PROCEEDINGS OF SCIENCE

# Baryon masses with dynamical twisted mass fermions

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We present results on the mass of the nucleon and the $\Delta$ using two dynamical degenerate twisted mass quarks. The evaluation is performed at four quark masses corresponding to a pion mass in the range of 690-300 MeV on lattices of size 2.1 fm and 2.7 fm. We check for cutoff effects by evaluating these baryon masses on lattices of spatial size 2.1 fm with lattice spacings $a(\beta = 3.9) = 0.0855(6)$ fm and $a(\beta = 4.05) = 0.0666(6)$ fm, determined from the pion sector and find them to be within our statistical errors. Lattice results are extrapolated to the physical limit using continuum chiral perturbation theory. The nucleon mass at the physical point provides a determination of the lattice spacing. Using heavy baryon chiral perturbation theory at $\mathscr{O}(p^3)$ we find $a(\beta = 3.9) = 0.0879(12)$ fm, with a systematic error due to the chiral extrapolation estimated to be about the same as the statistical error. This value of the lattice spacing is in good agreement with the value determined from the pion sector. We check for isospin breaking in the $\Delta$-system. We find that $\Delta^{++,-}$ and $\Delta^{+,0}$ are almost degenerate pointing to small flavor violating effects.

*The XXV International Symposium on Lattice Field Theory*
*July 30 - August 4 2007*
*Regensburg, Germany*

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PoS(LATTICE 2007)087

## 1. Introduction

Twisted mass fermions provide a promising formulation of lattice QCD that allows for automatic $\mathcal{O}(a)$ improvement, infrared regularization of small eigenvalues and fast dynamical simulations [1]. We use the tree-level Symanzik improved gauge action and work at maximal twist to realize $\mathcal{O}(a)$-improvement. Recent results obtained in the pion sector give an accurate evaluation of the low energy constants $\bar{l}_3$ and $\bar{l}_4$ [2, 3], which lead to the most accurate determination of the S-wave $\pi\pi$ scattering lengths [4]. In this work we study the light baryon sector.

The fermionic action for two degenerate flavors of quarks in twisted mass QCD is given by

$$S_F = a^4 \sum_x \bar{\psi}(x)\left(D_W[U] + m_0 + i\mu\gamma_5\tau^3\right)\psi(x) \tag{1.1}$$

with $D_W[U]$ the massless Dirac operator, $m_0$ the bare untwisted quark mass and $\mu$ the bare twisted mass. The twisted mass term in the fermion action of Eq. (1.1) breaks isospin symmetry since the u- and d-quarks differ by having opposite signs for the $\mu$-term. This isospin breaking is a cutoff effect of $\mathcal{O}(a^2)$. However the up- and down-propagators satisfy $G_u(x,y) = \gamma_5 G_d^\dagger(y,x)\gamma_5$, which means that two-point correlators are equal with their hermitian conjugate with u- and d-quarks interchanged. Since the masses are computed from real correlators this leads to the following pairs being degenerate: $\pi^+$ and $\pi^-$, proton and neutron and $\Delta^{++}(\Delta^+)$ and $\Delta^-(\Delta^0)$. A theoretical analysis [5] shows that potentially large $\mathcal{O}(a^2)$ effects that appear in the $\pi^0$-mass are suppressed in all other quantities. Calculation of the mass of $\pi^0$, which requires the evaluation of disconnected diagrams, has been carried out confirming large $\mathcal{O}(a^2)$-effects. In the baryon sector we can study isospin breaking by evaluating the mass difference between $\Delta^{++}(\Delta^-)$ and $\Delta^+(\Delta^0)$. Since no disconnected contributions enter we can obtain an accurate evaluation of isospin splitting and its dependence on the lattice spacing.

## 2. Lattice techniques

The parameters of the calculation are collected in Table 1. They span a pion mass range from 300-690 MeV. At a pion mass of about 300 MeV we have simulations for lattices of spatial size, $L_s = 2.1$ fm and $L_s = 2.7$ fm at $\beta = 3.9$ allowing to check finite size effects. We provide a first check of finite $a$-effects by comparing results at $\beta = 3.9$ and $\beta = 4.05$.

The masses of the nucleon and the $\Delta$'s are extracted from two-point correlators using the standard interpolating fields, which for the proton, the $\Delta^{++}$ and $\Delta^+$, are given by

$$\begin{aligned} J_p &= \epsilon_{abc}\left(u_a^T C\gamma_5 d_b\right)u_c, \qquad J_{\Delta^{++}} = \epsilon_{abc}\left(u_a^T C\gamma_\mu u_b\right)u_c \\ J_{\Delta^+} &= \frac{1}{\sqrt{3}}\epsilon_{abc}\left[2\left(u_a^T C\gamma_\mu d_b\right)u_c + \left(u_a^T C\gamma_\mu u_b\right)d_c\right] \quad . \end{aligned} \tag{2.1}$$

Local interpolating fields are not optimal for suppressing excited state contributions. We instead apply Gaussian smearing to each quark field, $q(\mathbf{x},t)$: $q^{\rm smear}(\mathbf{x},t) = \sum_{\mathbf{y}} F(\mathbf{x},\mathbf{y};U(t))q(\mathbf{y},t)$ using the gauge invariant smearing function

$$F(\mathbf{x},\mathbf{y};U(t)) = (1+\alpha H)^n(\mathbf{x},\mathbf{y};U(t)), \tag{2.2}$$

| $\beta = 3.9$, $a = 0.0855(6)$ fm from $f_\pi$ [3] | | | | | |
|---|---|---|---|---|---|
| $24^3 \times 48$, $L_s = 2.1$ fm | $\mu$ | 0.0040 | 0.0064 | 0.0085 | 0.010 |
| | $m_\pi$ (GeV) | 0.3131(16) | 0.3903(9) | 0.4470(12) | 0.4839(12) |
| $32^3 \times 64$, $L_s = 2.7$ fm | $\mu$ | 0.004 | | | |
| | $m_\pi$ (GeV) | 0.3082(55) | | | |
| $\beta = 4.05$, $a = 0.0666(6)$ fm from $f_\pi$ [3] | | | | | |
| $32^3 \times 64$, $L_s = 2.1$ fm | $\mu$ | 0.0030 | 0.0060 | 0.0080 | 0.010 |
| | $m_\pi$ (GeV) | 0.3070(18) | 0.4236(18) | 0.4884(15) | 0.6881(18) |

**Table 1:** The parameters of our calculation.

constructed from the hopping matrix, $H(\mathbf{x},\mathbf{y};U(t)) = \sum_{i=1}^{3}\left(U_i(\mathbf{x},t)\delta_{\mathbf{x},\mathbf{y}-i} + U_i^\dagger(\mathbf{x}-i,t)\delta_{\mathbf{x},\mathbf{y}+i}\right)$. The parameters $\alpha$ and $n$ are varied so that the root mean square (r.m.s) radius obtained using the proton interpolating field is in the range of 0.3-0.4 fm. In Fig. 1 we show lines of constant r.m.s radius as we vary $\alpha$ and $n$. The larger the $n$ the more time consuming is the smearing procedure. On the other hand, for $\alpha \gtrsim 1$, increasing further $\alpha$ does not reduce $n$ significantly. Therefore, we choose a value of $\alpha$ large enough so that the weak $n$-dependence sets in, and we adjust $n$ to obtain the required value of the r.m.s radius. We consider two sets for these parameters giving r.m.s radius 0.31 fm and 0.39 fm, as shown in Fig. 1. In Fig. 2, we show the nucleon effective mass, $m^N_{\rm eff} = -\log(C(t)/C(t-1))$ with $C(t)$ the nucleon correlator, for 10 configurations at $\beta = 3.9$ and $\mu = 0.0085$. For the optimization of the parameters we apply Gaussian smearing at the source, whereas for the sink we use local interpolating fields so that no additional inversions are needed when we change $\alpha$ and $n$. As can be seen, for both sets of smearing parameters, the excited state contributions are suppressed with the set $\alpha = 4$, $n = 50$ producing a plateau a couple of time slices earlier. If, in addition, we apply APE smearing to the spatial links that enter the hopping matrix, then gauge noise is reduced resulting in a better identification of the plateau. Therefore for all computations at $\beta = 3.9$ we use Gaussian smearing with $\alpha = 4$ and $n = 50$. We apply smearing at the source and compute the mass using both local (LS) and smeared sink (SS). For $\beta = 4.05$ we readjust the parameters so that the nucleon r.m.s radius is still about 0.39 fm, obtaining $\alpha = 4$ and $n = 70$. In all cases we apply APE smearing to the gauge links that are used in $F(\mathbf{x},\mathbf{y};U(t))$. We note that Gaussian smearing is very effective as compared to, for example, fuzzing on links joining quarks at different sites.

The nucleon effective masses obtained using correlators with smeared source and local or smeared sink for the four $\mu$-values at $\beta = 3.9$ are shown in Fig. 3, where we average over the proton and neutron correlators. In Fig. 4 we show, for the same $\mu$-values, the $\Delta$ effective masses after averaging the correlators obtained using smeared source and sink over the degenerate pairs $\Delta^{++}$, $\Delta^-$ and $\Delta^+$, $\Delta^0$ . As can be seen, the quality of the plateaus in the nucleon case is better than in case of the $\Delta$. This explains why results on the $\Delta$ mass have larger errors requiring more statistics for a reliable determination. The errors are evaluated using jackknife and the $\Gamma$-method [6] to check consistency. In all the figures we show the errors obtained with the latter method.

## 3. Results

We show our results for the nucleon mass as a function of $m_\pi^2$ in Fig. 5, where we use $a =$

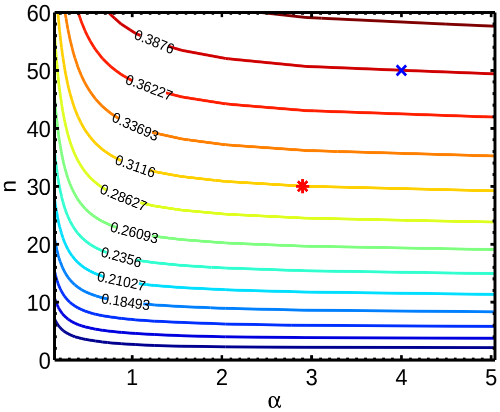



**Figure 1:** Lines of constant r.m.s radius as function of the smearing parameters $\alpha$ and $n$. The asterisk shows the values $\alpha = 2.9$, $n = 30$ and the cross $\alpha = 4.0$, $n = 50$.

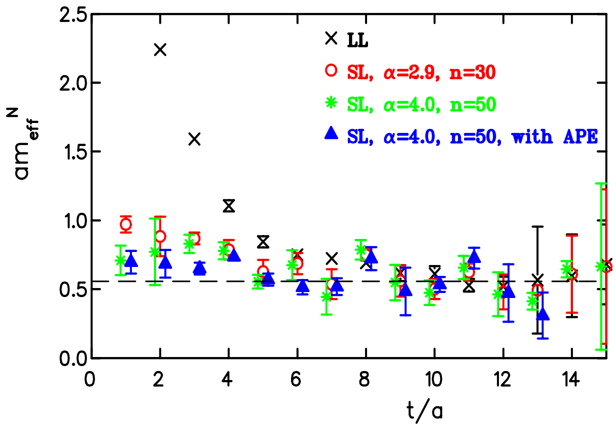



**Figure 2:** $m^N_{\rm eff}$ versus time separation both in lattice units. Crosses show results using local sink and source (LL), circles (asterisks) using Gaussian smearing at the sink (SL) with $\alpha = 2.9$ and $n = 30$ ( $\alpha = 4$ and $n = 50$), and filled triangles with $\alpha = 4$ and $n = 50$ and APE smearing. The dashed line is the plateau value when APE smearing is used.

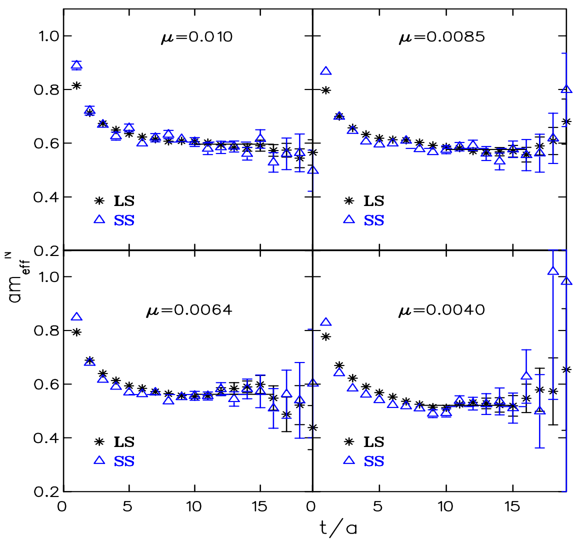



**Figure 3:** Nucleon effective mass (LS: asterisks, SS: open triangles) for $\beta = 3.9$ versus time separation in lattice units.

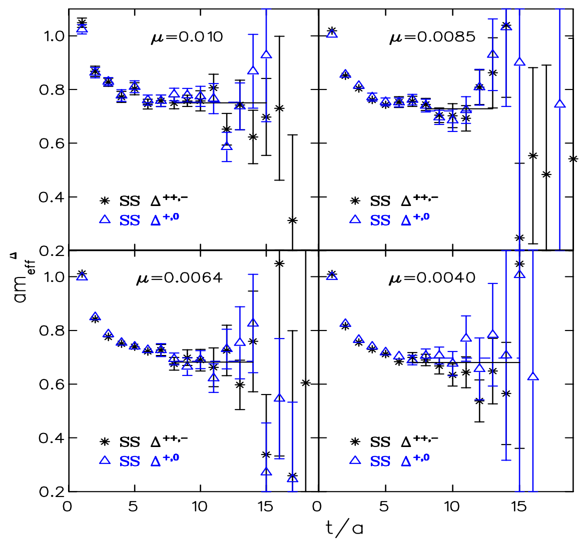



**Figure 4:** $\Delta^{++,-}$ (asterisks) and $\Delta^{+,0}$ (open triangles) effective masses for $\beta = 3.9$ versus time separation in lattice units.

0.0855 at $\beta = 3.9$ and $a = 0.0666$ at $\beta = 4.05$, determined from $f_\pi$ [2], to convert lattice results to physical units. As can be seen, the results at these two $\beta$-values show good scaling pointing to small cutoff effects. For the three larger pion masses $m_\pi L_s \geq 4$, whereas for the smallest value $m_\pi(\mu = 0.004)L_s \sim 3.2$. Applying the resummed Lüscher formula to the nucleon mass and using the knowledge of the $\pi N$ scattering amplitude to $\mathscr{O}(p^2)$ and $\mathscr{O}(p^4)$ it was shown that, for $L_s \sim 2$ fm and $m_\pi \sim 300$ MeV, the volume corrections are small being estimated to be about (3-5)% [7]. We calculate the nucleon mass increasing the spatial length of the lattice from 2.1 fm to 2.7 fm so that $m_\pi(\mu = 0.004)L_s \sim 4.3$. If $\Delta m_N \equiv m_N(L_s/a = 24) - m_N(L_s/a = 32)$ then we find that $\Delta m_N/m_N(L_s/a = 32) = 0.01 \pm 0.02$ at our smallest quark mass, i.e. consistent with zero within our statistical error but also within the estimated error range of Ref. [7]. In Fig. 5 we include, for comparison, results obtained with dynamical staggered fermions from Ref. [8]. As can be seen, the results using these two formulations are consistent with each other.

In Fig. 6 we show our results for the mass difference between the averaged mass of the pairs $\Delta^{++}$, $\Delta^{-}$ and $\Delta^{+}$, $\Delta^{0}$. As can be seen, the splitting is consistent with zero, indicating that isospin breaking in the $\Delta$ system is small.

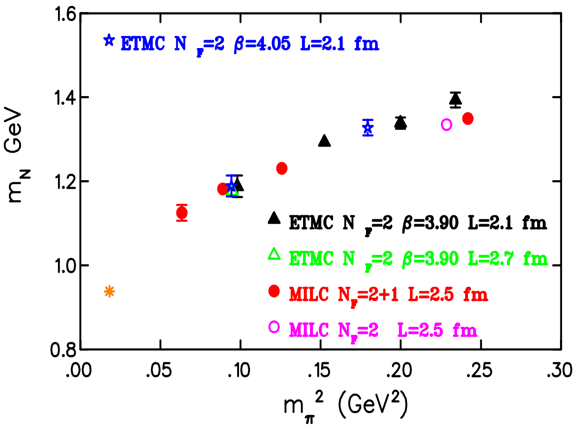



**Figure 5:** The nucleon mass as a function of $m_\pi^2$ for $\beta = 3.9$ on a lattice of size $24^3 \times 48$ (filled triangles) and on a lattice of size $32^3 \times 64$ (open triangles). Results at $\beta = 4.05$ are shown with the stars. The physical nucleon mass is shown with the asterisk. Results with dynamical staggered fermions for $N_F = 2+1$ (filled circles) and $N_F = 2$ (open circle) on a lattice of size $20^3 \times 64$ with $a = 0.125$ fm are from Ref. [8].

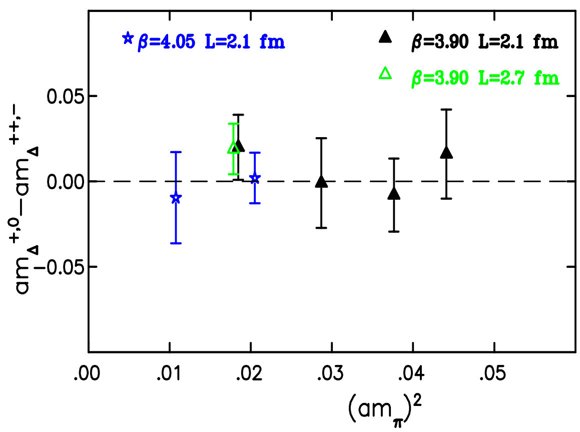



**Figure 6:** The mass splitting between $\Delta^{+,0}$ and $\Delta^{++,-}$ as a function of $m_\pi^2$ both in lattice units. The notation is the same as in Fig. 5.

Having checked that $\Delta m_N$ at the smallest pion mass is consistent with zero within our statistical errors and that cut-off effects are small, we use, in what follows, continuum chiral perturbation theory in an infinite volume to perform the chiral extrapolation to the physical point. The leading one-loop result in heavy baryon chiral perturbation theory (HB$\chi$PT) [9] is well known:

$$m_N = m_N^0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3 \tag{3.1}$$

with $m_N^0$, the nucleon mass at the chiral limit, and $c_1$ treated as fit parameters. We find that this $\mathscr{O}(p^3)$ result provides a very good fit to our lattice data at $\beta = 3.9$, yielding $m_N^0 = 0.875(10)$ GeV and $c_1 = -1.23(2)$ GeV$^{-1}$ with $\chi$/d.o.f. $= 0.2$. In this determination we use $a = 0.0855$ and results obtained on both lattice volumes. The value extracted for $c_1$ can be compared to the value $c_1 = -0.9 \pm 0.5$ GeV$^{-1}$ extracted from various partial wave analyses of elastic $\pi N$ scattering data for the $\pi N$-sigma term. We would like to stress that, despite the fact that the physical point is not included in the fit as customary done in other chiral extrapolations of lattice data, the nucleon mass that we find at the physical pion mass is 0.955(10) GeV. Given that the error is only statistical, the fact that this value is so close to the experimental value is very satisfactory. Chiral corrections to the nucleon mass are known to $\mathscr{O}(p^4)$ within several expansion schemes. In this work to $\mathscr{O}(p^4)$ we use the results obtained in HB$\chi$PT with dimensional regularization [10] and in the so called small scale expansion (SSE) [11]. HB$\chi$PT with dimensional regularization is in agreement with covariant baryon $\chi$PT with infrared regularization up to a recoil term that is of no numerical significance. In SSE the $\Delta$-degrees of freedom are explicitly included in covariant baryon $\chi$PT by introducing as an additional counting parameter the $\Delta$-nucleon mass splitting, $\Delta \equiv m_\Delta - m_N$, taking $\mathscr{O}(\Delta/m_N) \sim \mathscr{O}(m_\pi/m_N)$. A different counting scheme, known as $\delta$-scheme, takes $\Delta/m_N \sim \mathscr{O}(\delta)$ and $m_\pi/m_N \sim \mathscr{O}(\delta^2)$ [12]. Using the $\delta-$scheme in a covariant chiral expansion to order $p^3$, $p^4/\Delta$ one obtains an expansion that has a similar form for the nucleon and $\Delta$ mass. Here we use the variant of the $\delta$-scheme that includes the $\pi\Delta$-loop and adds the fourth order term $c_2 m_\pi^4$ as an estimate of higher order effects, since the complete fourth order result is not available. The parameter $c_2$ is to be determined from the lattice data. The fits using these different formulations are shown in Fig. 7. All formulations provide a good description of the lattice results and yield a nucleon mass at the physical point that is close to the experimental value. The physical nucleon mass is not including in the fits. We can use these chiral expansions to fix the lattice spacing using the nucleon mass at the physical point and compare with the value determined from the pion sector. The results of the fits in HB$\chi$PT to $\mathscr{O}(p^3)$ and $\mathscr{O}(p^4)$ are shown in Fig. 8. Using the leading one-loop result we find $a = 0.0879(12)$ fm, whereas to $\mathscr{O}(p^4)$ we obtain $a = 0.0883(9)$ fm. Both SSE and the $\delta-$scheme, which include explicitly $\Delta$-degrees of freedom, yield values that are consistent with those obtained in HB$\chi$PT. The variation in the value of $a$ in the different chiral extrapolation schemes gives an estimation of the systematic error involved in the chiral extrapolation. A proper determination of the systematic error is in progress.



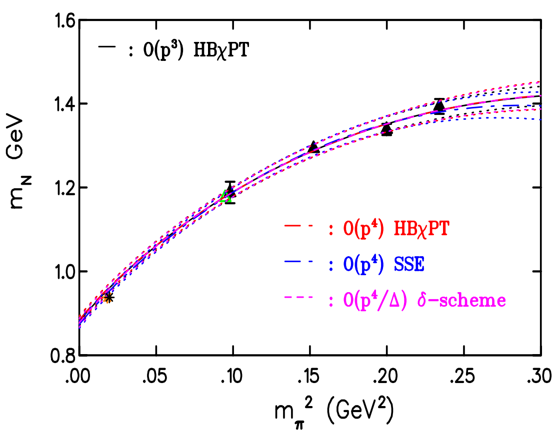



**Figure 7:** Chiral fits to the nucleon mass using $a = 0.0855$. The physical point shown by the asterisk is not included in the fits.

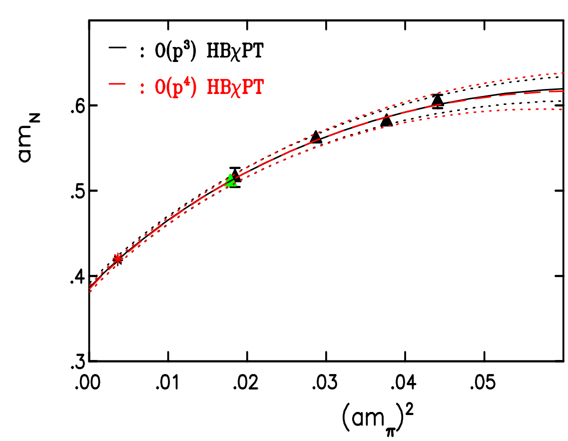
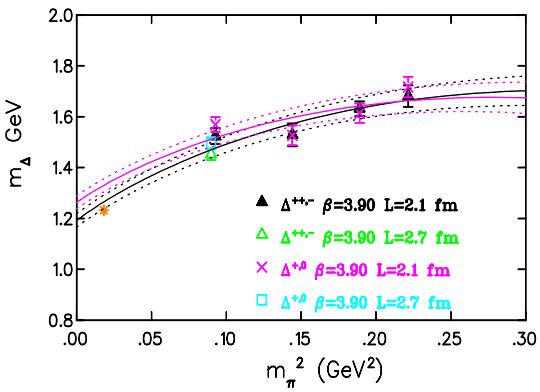



**Figure 8:** Chiral fits using HB$\chi$PT to $\mathscr{O}(p^3)$ and $\mathscr{O}(p^4)$ determining $a$ from the nucleon mass.

The leading one-loop HB$\chi$PT result in the case of the $\Delta$ mass has the same form as that for the nucleon mass given in Eq. (3.1) with $M_N^0 \to M_\Delta^0$ and $c_1 \to c_{1\Delta}$. Assuming SU(6) symmetry, the one-loop contribution has the same numerical value as in the nucleon case. It is useful to chirally extrapolate the $\Delta$ mass to see how close current results are to $\Delta(1232)$ taking $a = 0.0879$ as determined from the nucleon mass. In Fig. 9 we show the resulting fit.




Lattice data on the $\Delta^{++,-}$ mass, chirally extrapolated using the $\mathscr{O}(p^3)$-result in HB$\chi$PT, yield, at the physical point, $m_{\Delta^{++,-}} = 1.265(26)$ GeV, consistent with the resonant $\Delta$ mass. A similar chiral fit to the $\Delta^{+,0}$ mass yields a curve that lies above the physical point but with an overall statistical error band that overlaps the one obtained from the chiral fit to the $\Delta^{++,-}$ mass.

**Figure 9:** Chiral fits to the $\Delta^{++,-}$ and $\Delta^{+,0}$ mass using HB$\chi$PT to $\mathscr{O}(p^3)$ with $a$ set from the nucleon mass.

## 4. Conclusions

We have shown that twisted mass QCD yields accurate results on the nucleon mass close to the chiral regime. The quality of the results for pion masses in the range of 300-500 MeV allows a chiral extrapolation using heavy baryon chiral perturbation theory to $\mathscr{O}(p^3)$. The nucleon mass at the physical point provides a good physical quantity for setting the scale. Using the leading one-loop result in HB$\chi$PT we find $a(\beta = 3.9) = 0.0879(12)$ fm. Comparing this value to the results obtained using higher order terms in the chiral expansion, gives a first estimate of the systematic uncertainty due to the chiral extrapolation, that is of the same order of magnitude as the statistical error. Within this estimated uncertainty of the chiral extrapolation, the value we find for $a(\beta = 3.9)$ at leading order in HB$\chi$PT is consistent with the value determined from $f_\pi$. The mass splitting in the $\Delta$ isospin multiplets calculated with two lattice spacings on two volumes is consistent with zero showing that isospin breaking effects are not severe in this channel.

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