

## $D_s$ spectrum from $N_f = 2$ anisotropic lattices

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We present a pilot study of the  $D_s$  spectrum from  $N_f = 2$  dynamical QCD simulations on  $12^3 \times 80$  lattices. Using all-to-all quark propagators, we study the ground and excited states of S and P waves.

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## 1. Introduction

In recent years there has been renewed interest in charm physics. A new state  $D_{sJ}$  has been found by BABAR and CLEO collaborations [1, 2] with a mass of around 2.32 GeV and its discovery has been an important topic both experimentally and theoretically.

In principle, lattice QCD should be able to determine the spectrum of the  $c\bar{s}$  system, but to do this in practice requires high precision numerical simulations.

In this paper, we study the  $D_s$  spectrum with dynamical anisotropic lattices. We use all-to-all propagators and construct a spatially extended interpolation operator basis for the excited states. We also compare our results with previous lattice studies [3, 4]

## 2. The dynamical anisotropic actions

The gauge action employed is a Two-Plaquette Symanzik improved action which has been designed to reduce the cut-off effects for the scalar glueball [5]. It is given by

$$S_G = \frac{\beta}{\xi_g^0} \left\{ \frac{5(1+w)}{3u_s^4} \Omega_s - \frac{5w}{3u_s^8} \Omega_s^{(2t)} - \frac{1}{12u_s^6} \Omega_s^R \right\} + \beta \xi_g^0 \left\{ \frac{4}{3u_s^2 u_t^2} \Omega_t - \frac{1}{12u_s^4 u_t^2} \Omega_t^{(R)} \right\} \quad (2.1)$$

where  $\Omega_{s,t}^{R,2t}$  refers to simple and rectangular plaquettes in the spatial and temporal directions and  $\xi_g^0$  is the bare anisotropy.

The  $N_f = 2$  anisotropic fermion action [6] is given by

$$S_q = \bar{\Psi} \left[ \gamma_0 \nabla_0 + \sum_i \mu_r \gamma_i \nabla_i \left( 1 - \frac{1}{\xi_q^0 a_s^2} \Delta_i \right) - \frac{ra_t}{2} \Delta_{i0} + sa_s^3 \sum_i \Delta_i^2 + m_0 \right] \Psi \quad (2.2)$$

where  $a_s$  and  $a_t$  ( $a_s \gg a_t$ ) are the spatial and temporal lattice spacings respectively and  $\xi_q^0$  is the bare anisotropy. The links are fattened using stout links [7] that maximise the plaquette. The same anisotropic fermion action is used for both light sea quarks and heavy valance quarks.

In a dynamical simulation, the bare anisotropies  $\xi_q^0$  and  $\xi_g^0$ , which are input parameters, must be tuned simultaneously such that one reaches the target anisotropy ( $\xi_r \sim 6$ ). This procedure is explained in details in Refs. [6, 8, 9].

## 3. Lattice Parameters

We performed our simulation at  $N_f = 2$  on  $N_s \times N_t = 12^3 \times 80$  lattices using 250 configurations with a sea quark mass close to that of the strange quark mass. The simulation parameters are summarized in Table 1.

We set our target anisotropy to be 6 and tune the anisotropies in the charm and strange sectors separately in order to obtain the same physical anisotropy. The charm quark mass used in our simulation,  $a_t m_c = 0.117$ , was tuned so as to obtain the correct  $J/\Psi$  mass. Our lattice spacing is determined from the spin averaged (1P-1S) splitting in charmonium [10, 11] and found to be  $a_t \sim 0.028$  fm. We use all-to-all propagators with the ‘‘dilution’’ hybrid method of Ref. [12] using no eigenvectors in the charm sector and  $N_\lambda = 20$  eigenvectors in the light sector with two independent

Configurations	250 ( $a_t m_c = 0.117$ , $a_t m_{sea} = a_t m_{light} = -0.057$ )
Dilution	Time+space even/odd
Physics	S and P waves ground and excited states
Volume	$12^3 \times 80$
$N_f$	2
$a_s$	$\sim 0.17$ fm
$a_t^{-1}$	$\sim 7.2$ GeV
$m_\pi/m_r ho$	$\sim 0.55$
$\xi_r$	$\sim 6$

**Table 1:** Simulation parameters used for this study of  $D_s$  spectrum.

noise vectors in each case. In this first study, we limit the dilution to time and space (even/odd). An important advantage of all-to-all propagators is that it allows us to easily implement a spatially extended operator basis so that we can search for better overlap with the states of interest than when restricted to point-like operators. Our list of operators used in this simulation is listed in Table 2.

$0^-$	$\gamma_5, \gamma_5(s_1 + s_2 + s_3), \gamma_5 \gamma_4$
$1^-$	$\vec{\gamma}, \gamma_j(s_1 + s_2 + s_3), \gamma_i \gamma_4$
$0^+$	$1, \gamma_4, \vec{\gamma} \cdot \vec{p}$
$1^+$	$\gamma_5 \gamma_i, \gamma_i \gamma_j, \vec{\gamma} \times \vec{p}, \gamma_5 p_i$
$2^+$	$\gamma_k p_i + \gamma_i p_k, \gamma_1 p_1 - \gamma_2 p_2, 2\gamma_3 p_3 - \gamma_1 p_1 - \gamma_2 p_2$

**Table 2:** The basis of operators. The notation for the gluonic paths  $p_i$  and  $s_i$ ,  $i=1,2,3$ , are defined in Ref. [13].

#### 4. Analysis

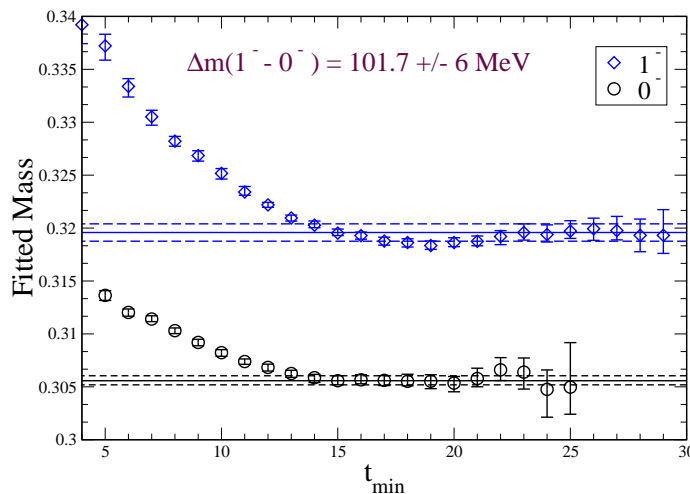
Due to their stochastic nature, time diluted all-to-all propagators introduce random noise at each time slice. As a result, effective masses calculated from correlators derived in this manner fluctuate more than what is observed when using point-to-all propagators. Effective mass plots are no longer an accurate visual aid when searching for good plateau regions if this method is employed. These introduced fluctuations do not, however, affect the quality of the exponential (cosh) fits that are performed on the original correlators.

A more accurate visual picture (to replace effective mass plots) can be obtained by introducing the “ $t_{min}$ ” or sliding window plots. For a fixed value of  $t_{max}$  (or alternatively  $t_{min}$ ) the fitting window,  $(t_{min}, t_{max})$ , is varied by changing  $t_{min}$  (or  $t_{max}$ ). This is illustrated in Fig. 1 for the  $0^-$  and  $1^-$  ground states. Our selection of the fits is based on stable regions of the sliding window plots together with a good  $\chi^2/n.d.f$  value. We observed these stable fits for all the ground and excited states we show here. Based on this analysis, we calculate the mass differences  $\Delta m(1^- - 0^-) = 102 \pm 6$  and  $\Delta m(1^+ - 0^+) = 118 \pm 26$  MeV in contrast to experimental values of 144 and 142 MeV, respectively. The possible reasons for the discrepancies are discussed in the conclusions.

In order to extract the excited states, we adopt the use of the variational analysis method [14, 15]. Using the operator basis given in Table 2 combined with two different smearings for each quark field, we can obtain the correlation matrix,

$$C_{\alpha\beta} = \langle 0 | O_\alpha O_\beta^\dagger | 0 \rangle \quad (4.1)$$

where  $\alpha, \beta = 1, \dots, n$  represent the different interpolating operators constructed. We performed single-state fits to the diagonal elements of this correlation matrix. Our results for the ground states obtained from the variational analysis and the cosh fits agrees within three percent. Our preliminary  $D_s$  spectrum for the S and P waves are shown in Figure 2



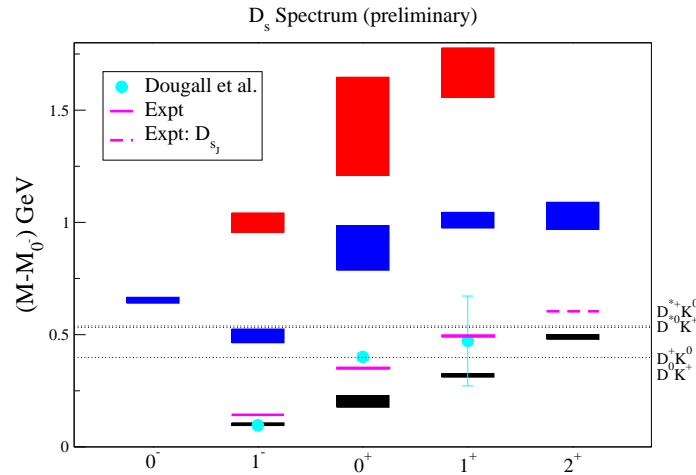
**Figure 1:** Sliding window plots for the ground states of  $0^-$  and  $1^-$  at zero momentum. The solid lines show the best fitted masses chosen.

## 5. Conclusions

We have presented our preliminary results for the  $D_s$  spectrum on  $N_f = 2$  dynamical  $12^3 \times 80$  anisotropic lattices for the S and P waves. We used all-to-all propagators and a variational basis of operators in order to extract the excited states. Our initial simulation was performed at a low level of dilution (time and space even-odd). Increasing the dilution level to obtain better data is currently underway. In addition, we plan on studying the D waves and hybrids and to repeat the simulation at finer lattices.

## 6. Acknowledgements

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**Figure 2:** The  $c\bar{s}$  spectrum which is normalised to  $0^-$  state. The scale is determined from the charmonium ( $1P - 1\bar{S}$ ) splitting. Dashed lines represent the experimental thresholds and blue circles represent the UKQCD [3] results.

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