PROCEEDINGS OF SCIENCE

Glueball spectroscopy from $N_f = 2$ anisotropic lattice QCD

Andrew Nolan*
School of Mathematics,
Trinity College, Dublin 2, Ireland.
E-mail: nolana@maths.tcd.ie

Alan Ó Cais
Special Research Centre for the Subatomic Structure of Matter and Department of Physics,
University of Adelaide 5005, Australia.
E-mail: alan.ocais@adelaide.edu.au

Mike Peardon
School of Mathematics,
Trinity College, Dublin 2, Ireland.
E-mail: nolana@maths.tcd.ie

The spectrum of glueballs in $N_f = 2$ dynamical QCD is studied on anisotropic lattices and preliminary results are presented. The mixings of the glueballs with isoscalar quarkonium and multi-hadron states using all-to-all quark propagator methods are investigated.

The XXV International Symposium on Lattice Field Theory
July 30-4 August 2007
Regensburg, Germany

*Speaker.

PoS(LATTICE 2007)127

1. Introduction

Clear evidence for the existence of glueballs in QCD remains elusive. While the spectrum of the Yang-Mills theory has been established [1, 2] , the properties of these states in the presence of light, dynamical quark fields is not well understood. The difficulty for these calculations arises not only because the Monte Carlo sampling of full QCD is much more expensive than $SU(3)$ Yang-Mills theory but also because the glueballs can both mix with quarkonium states and decay into light mesons [3].

In relating the decay width of a state calculated on the lattice to QCD, many problems arise [4]. Primarily, LQCD calculations are carried out in a Euclidean framework to ensure that a probability measure can be well defined for importance sampling. However, the analytic continuation to Minkowski space is not trivial for unstable particles. The decay width of a resonance can be related to the spectrum of states with the appropriate quantum numbers in a finite volume. The theoretical framework for this has been known for some time, but direct attempts to compute widths from Monte Carlo data are few and far between.

To make a first attempt at studying the decay of the glueball, we are comparing the LQCD spectrum with the spectrum of an appropriate Euclidean field theory, so chosen that this model theory reproduces the LQCD data where necessary and can be subsequently mapped to Minkowski space. A model with a simple perturbative expansion is chosen to help theoretical development. One other goal of this investigation is to develop expertise in the statistical analysis of Monte Carlo data related to unstable states in Euclidean field theory simulations.

To proceed, we must first define our model field theory and establish a method for extracting the decay widths from this theory. We choose scalar $\phi \to 2\chi$ theory. In Ref. [5], Lüscher proposed a method for calculating scattering phase shifts from the volume dependence of the energy spectrum. This was applied in Ref. [6] to a simple lattice model of two fields with a three point coupling, in both the rest and non-rest frames. We intend to use a model theory similarly to investigate the numerical requirements to perform such a calculation for decays in full QCD.

2. Model Field Theory

Since we are interested in decays such as $G \to \pi\pi$, so we would like to have a model field theory which resembled this decay in some simple way. We use a scalar $\phi \to \chi\chi$ theory. Our Lagrangian for this theory is

$$\mathscr{L} = \int d^4x \;\; \phi(-\partial^2 + m_\phi^2)\phi + \chi(-\partial^2 + m_\chi^2)\chi + \lambda\phi\chi^2 + \mu\chi^4 \tag{2.1}$$

When $m_\phi > 2m_\chi$, the ϕ particle is unstable and can decay down to a $\chi\chi$ state. We include a χ^4 term to remove a vacuum instability in the χ field. The coefficient μ is governed by the values of the coupling, λ and m_ϕ. We make an association between the ϕ and the G, and the χ and the π.

Some Monte Carlo investigations of the theory at both strong and weak coupling are underway. For quite moderate values of the coupling constant, the $\phi \to \phi$ correlator varies dramatically in the region of the $m_\phi = 2m_\chi$ threshold. In Fig. (2), we see the difference in the effective mass plot of the $\phi \to \phi$ correlator either side of this threshold. Two measurements were made with $m_\phi = 0.1$. The χ masses for these measurements were $m_\chi^2 = 0$ (below threshold) and $m_\chi^2 = -0.000625$ (above

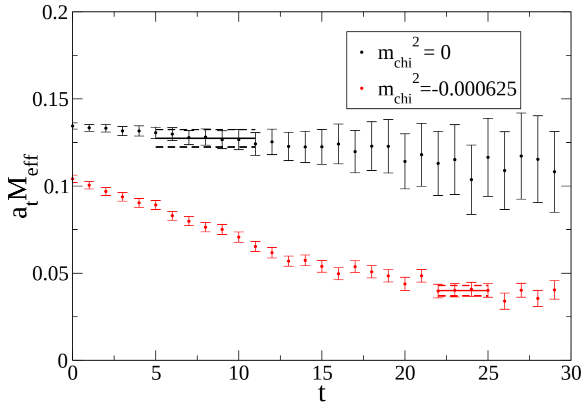


Figure 1: Effective mass plot for the correlator $\langle\phi|\phi\rangle$ for simulations above and below threshold.

threshold). Below the threshold, A stable ground state occurs almost instantly, whereas above the threshold, the effective mass includes many excited states before reaching a ground state plateau. This gives us confidence that quantities such as the $\phi \rightarrow \phi$ correlator in this theory are sensitive to the $m_\phi = 2m_\chi$ threshold.

It is our intention to use Monte Carlo simulations on this scalar $\phi \longrightarrow \chi\chi$ theory to provide us with an insight into the computational mechanism and precision needed to adequately investigate spectra in LQCD, such as the glueball spectrum. Specifically, we are interested in observing avoided level crossings in our theory, from which we can estimate the ϕ decay width. From there, we hope to have a platform to examine the glueball spectrum and decay in QCD to compliment existing data.

3. Glueball Data

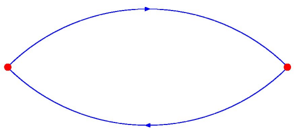
We present measurements of preliminary determinations of two correlation functions: $\langle G|G\rangle$ and $\langle G|\sigma\rangle$. Here, G denotes a state created by a large, smeared Wilson loop while σ is used to denote a meson created by a quark-anti-quark operator. The simulations were performed on $N_f = 2$ dynamical quark configurations, on anisotropic lattices with a two plaquette Symanzik improved gauge action and an improved quark action coupled to the gauge fields through stout-link smeared [8] links. Measurements were made on $8^3 \times 80$ and $8^3 \times 48$ anisotropic lattices with $N_f = 2$. The spacial lattices spacing was $a_s \approx 0.2\,fm$, with aspect ratio $\xi = 6$, giving a temporal spacing of $a_t^{-1} = 7.22(3)GeV$. The bare sea quark mass was $a_t m_q = -0.057$, corresponding to about half the strange quark mass [7].

3.1 Measuring QCD isoscalar correlation functions

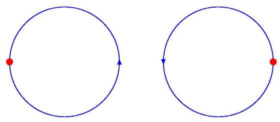
The glueballs will appear as a state in the isoscalar spectrum of mesons, and can mix with ordinary quark-anti-quark mesons as well as with multi-meson states via their decays. In order to make precise measurements of the spectrum with these quantum numbers, efficient all-to-all techniques are needed. Consider a meson in N_f-flavour QCD, created by the isoscalar interpolating operator $P = \sum_{i=1}^{N_f} \bar{\psi}_i \Gamma \psi_i$. The time dependence of its propagator is given by

$$C_P(t',t) = \sum_{x,y} \langle \sum_i \bar{\psi}_i(y,t')\Gamma^\dagger \psi_i(y,t') \sum_j \bar{\psi}_j(x,t)\Gamma\psi_j(x,t)\rangle.$$

Performing Wick contractions gives two types of term, connected and disconnected;



and



The connected contribution is identical to the term that would appear in an isospin 1 meson correlator, while for isospin 0, a disconnected contribution appears;

$$\begin{aligned} C_P(t',t) &= \sum_{x,y} \big(N_f \mathrm{Tr}\, M^{-1}(x,t;y,t')\Gamma^\dagger M^{-1}(y,t';x,t)\Gamma \\ &- N_f^2 \ \mathrm{Tr}\, M^{-1}(x,t;y,t')\Gamma^\dagger \ \mathrm{Tr}\, M^{-1}(y,t';x,t)\Gamma \big)\,. \end{aligned} \tag{3.1}$$

If this quantity is calculated using point-to-all progagators, only a small fraction of the data in each configuration is sampled, and the subsequent statistical noise dominates the signal. Calculation of these disconnected loops requires propagators from all points in space to all other points. The method described in Ref. [9] is used, where a single noise vector per quark line per gauge configuration is used, and the variance in the estimator is reduced via dilution. In some tests, 50 eigenvectors of the light quark matrix are computed and used to reduce noise.

4. Preliminary Results

Data from a range of different irreps of the lattice rotation group have been investigated to date in some preliminary tests. The lightest three glueballs in the Yang-Mills theory are the scalar 0^{++}, tensor 2^{++} and pseudoscalar 0^{-+} states. On the lattice, the relevant irreps of the lattice rotation group are the A_1 for spin 0 states and the E and T_2 irreps for spin 2. Glueball operators are made up of appropriate linear combinations of rotated Wilson loops measured on stout-smeared links. For the quark-anti-quark states, the simplest bilinear operators are measured on Jacobi-smeared quark fields. For the pseudoscalar and scalar, these operators are $\bar{\psi}\gamma_5\psi$ and $\bar{\psi}\psi$ respectively. In this preliminary study, 250 gauge field configurations have been analysed.

4.1 Scalar

The correlation function of the scalar glueball operator with a meson created by the scalar operator $\bar{\psi}\psi$ shows a clear mixing signal. This data is plotted in Fig. 2, along with the effective mass for the glueball correlator. In fits to this data, different energies are found, with the mass in the mixing channel substantially lighter. Understanding the mixing and the nature of the two distinct energies requires further investigation. Initial attempts to measure the correlation function of a scalar glueball with a two-pion state have not yielded a discernable signal. This measurement has only been made with very low levels of variance reduction in the all-to-all propagator determination. Further investigation with higher levels of variance reduction are on-going.

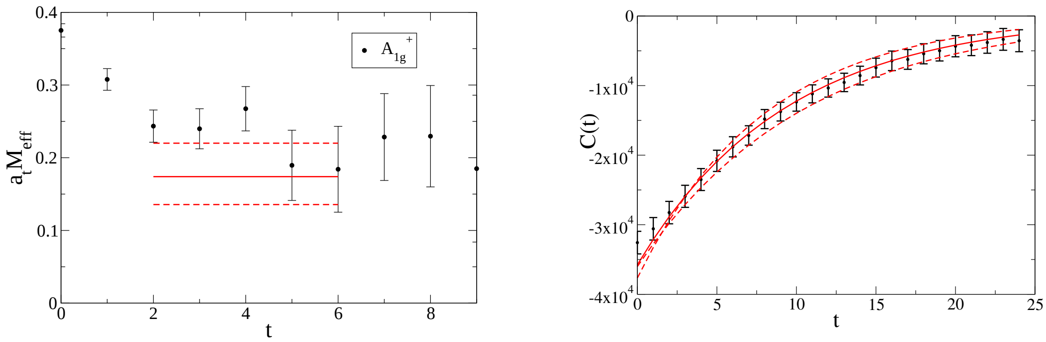


Figure 2: The effective mass and glueball-$\overline{\psi}\psi$ meson mixing correlation function in the A_{1g} (scalar) channel.

4.2 Pseudoscalar

A good signal in the correlation function of Wilson loops on the dynamical configurations is seen, allowing a single exponential fit to be performed. The mixing of the quark-anti-quark operator that creates the η' with the pseudoscalar glueball is small and no discernable signal for the matrix element $\langle G|\eta'\rangle$ has been seen in this study. This data is presented in Fig. 3

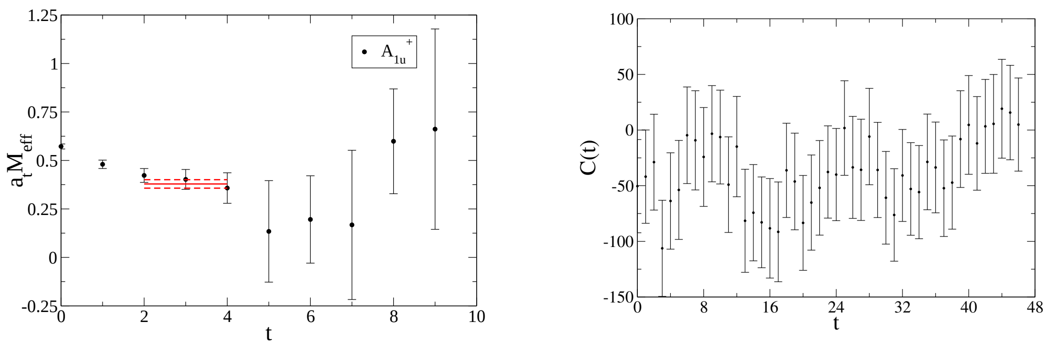


Figure 3: The effective mass and glueball-meson mixing correlation function in the A_{1u} (pseudoscalar) channel. The line in the effective mass plot indicates a fit to a single exponential.

4.3 Tensor

Good signals can be found in the Wilson loop correlators for both the E and T_2 lattice irreps. Fitting the correlation function to a single exponential gives degenerate states, at an energy consistent with the Yang-Mills mass. Our data are shown in Fig. 4

No attempt to investigate mixing between Wilson loop and quark-anti-quark operators with these quantum numbers has been made on our data set.

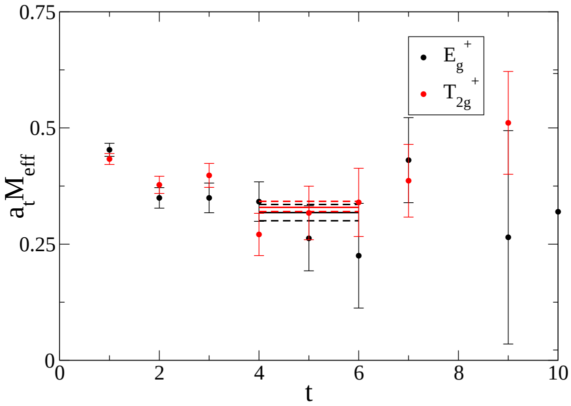


Figure 4: The effective masses for the E_g and T_{2g} (tensor) channels.

All fit results from this preliminary study are presented in the table below. Statistical precision of about 5% was achieved on this small ensemble. A larger set of gauge field configurations is being generated to allow a better statistical determination to be made.

Correlation function	$a_t M_{fit}$	$\chi^2_{/dof}$	$M_{fit}(GeV)$
$\langle G\|G\rangle$ (Scalar A_{1g}^+)	$0.174^{+0.046}_{-0.038}$	0.014	$1.26^{+0.33}_{-0.27}$
$\langle G\|G\rangle$ (Pseudoscalar A_{1u}^+)	$0.378^{+0.022}_{-0.021}$	0.3	$2.73^{+0.16}_{-0.15}$
$\langle G\|G\rangle$ (Tensor E_g^+)	0.317 ± 0.017	0.48	2.29 ± 0.12
$\langle G\|G\rangle$ (Tensor T_{2g}^+)	0.329 ± 0.009	0.65	2.38 ± 0.06
$\langle G\|\bar{\psi}\psi\rangle$ (Scalar)	$0.107^{+0.016}_{-0.013}$	0.2	$0.773^{+0.12}_{-0.09}$

5. Conclusions and Outlook

We are investigating a scalar $\phi \to \chi\chi$ theory as a model for $G \to \pi\pi$ decay in QCD. A marked variation is seen in the behaviour of the $\phi \to \phi$ correlator in this theory about the decay threshold, $m_\phi = 2m_\chi$. The model is providing valuable data for testing statistical analysis of Monte Carlo data of unstable states, and we hope it will shed some more light on the decay of scalar states in strongly coupled theories by allowing us to model QCD data reliably.

Preliminary investigations of the glueballs in QCD were made on anisotropic lattice with two flavours of dynamical quarks. We use all-to-all propagators to control the noise in the disconnected contributions of the correlation function of a glueball operator with a quark-anti-quark operator. In the $\langle G|\bar{\psi}\Gamma\psi\rangle$ correlators, a strong signal was seen in the 0^{++} channel, but not in the 0^{-+} case.

On this small ensemble of dynamical gauge fields, fits of the glueball correlation functions to single exponentials gave energy determinations to to about 5% accuracy. Our future work in this area will be to explore the volume dependence of the spectrum of our model $\phi \to \chi\chi$ theory and use this as a template for continued studies of the glueball spectrum.

This work was supported by SFI basic research grant 04/BRG/P0266, and by the IITAC project funded by the Irish Higher Education Authority under PRTLI Cycle 3 of the National Development Plan.

References

[1] G. Bali, *et. al* (UKQCD Collaboration), *A Comprehensive lattice study of SU(3) glueballs*, Phys. Lett. B **309** (1993) 378

[2] C. Morningstar and M. Peardon, *The Glueball spectrum from an anisotropic lattice study*, Phys. Rev. D **60** (1999) 034509

[3] C. McNeile, *these proceedings*.

[4] L. Maiani and M. Testa, *Final state interactions from euclidean correlation functions*. Phys. Lett. **B245**, 585 (1990)

[5] M. Lüscher, *Volume Dependence of the Energy Spectrum in Massive Quantum Field Theories, II. Scattering States*. Commun. Math. Phys. **105**, 153 (1986).

[6] K. Rummukainen and S. Gottlieb, *Resonance scattering phase shifts on a non-rest frame lattice*. Nucl.Phys. **B450**, 397(1995).

[7] R. Morrin, A. O'Cais, M. Peardon, S. Ryan and J-I. Skullerud, *Dynamical QCD simulations on anisotropic lattices*. Phys.Rev. **D74**, (2006).

[8] C. Morningstar and M. J. Peardon, *Analytic smearing of SU(3) link variables in lattice QCD*, Phys. Rev. D **69** (2004) 054501

[9] J. Foley, K. J. Juge, A. O'Cais, M. Peardon, S. Ryan, J-I. Skullerud *Practical all-to-all propagators for lattice QCD*. Comput.Phys.Commun. **172**, 145(2006).