm: the two-flavor gauge ensemble generated by the JLQCD collaboration [6, 7, 8]. The sea quark mass covers the region $m_s/6$–m_s (m_s is the physical strange quark mass), with which the chiral extrapolation should be reliable. In the following we report some preliminary results and a first attempt to test the consistency with ChPT.

2. Calculation setup and methods

We employ the overlap fermion [9, 10], whose Dirac operator with a quark mass m is defined by

$$D(m) = \left(m_0 + \frac{m}{2}\right) + \left(m_0 - \frac{m}{2}\right)\gamma_5 \mathrm{sgn}[H_W(-m_0)], \tag{2.1}$$

with the standard hermitian Wilson-Dirac operator $H_W(-m_0)$ with a large negative mass $-m_0$ ($m_0 a = 1.6$ throughout this work). For the gauge sector the Iwasaki gauge action is used at $\beta = 2.30$ together with extra (irrelevant) Wilson fermions to suppress the near-zero modes of $H_W(-m_0)$ [11]. With this choice, the global topological charge is conserved during the HMC simulations. Its effect on the physical quantities can be understood as a finite volume effect of $O(1/V)$ [12], and the effect in this particular calculation will be briefly discussed below.

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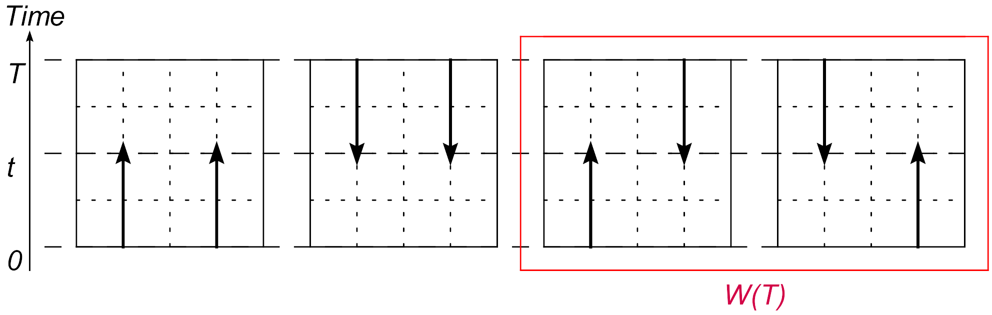

Figure 1: Contributions to the two-pion correlator in the periodic box in t direction. Extra terms due to the *wrap-around* effect is shown by $W(T)$.

The sea and valence quark masses are set equal in this calculation at 0.015, 0.025, 0.035, 0.050, 0.070, and 0.100 in the lattice unit, which correspond to the physical range $m_s/6$–m_s. For each sea quark mass, we picked $\sim$ 100 gauge configurations every 100 HMC trajectories in the JLQCD runs. The auto-correlation time for these runs is $\lesssim 100$.

For the extraction of the scattering length a_0 we utilize Lüscher's formula [1, 2]

$$E_{\pi^+\pi^+} - 2m_{\pi^+} = -\frac{4\pi a_0}{m_\pi L^3}\left\{1 + c_1\frac{a_0}{L} + c_2\frac{{a_0}^2}{L^2}\right\} + O(L^{-6}), \tag{2.2}$$

where $E_{\pi^+\pi^+}$ is the energy of two-pion system in a box of length L and m_{π^+} is the pion mass. The numerical constants c_1 and c_2 are -2.837297 and 6.375183 respectively.

For the interpolating operator of two pions, we simply use the wall source at a time slice $t = 0$. The $I = 2$ two-pion state is constructed by combining the direct (D) and crossed (C) topology of quark lines according to [4]. The sink operator is two local pseudo-scalar density projected onto zero spatial momentum.

The JLQCD group calculated and stored the lowest 50 pairs of eigenvalues and eigenvectors of the overlap-Dirac operator for their dynamical configurations. We utilize them to precondition the overlap solver, which makes the calculation faster by an order of magnitude. In addition, we use them to improve the statistics through the low mode averaging [13]. In this technique, the quark propagator is decomposed to the low-lying mode contribution ("L") and high mode one ("H"). Then, for example, the meson correlator is written as

$$C(t) = C_{\rm HH}(t) + C_{\rm HL}(t) + C_{\rm LH}(t) + C_{\rm LL}(t). \tag{2.3}$$

Among these four contributions, an average over source points is taken for the "LL" piece $C_{\rm LL}(t)$, which is actually the dominant contribution for small quark masses. This averaging greatly improves the statistical signal as demonstrated in [8]. We use the same technique for the two-pion state.

3. Calculation of the energy shift

Because of the periodic boundary condition in the temporal direction, the two-pion correlator does not simply behave as $\exp(-E_{\pi\pi}t)$ but contains some *wrap-around* effects. For the single pion

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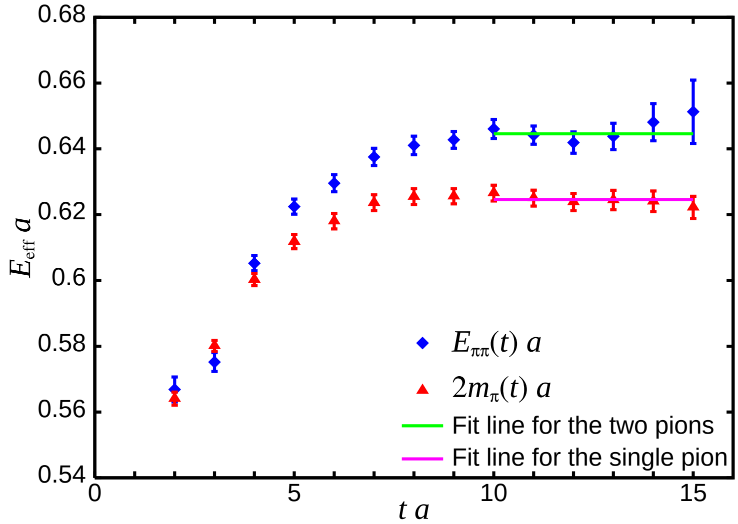

Figure 2: Effective mass plot for the two-pion state (blue squares) at $ma = 0.050$. Twice the pion effective mass is also shown for comparison (red triangles).

case, it is simply written as

$$C_{\pi^+}(t) \propto \exp[-m_\pi t] + \exp[-m_\pi(T-t)] \tag{3.1}$$

with T the temporal length of the lattice. For the two-pion correlator we obtain

$$C_{\pi^+\pi^+}(t) \propto \exp[-E_{\pi\pi}t] + \exp[-E_{\pi\pi}(T-t)] + W(T), \tag{3.2}$$

where $W(T)$ is the *wrap-around* effect and is approximately expressed as

$$W(T) \propto \exp[-m_\pi t] \cdot \exp[-m_\pi(T-t)] = \exp(-m_\pi T). \tag{3.3}$$

(See Fig. 1.) Namely, the two-pion correlator contains a small but significant constant term, that has to be taken into account especially when pion mass is small.

We fit the two-pion correlator to a form

$$C_{\pi^+\pi^+}(t) = A\cosh[-E_{\pi\pi}(t-T/2)] + B \tag{3.4}$$

with two free parameters A and B. In order to identify the region where the excited state contamination can be neglected, we consider a ratio

$$R_{\pi^+\pi^+}(t) = \frac{C_{\pi^+\pi^+}(t+1) - C_{\pi^+\pi^+}(t)}{C_{\pi^+\pi^+}(t) - C_{\pi^+\pi^+}(t-1)}, \tag{3.5}$$

in which the parameters A and B cancel out when the ground state dominates as in (3.4). We plot a variant of the effective mass for this ratio $E_{\rm eff}(t) \equiv \ln R_{\pi^+\pi^+}(t)$ in Fig. 2 for $ma = 0.050$. Since we are using the wall source, the plateau is reached rather slowly, but we can still observe a nice plateau in the region $ta \in$[10,15], which is chosen as the fit range. As the plot shows the energy difference between $E_{\pi\pi}$ and $2m_\pi$ is clearly extracted.

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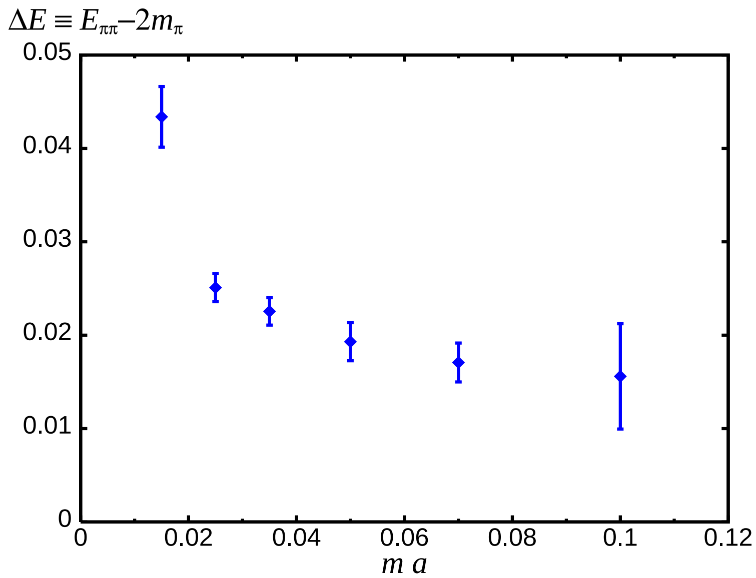

Figure 3: Energy shift due to the $\pi\pi$ interaction.

4. Finite size effect

On our lattice the spatial extent L is about 1.9 fm, which is not sufficiently large to neglect finite size effects (FSE). A complete analysis of FSE for pion mass and decay constant using the next-to-next-to-leading order (NNLO) ChPT [14, 15] has been done by the JLQCD collaboration [8], which we use in this work. Similarly, the modification of Lüscher's formula is calculated at next-to-leading order (NLO) ChPT in [16], that is also taken into account in the following analysis. In general, this type of FSE behaves as $e^{-m_\pi L}$, so that the effect is more significant for smaller quark masses. At the smallest quark mass in our calculation, the numerical size of FSE is approximately 2%, 6%, 10% for the pion mass, decay constant, and scattering length, respectively.

In addition to these standard FSE, there is an artifact due to fixing the global topological charge in our calculation. This type of FSE is of $O(1/V)$ in general, and can be estimated once the topological susceptibility and the θ dependence of the physical quantity of interest are known [12]. The topological susceptibility has been calculated on the JLQCD configurations recently [17]. The θ dependence is known through ChPT. At the leading order of ChPT, only the pion mass has the θ dependence. At this stage of the analysis, we therefore include this effect for the pion mass as done in [8] but neglect it for the scattering length. An NLO calculation is to be done to include the fixed topology artifact.

5. Result

Fig. 3 shows the interaction energy between two pions for each quark mass. The FSE corrections are included as described in the previous section. We clearly observe that the energy shift $\Delta E \equiv E_{\pi\pi} - 2m_\pi$ increases rapidly as quark mass approaches the chiral limit as expected from ChPT.

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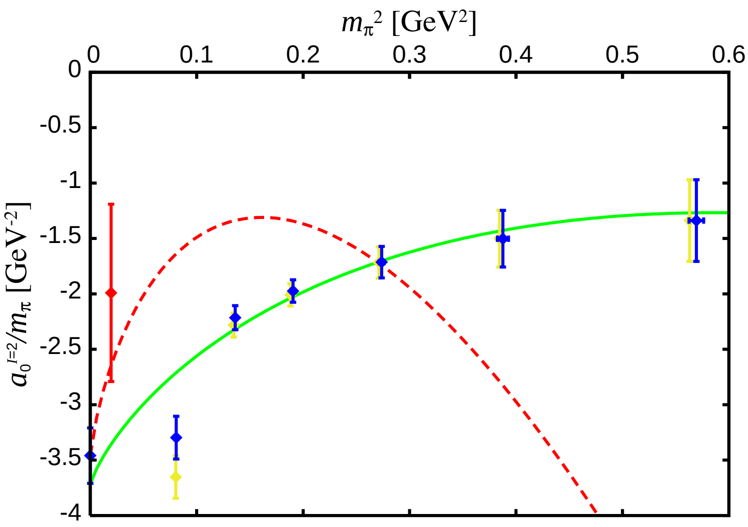

Figure 4: $I = 2$ $\pi\pi$ scattering length divided by m_π as a function of m_π^2. The yellow points are data before the FSE corrections, while the blue points are after the FSE corrections. The data point in the massless limit denotes $1/(8\pi F^2)$ with the decay constant F in the chiral limit calculated separately. The red point is the phenomenological value. Dashed and solid curves are the NLO ChPT fits with or without the constraint $F = F_0$.

By converting these values to the scattering lengths using Lüscher's formula (2.2) we obtain the result for the scattering length a_0 as shown in Fig. 4. The impact of FSE can be seen from the difference between yellow (without FSE) and blue (with FSE) symbols.

In the following, we attempt a fit of the data with the NLO ChPT formula

$$\frac{a_0^{I=2}}{m_\pi} = -\frac{1}{8\pi F^2}\left[1+\frac{m_\pi^2}{8\pi^2 F_0^2}\left(\frac{7}{2}\log\frac{m_\pi^2}{\mu^2}+l_{\pi\pi}(\mu)\right)\right] \tag{5.1}$$

where F and F_0 are the decay constants in the chiral limit (in the 132 MeV normalization). F and F_0 must be the same in this formula, but here we introduce separate parameters for the overall constant (F) and for the strength of the chiral logarithm term (F_0). (For a discussion, see below.) $l_{\pi\pi}(\mu)$ is a linear combination of the scale-dependent low energy constants in the chiral lagrangian at $O(p^4)$.

In the massless limit, a_0/m_π is solely given by F. In addition to the data of this calculation, we plot the data for $-1/(8\pi F^2)$ with F obtained from the standard analysis of the pion decay constant [8]. Then, all the data points are fitted including the massless limit (green line). From this fit leaving F and F_0 as independent free parameters, we obtain F = 103(5) MeV and F_0 = 199(35) MeV, which indicate that the data are not consistent with (5.1) if all the data points are included. Instead, if we put a constraint $F = F_0$, then we obtain the red curve, which clearly indicate that the chiral logarithm term in (5.1) is too strong.

From this analysis, it is likely that the one-loop ChPT formula can be applied only in the very small quark mass region and the two-loop effect is significant already at around $m \sim m_s/2$. Also, the remaining finite size effect could be important, since the effect at NLO is already significant. Analysis is on-going to understand these points.

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References

[1] M. Lüscher, Commun. Math. Phys. **105**, 153 (1986).

[2] M. Lüscher, Nucl. Phys. B **354**, 531 (1991).

[3] S. R. Sharpe, R. Gupta and G. W. Kilcup, Nucl. Phys. B **383**, 309 (1992).

[4] Y. Kuramashi, M. Fukugita, H. Mino, M. Okawa and A. Ukawa, Phys. Rev. Lett. **71**, 2387 (1993).

[5] S. R. Beane, T. C. Luu, K. Orginos, A. Parreno, M. J. Savage, A. Torok and A. Walker-Loud, `arXiv:0706.3026[hep-lat]`.

[6] T. Kaneko *et al.* [JLQCD Collaboration], `PoS(LAT2006)054` [`arXiv:hep-lat/0610036`].

[7] H. Matsufuru, a plenary talk at Lattice 2007, in these proceedings, `PoS(LATTICE 2007)018`.

[8] J. Noaki *et al.*, `PoS(LATTICE 2007)126` [`arXiv:0710.0929[hep-lat]`].

[9] H. Neuberger, Phys. Lett. B **417**, 141 (1998) [`arXiv:hep-lat/9707022`].

[10] H. Neuberger, Phys. Lett. B **427**, 353 (1998) [`arXiv:hep-lat/9801031`].

[11] H. Fukaya, S. Hashimoto, K. I. Ishikawa, T. Kaneko, H. Matsufuru, T. Onogi and N. Yamada [JLQCD Collaboration], Phys. Rev. D **74**, 094505 (2006) [`arXiv:hep-lat/0607020`].

[12] S. Aoki, H. Fukaya, S. Hashimoto and T. Onogi, Phys. Rev. D **76**, 054508 (2007) [`arXiv:0707.0396[hep-lat]`].

[13] T. A. DeGrand and S. Schaefer, Comput. Phys. Commun. **159**, 185 (2004) [`arXiv:hep-lat/0401011`].

[14] G. Colangelo, S. Durr and C. Haefeli, Nucl. Phys. B **721**, 136 (2005) [`arXiv:hep-lat/0503014`].

[15] G. Colangelo and C. Haefeli, Nucl. Phys. B **744**, 14 (2006) [`arXiv:hep-lat/0602017`].

[16] P. F. Bedaque, I. Sato and A. Walker-Loud, Phys. Rev. D **73**, 074501 (2006) [`arXiv:hep-lat/0601033`].

[17] S. Aoki *et al.* [JLQCD and TWQCD Collaborations], `arXiv:0710.1130[hep-lat]`.

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