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Magnetic Moment of Vector Mesons in the Background Field Method

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We report some results for the magnetic moments of vector mesons extracted from mass shifts in the presence of static external magnetic fields. The calculations are done on 24^4 quenched lattices using standard Wilson actions, with β=6.0 and pion mass down to 500 MeV. The results are compared to those from the form factor method.

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1. Introduction

Magnetic moment is a fundamental property of particles. It determines the dynamical response of a bound system to a soft external stimulus, and provides valuable insight into internal strong interaction structure. Efforts to compute the magnetic moment on the lattice come in two categories. One is the form factor method which involves three-point functions [1, 2, 3, 4, 5, 6, 7]. The other is the background field method using only two-point functions (mass shifts) [8, 9, 10, 11]. The form factor method requires an extrapolation to zero momentum transfer $G_M(Q^2 = 0)$ due to the non-vanishing minimum discrete momentum on the lattice [12]. The background field method, on the other hand, accesses the magnetic moment directly and cleanly but is limited to static properties due to the use of a static field. Here we report a calculation of the vector meson magnetic moments in this method, in parallel to a recent calculation in the form factor method [13]. It is an extension of our earlier work on baryon magnetic moments [14] and electric [15] and magnetic polarizabilities [16] in the same method.

2. Method

For a particle of spin s in uniform fields,

$$E_\pm = m \pm \mu B \tag{2.1}$$

where the upper sign means spin up and the lower sign means spin-down relative to the magnetic field, and $\mu = g\frac{e}{2m}s$. We use the following method to extract the g factors,

$$g = m\frac{(E_+ - m) - (E_- - m)}{eBs}. \tag{2.2}$$

In order to place a magnetic field on the lattice, we construct an analogy to the continuum case. The fermion action is modified by the minimal coupling prescription

$$D_\mu = \partial_\mu + gG_\mu + qA_\mu \tag{2.3}$$

where q is the charge of the fermion field and A_μ is the vector potential describing the background field. On the lattice, the prescription amounts to multiplying a U(1) phase factor to the gauge links. Choosing $A_y = Bx$, a constant magnetic field B can be introduced in the z-direction. Then the phase factor is in the y-links

$$U_y \to \exp(iqa^2Bx)U_y. \tag{2.4}$$

The computational demand can be divided into three categories. The first is a *fully-dynamical* calculation. For each value of external field, a new dynamical ensemble is needed that couples to u-quark (q=1/3), d-and s-quark (q=-2/3). This requires a Monte Carlo algorithm that can treat the three flavors separately. Quark propagators are then computed on the ensembles with matching values. This has not been attempted. The second can be termed *re-weighting* in which a perturbative expansion of action in terms of external field is performed (see Ref. [17] for a calculation of the neutron electric polarizability in this method). The third is *U(1) quenched*. No field is applied in the Monte-Carlo generation of the gauge fields, only in the valence quark propagation in the given gauge background. In this case, any gauge ensemble can be used to compute valence quark propagators.

We use standard Wilson actions on 24^4 lattice at $\beta = 6.0$, both SU(3) and U(1) quenched, and six kappa values κ=0.1515, 0.1525, 0.1535, 0.1540, 0.1545, 0.1555, corresponding to pion mass of 1015, 908, 794, 732, 667, 522 MeV. The critical value of kappa is κ_c=0.1571. The strange quark mass is set at κ=0.1535. The source location for the quark propagators is (x,y,z,t)=(12,1,1,2). We analyzed 87 configurations. The following five dimensionless numbers $\eta = qBa^2$=+0.00036, -0.00072, +0.00144, -0.00288, +0.00576 give four small B fields (two positive, two negative) at eBa^2=-0.00108, +0.00216, -0.00432, +0.00864 for both u and d (or s) quarks. These field values do not obey the quantization condition for periodicity since the values given by the condition cause too large a distortion to the system. To minimize the boundary effects, we work with fixed (or Dirichlet) b.c. in the x-direction and large N_x, so that quarks originating in the middle of the lattice have little chance of propagating to the edge. To eliminate the contamination from the even-power terms, we calculate mass shifts both in the field B and its reverse $-B$ for each value of B, then take the difference and divide by 2. Another benefit of repeating the calculation with the field reversed is that by taking the average of $\delta m(B)$ and $\delta m(-B)$ in the same dataset, one can eliminate the odd-powered terms in the mass shift. The coefficient of the leading quadratic term is directly related to the magnetic polarizability [16].

3. Interpolating field

For the ρ^+ meson, we use the polarized forms

$$\eta_\pm = \frac{1}{\sqrt{2}}\bar{d}\,(\mp\gamma_x - i\gamma_y)\,u = \frac{1}{\sqrt{2}}\,(\eta_x \pm i\eta_y) \tag{3.1}$$

The interaction energies $E_\pm$ are extracted from the correlation functions

$$\langle \eta_\pm \eta_\pm^\dagger \rangle = \frac{1}{2}\left[\langle \eta_x \eta_x^\dagger \rangle \pm i\left(\langle \eta_y \eta_y^\dagger \rangle - \langle \eta_x \eta_y^\dagger \rangle\right) + \langle \eta_y \eta_y^\dagger \rangle\right]. \tag{3.2}$$

Eq. (3.2) implies that the polarization comes from the imaginary parts of the off-diagonal correlation between x and y components. These imaginary parts are zero in the absence of the external field. Other vector mesons have similar forms with different quark content $\rho^- = \bar{u}d$, $\rho^0 = \bar{u}u - \bar{d}d$, $\phi = \bar{s}s$, $K^{*+} = \bar{s}u$, $K^{*-} = \bar{u}s$, $K^{*0} = \bar{d}s - \bar{s}d$. By symmetry, the magnetic moments are expected to be related by

$$\mu_{\rho^-} = -\mu_{\rho^+}, \;\; \mu_{\rho^0} = 0; \tag{3.3}$$

and

$$\mu_{K^{*-}} = -\mu_{K^{*+}}, \;\; \mu_{K^{*0}} \;\; \text{small}. \tag{3.4}$$

These relations are borne out numerically in our calculations.

4. Results and discussion

Fig. 1 displays a typical effective mass plot for both the mass and the mass shifts. Good plateaus exist for all six quark masses. Our results are extracted from the time window 10 to 13.

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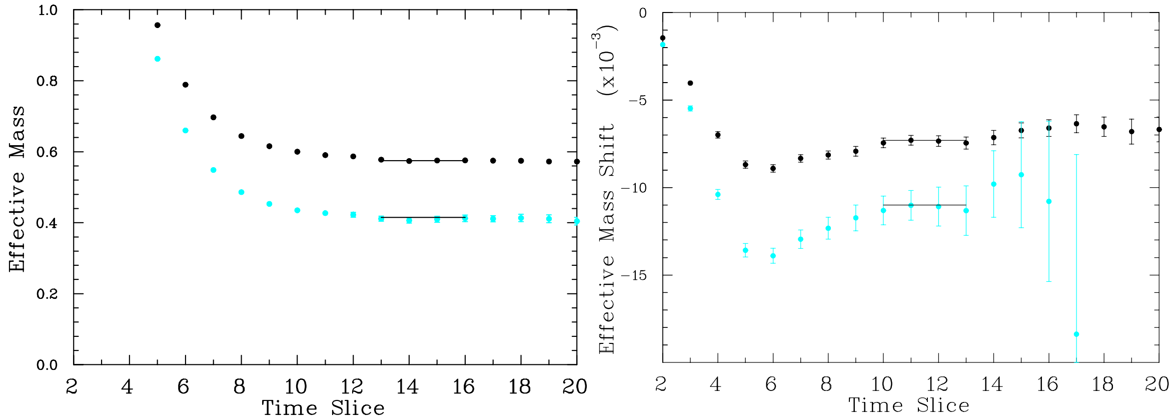

Figure 1: Effective mass plot for the ρ^+ mass (left) at zero field and mass shifts (right) at the weakest magnetic field in lattice units, corresponding to the heaviest and lightest quark masses.

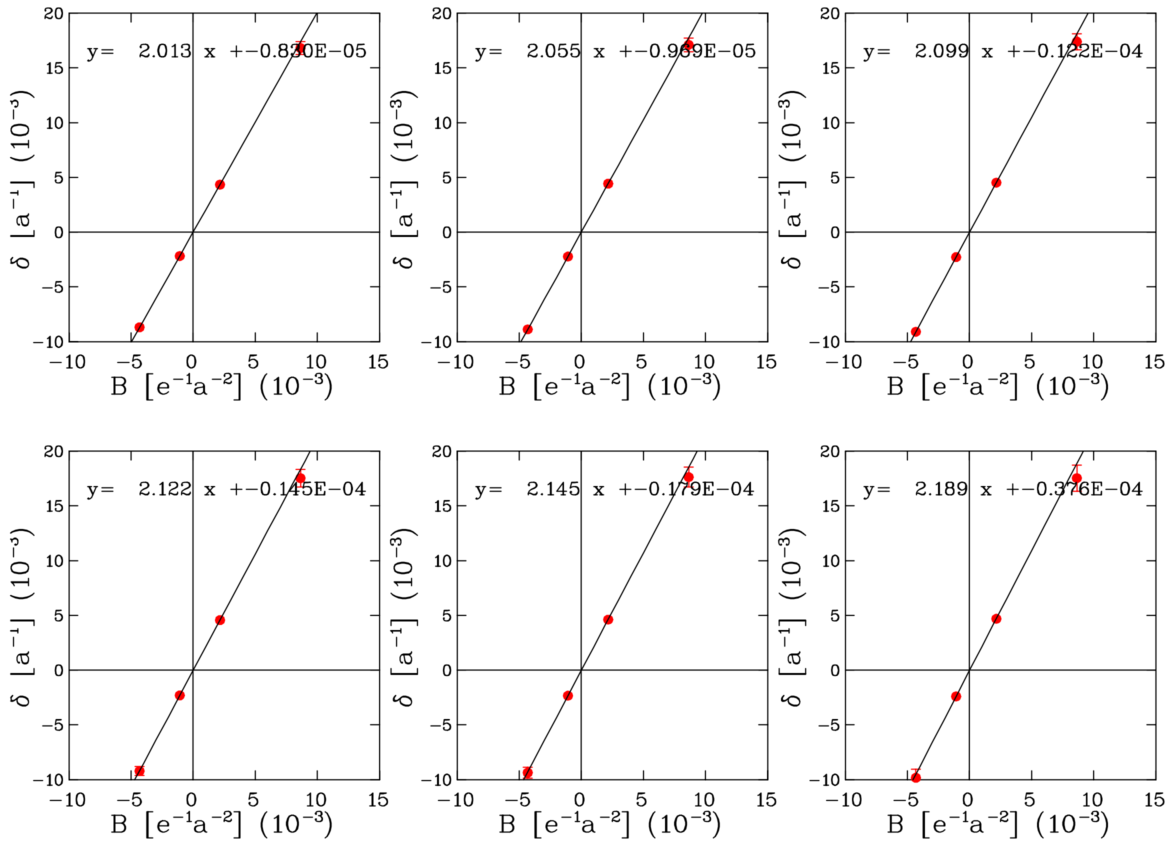

Figure 2: Mass shifts for the ρ^+ meson as a function of the magnetic field in lattice units at the six quark masses (lightest in the lower right corner). The slope of the mass shift at each quark mass gives the g factor corresponding to that quark mass. The line is a fit using only the two smallest B values.

Fig. 2 shows the mass shifts, defined as $\delta = g(eBs)$ from Eq. (2.2), as a function of the field for the ρ^+ meson. The slope gives the g-factor. There is good linear behavior going through the origin for all the fields when the quark mass is heavy, an indication that contamination from the higher-power terms has been effectively eliminated by the $(\delta(B) - \delta(-B))/2$ procedure. This is also confirmed numerically by the smallness of intercept as shown in the same figure. At the lightest quark mass, there is a slight deviation from linear behavior at the stronger fields. For this reason, we only use the two smallest field values to do the linear fit at all the quark masses.

Fig. 3 shows the g-factors for the vector mesons as a function of pion mass squared. The lines are simple chiral fits using the ansatzs

$$g = a_0 + a_1 m_\pi, \tag{4.1}$$

and

$$g = a_0 + a_1 m_\pi + a_2 m_\pi^2. \tag{4.2}$$

They serve to show that there is onset of non-analytic behavior as pion mass is lowered, so a linear extrapolation is probably not a good idea. But overall the g-factors have a fairly weak quark mass dependence. At large quark masses, the g-factor of ρ^+ approaches 2, consistent with a previous lattice calculation using the charge-overlap method [18]. Our results for ρ^+ are slightly higher than those from the form factor method (see Fig.8 in [13]). The results confirmed that $g_{\rho^-} = -g_{\rho^+}$ and $g_{K^{*-}} = -g_{K^{*+}}$. We confirmed $g_{\rho^0} = 0$ numerically (not shown). The results also show that as far as g-factors are concerned the ρ mesons are quite similar to their strange counterparts K^* mesons.

Note that the extracted g-factors are in the particle's natural magnetons. To convert them into magnetic moments in terms of the commonly-used nuclear magnetons (μ_N), we need to scale the results by the factor $938/M$ where M is the mass of the particle measured in the same calculation at each quark mass. Fig. 4 shows the results for ρ^+ and K^{*+}. The different quark-mass dependence between ρ^+ and K^{*+} mostly comes from that in their masses that are used to convert the g-factors to magnetic moments. The values at the chiral limit extrapolated from Eq. (2.2) are $\mu_{\rho^+} = 3.25(3)\mu_N$ and $\mu_{K^{*+}} = 2.81(1)\mu_N$. There is no experimental information. Compared to the form factor method (see Fig.7 in [13]), our results are again a little higher. At the strange quark mass point (the 3rd data point from the left), the two coincide to give a prediction for the magnetic moment of the $\phi(1020)$ meson, $\mu_\phi = 2.07(7)\mu_N$.

Fig. 5 shows the results for K^{*0}. Our results confirm the expectation that $\mu_{K^{*0}}$ is small. It is positive when the d-quark is heavier than the s-quark, exactly zero when they are equal, and turns negative when the d-quark is lighter than the s-quark. The same behavior has been observed in the form factor method (see Fig.11 in [13]).

5. Conclusion

In conclusion, we have computed the magnetic moment of vector mesons on the lattice using the background field method and standard lattice technology. Our results are consistent with those from the form factor method. There is no experimental information so the lattice results can serve as first-principles predictions. The calculation can be improved by providing a full account of systematic errors present in the results, such as finite-volume effects. In addition, there is a need to

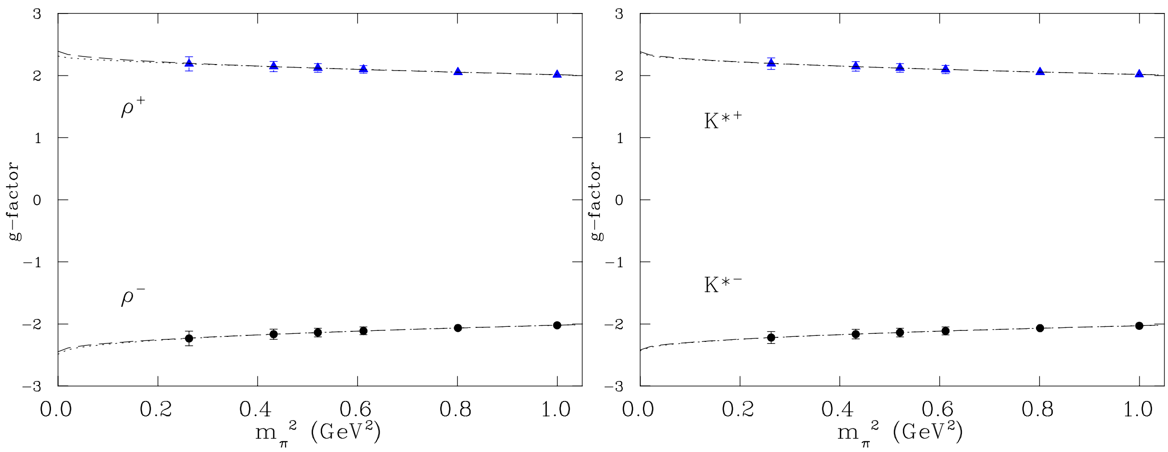

Figure 3: G-factors for the $\rho^{\pm}$ (left) and K^* (right) mesons as a function of pion mass squared. The 2 lines are chiral fits according to Eq. (4.1) (dashed), Eq. (4.2) (dotted).

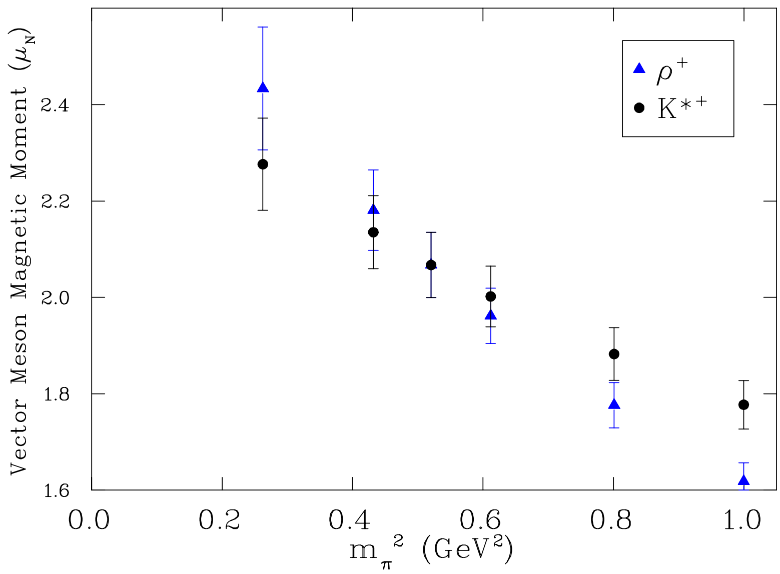

Figure 4: Magnetic moments (in nuclear magnetons) for ρ^+ and K^{*+}.

push the calculations to smaller pion masses so that reliable chiral extrapolations can be applied. Nonetheless, our results demonstrate that the method is robust and relatively cheap. Only mass shifts are required. This may facilitate the push to smaller pion masses, perhaps with the help of chiral fermions (overlap, domain-wall, twisted mass, ...). Finally, we await fully-dynamical background-field calculations in order to see the effects of the quenched approximation.

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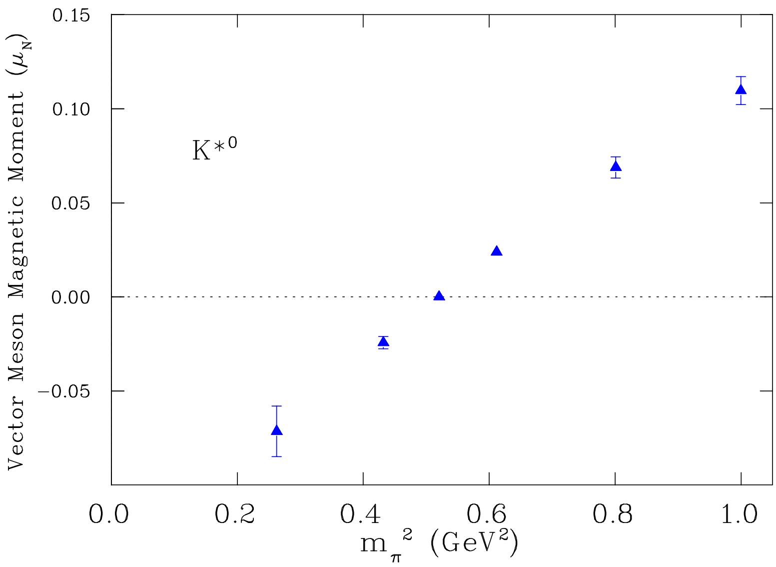

Figure 5: Magnetic moments (in nuclear magnetons) for K^{*0}.

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