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Topological defects and equation of state of gluon plasma

M. N. Chernodub*

ITEP, B. Cheremushkinskaya 25, Moscow, 117218, Russia

Katsuya Ishiguro

Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan
RIKEN, Radiation Laboratory, Wako 351-0158, Japan

Atsushi Nakamura

RIISE, Hiroshima University, Higashi-Hiroshima, 739-8527, Japan

Toru Sekido

Institute for Theoretical Physics, Kanazawa University, Kanazawa, 920-1192, Japan
RIKEN, Radiation Laboratory, Wako 351-0158, Japan

Tsuneo Suzuki

Institute for Theoretical Physics, Kanazawa University, Kanazawa, 920-1192, Japan
RIKEN, Radiation Laboratory, Wako 351-0158, Japan

V. I. Zakharov

INFN, Sezione di Pisa, Largo Pontecorvo 3, 56127, Pisa, Italy
ITEP, B. Cheremushkinskaya 25, Moscow, 117218, Russia

We show that the degrees of freedom associated with magnetic monopole– and vortexlike gluonic configurations make a strong contribution to the anomaly of the energy–momentum tensor of Yang–Mills theory in the deconfinement phase immediately above the critical temperature. As is well known in zero–temperature Yang–Mills theory, the monopoles and vortices are constituents of a generic gluonic object in which the two neighbor monopoles are connected together by a segment of vortex string. Our results provide evidence that the monopole-vortex chains in SU(2) gauge theory and their SU(3) counterparts, the monopole-vortex nets, are thermodynamically relevant degrees of freedom in the gluonic plasma.

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1. Introduction

The properties of thermal quark–gluon plasma in QCD have attracted great interest in recent years [1, 2]. The plasma can be studied by both heavy–ion collision experiments and numerical lattice simulations. The conventional theoretical approach to thermal plasma is to treat it, in a zero approximation, as a gas of free gluons and quarks supplemented with perturbative corrections. On the theoretical side, the bulk characteristics of the plasma such as pressure and energy density can then be represented in terms of perturbative series in the effective coupling constant $g^2(T)$. The perturbative predictions for bulk quantities turn to be in reasonable agreement with the available lattice data [2] for sufficiently high temperatures.

On the other hand, some particular properties of the plasma such as viscosity [3] indicate that in the zero approximation the plasma at temperatures slightly above the critical temperature T_c can be considered as an ideal liquid rather than an ideal gas. There is not yet a coherent picture that unifies both perturbative and nonperturbative features of the QCD plasma.

It was speculated in Refs. [4, 5] that there is a magnetic component of Yang-Mills plasma that is crucial for determining the plasma properties. In Ref. [5] the constituents of the magnetic component are thought to be classical magnetic monopoles. In Ref. [4] the magnetic component is identified with so-called magnetic strings which join (nonclassical) monopoles constituting chainlike structures. The Abelian monopoles and the center vortices are constituents of a generic gluonic object in which the two neighbor monopoles are connected together by a segment of the vortex [6, 7]. In $SU(2)$ gauge theory this object is considered as a monopole-vortex chain, while in the $SU(3)$ case the objects form the monopole-vortex 3-nets. The formation of the chains and nets is essential for the self–consistent treatment of the monopoles in the quark-gluon plasma [4].

Both the magnetic (center) strings and the (Abelian) monopoles as well as their role in the color confinement have been discussed in the lattice community for more than a decade [7, 8]. The properties of these defects change markedly once the temperature is increased above the critical value T_c. In particular, these defects become predominantly time–oriented in accordance with the assumption that they become a light component of the thermal gluon plasma [4].

Once the magnetic component of the plasma is identified with the topological defects, further information on its properties can be obtained by direct numerical calculations on the lattice at a finite temperature. Below we report the results of the first lattice measurements of the contribution of the magnetic strings and magnetic monopoles to the equation of state of the thermal Yang-Mills plasma.

2. Equation of state and trace anomaly

The free energy F of the gauge system is expressed via a partition function $\mathcal{Z}$ as follows:

$$F = -T \log \mathcal{Z}(T,V)\,, \qquad \mathcal{Z} = \int \mathrm{D}A \exp\left\{-\frac{1}{2g^2}\mathrm{Tr}\, G^2_{\mu\nu}\right\}\,, \tag{2.1}$$

where $G_{\mu\nu} = G^a_{\mu\nu} t^a$ is the field strength tensor of the non-Abelian field A and t^a are the generators normalized in the standard way, $\mathrm{Tr}\, t^a t^b = \frac{1}{2}\delta^{ab}$. The pressure p and the energy density ε are given by the derivatives of the partition function with respect to the spatial volume of the system and with respect to the temperature:

$$p=\frac{T}{V}\frac{\partial\log Z(T,V)}{\partial\log V}=-\frac{F}{V}=\frac{T}{V}\log\mathscr{Z}(T,V)\,,\qquad \varepsilon=\frac{T}{V}\frac{\partial\log Z(T,V)}{\partial\log T}\,. \tag{2.2}$$

The last two equalities for the pressure are valid for a sufficiently large and homogeneous system in thermodynamical equilibrium. The relation between the pressure and the energy in Eq. (2.2) constitutes the equation of state of the system.

According to Eq. (2.2) it is sufficient to determine the partition function of the system to calculate the corresponding equation of state. However, lattice simulations are suitable for calculation of quantum averages of operators rather than the partition function itself. On the other hand, both the energy and the pressure can be derived from the quantum average of a single quantity, which is the trace of the energy–momentum tensor $T_{\mu\nu}$.

In $SU(N)$ gauge theory the energy–momentum tensor is given by the formula

$$T_{\mu\nu}=2\mathrm{Tr}\left[G_{\mu\sigma}G_{\nu\sigma}-\frac{1}{4}\delta_{\mu\nu}G_{\sigma\rho}G_{\sigma\rho}\right]\,, \tag{2.3}$$

which is traceless because the *bare* Yang–Mills theory is a conformal theory. However, because of a dimensional transmutation the conformal invariance is broken at the quantum level and the energy–momentum tensor exhibits a trace anomaly. The thermodynamic relations in Eq. (2.2) give us

$$\theta(T)=\langle T^{\mu}_{\mu}\rangle\equiv\varepsilon-3p=T^5\frac{\partial}{\partial T}\frac{p(T)}{T^4}=-T^5\frac{\partial}{\partial T}\frac{\log\mathscr{Z}(T,V)}{T^3V}\,. \tag{2.4}$$

The pressure and energy density can be expressed via the trace anomaly as follows:

$$p(T)=T^4\int^{T}\frac{\mathrm{d}T_1}{T_1}\,\frac{\theta(T_1)}{T_1^4}\,,\qquad \varepsilon(T)=3\,T^4\int^{T}\frac{\mathrm{d}T_1}{T_1}\,\frac{\theta(T_1)}{T_1^4}+\theta(T)\,. \tag{2.5}$$

Thus the trace anomaly is a key quantity that allows us to reconstruct the whole equation of state.

Note that the trace anomaly should vanish in the case of free relativistic particles ($\varepsilon=3p$), or in the case when excitations are too massive compared with the temperature, $m\gg T$ (then $\varepsilon\sim p\sim\exp\{-m/T\}$). For Yang–Mills theory these statements imply that the dimensionless quantity θ/T^4 should approach zero at both high temperatures (the gluons form a weakly interacting gas) and low temperatures (the mass gap is much greater than the temperature).

The partition function of $SU(N)$ lattice gauge theory is written in the Wilson form,

$$\mathscr{Z}(T,V)=\int DU\,\exp\Bigl\{-\beta\sum_P S_P[U]\Bigr\}\,,\qquad S_P[U]=1-\frac{1}{N}\mathrm{Re}\,\mathrm{Tr}\,U_P\,. \tag{2.6}$$

The temperature $T=1/(N_t a)$ and the volume $V=(N_s a)^3$ of the system are related to the geometry of the asymmetric lattice, $N_s^3 N_t$ and to the lattice spacing a which is a function of the lattice coupling $\beta=2N/g^2$. Using the relation $T(\partial/\partial T)=-a(\partial/\partial a)$ one can adopt Eq. (2.4) to describe the lattice thermodynamics,

$$\frac{\theta(T)}{T^4}=6N_t^4\left(\frac{\partial\beta(a)}{\partial\log a}\right)\cdot\left(\langle S_P\rangle_T-\langle S_P\rangle_0\right)\,, \tag{2.7}$$

where the plaquette averages $\langle S_P \rangle_T$ and $\langle S_P \rangle_0$ are taken, respectively, in the thermal bath at $T > 0$ and in the zero–temperature case corresponding to the asymmetric $N_s^3 N_t$ and symmetric N_s^4 lattices. In Eq. (2.7) it is implied that the $T = 0$ plaquette expectation value is subtracted to remove the effect of quantum fluctuations, which lead to ultraviolet divergency of the quantum expectation value. As a result, the trace anomaly becomes an ultraviolet quantity, which is normalized to zero at $T = 0$ because of the existence of the mass gap. The trace anomalies and the equation of states for $SU(2)$ and $SU(3)$ gauge theories were calculated in Refs. [9] and [10], respectively.

3. Trace anomaly from monopoles in SU(3) lattice gauge theory

The magnetic monopoles are particle–like configurations appearing as singularities in the diagonal component of the gluonic field A_μ^{diag} in the so–called Maximal Abelian gauge, which makes the off-diagonal gluon components A_μ^{off} non–propagating [11] (a review can be found in Ref. [8]). The local gauge condition can formally be written as $D_\mu^{\mathrm{diag}} A_\mu^{\mathrm{off}} = 0$. The monopole trajectories k_μ are identified as sources of the Abelian magnetic field in the diagonal component of the gauge field, $k_\mu \sim \partial_\nu \tilde{F}_{\mu\nu}^{\mathrm{diag}}$. The magnetic charge is quantized and conserved quantity. Details of gauge fixing and the determination of the monopole trajectories on the lattice can be found in Ref. [12].

The partition function of Yang–Mills theory can be represented as a product of two parts: the first part is from the monopole contribution, while the second part is given by the remaining field fluctuations including a perturbative contribution. The trace of the energy–momentum tensor in Eq. (2.4) can consequently be represented as the sum of these parts,

$$\mathscr{Z} = \mathscr{Z}^{\mathrm{mon}} \mathscr{Z}^{\mathrm{rest}}, \qquad \theta = \theta^{\mathrm{mon}}(T) + \theta^{\mathrm{rest}}(T). \tag{3.1}$$

The monopole partition function is given by the sum over the closed monopole trajectories,

$$\mathscr{Z}^{\mathrm{mon}} = \sum_{\delta k = 0} \exp\left\{ - \sum_i f_i(\beta) S_i^{\mathrm{mon}}(k) \right\}, \qquad S_i^{\mathrm{mon}}(k) = \sum_{s,\mu} \sum_{s',\nu} k_\mu(s) K_{\mu\nu}^{(i)}(s,s') k_\nu(s'), \tag{3.2}$$

where the monopole action consists of the two-point interaction terms S_i^{mon} between the elementary segments of the closed ($\delta k = 0$) monopole trajectories. Some part of the interactions terms – defined by the kernels $K_{\mu\nu}^{(i)}(s,s')$ in Eq. (3.2) – are shown in Figure 1.

The coupling constants of the monopole action f_i are numerically determined as functions of β with the help of the inverse Monte Carlo (MC) method [12, 13]. The inverse MC algorithm uses an ensemble of the monopole trajectories as the input, which are located in the original gluonic configurations by the Abelian projection method. We used 400 statistically independent configurations of the $SU(3)$ gauge field generated by the usual MC procedure for $L_s = 16$ and $L_t = 4, 16$ lattices. The inverse MC algorithm searches for the best monopole action that can most consistently describe the ensemble of the monopole trajectories by the partition function in Eq. (3.2). In our simulations we truncated the monopole action after 11 terms allowed us to describe the available ensembles of the monopole trajectories with acceptable accuracy.

Having determined the monopole action we can calculate the contribution of the magnetic monopoles to the trace anomaly of the gluon plasma,

$$\theta^{\mathrm{mon}} = N_t^4 \left(a \frac{\partial \beta}{\partial a} \right) \sum_i \left(\frac{\partial f_i(\beta)}{\partial \beta} \right) \left[\langle \bar{S}_i^{\mathrm{mon}} \rangle_T - \langle \bar{S}_i^{\mathrm{mon}} \rangle_0 \right], \qquad \bar{S}_i^{\mathrm{mon}} = \frac{1}{N_s^3 N_t} S_i^{\mathrm{mon}}, \tag{3.3}$$

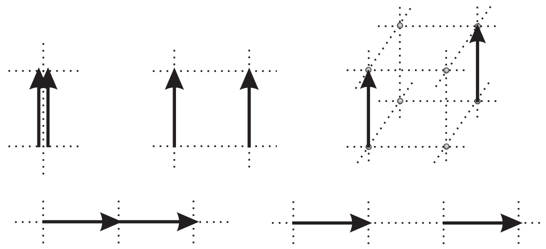
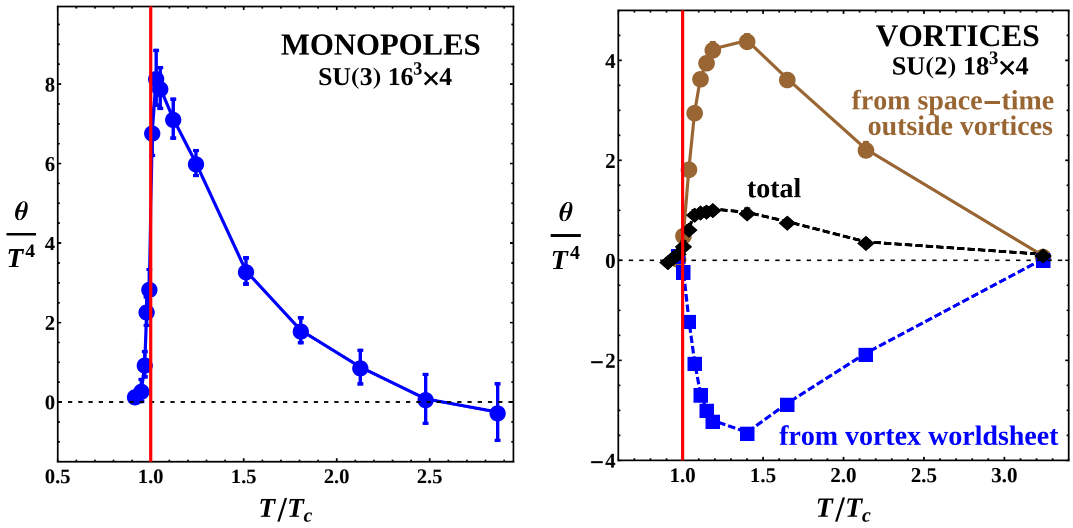


Figure 1: Schematic representation of typical terms in the lattice monopole action.

in which we used Eqs. (2.4), (3.1) and (3.2). Analogously to Eq. (2.7) we have normalized the finite–temperature expectation values of the parts S_i^{mon} of the monopole action, shifting these expectation values by the corresponding zero–temperature expectation values. The subtraction procedure is required to remove the ultraviolet divergencies and to correctly normalize the monopole trace anomaly.



(a) (b)

Figure 2: (a) Contribution of the magnetic monopoles to the trace anomaly in $SU(3)$ lattice gauge theory. (b) The trace anomaly in $SU(2)$ lattice gauge theory is divided into contributions from the vortex worldsheets (squares) and from the whole space-time located outside the center vortices (circles). Their sum gives the total contribution (diamonds). The lines connecting the data points are drawn to guide the eye. The vertical lines mark the deconfinement temperature T_c.

The contribution of the monopoles to the trace anomaly θ/T^4 in $SU(3)$ lattice gauge theory is shown in Figure 2(a). In the confinement region the trace anomaly is zero. The anomaly starts to increase at $T \sim T_c$ approaching a maximum at a temperature slightly above the deconfinement temperature[1]. One also notices that the monopole contribution to the anomaly is a positive quantity that increases at slower rate than T^4 for $T \gg T_c$. All these properties qualitatively match those of

[1]Note that this maximum is higher than that for the pure gluons [Eq. (2.7)] as calculated in Ref. [10]. We attribute this difference to the large finite–size corrections originating from the relatively small and rough lattice ($N_t = 4$) used in our numerical calculations. To improve our results at the quantitative level, one should check the scaling towards the continuum limit (this work is currently in preparation).

the original gluonic trace anomaly in Eq. (2.7) calculated numerically in Ref. [10]. We conclude that the monopoles do contribute to the equation of state of the gluon plasma.

4. Trace anomaly from vortex worldsheets in SU(2) gauge theory

The vortices are magnetic defects, which can be identified in gluon field configurations using the so–called Maximal Center gauge (a review and details can be found in Ref. [7]). The gauge fixes the non-Abelian gauge group up to its center subgroup. The magnetic vortices are then considered as stringlike defects in the center gauge variables.

In $SU(2)$ lattice gauge theory the position of the vortex is determined using the $\mathbb{Z}_2$ gauge field $Z_l = \mathrm{sign}\mathrm{Tr} U_l = \pm 1$. The lattice field–strength tensor of the $\mathbb{Z}_2$ gauge field, $Z_P = \prod_{l\in\partial P} Z_l$, takes the negative value $Z_P = -1$ if the plaquette P is pierced by the vortex worldsheets ${}^*\sigma_{\mu\nu}(s)$ on the dual lattice. If $Z_P = +1$ then the plaquette P is not pierced by the vortex.

The gluonic trace anomaly in Eq. (2.7) is proportional to the expectation values of the plaquette action. Therefore, the separation of space-time into two subspaces (occupied and not occupied by vortices) leads to the natural splitting of the gluonic contribution to the trace anomaly into that originating from the vortex worldsheets, θ_{vort}, and that from elsewhere, θ_{rest}:

$$\theta = \theta_{\mathrm{vort}} + \theta_{\mathrm{rest}} \quad \text{since} \quad \langle S_P\rangle = \langle S_P\rangle_{\mathrm{vort}} + \langle S_P\rangle_{\mathrm{rest}} \quad \text{and} \quad \sum_P S_P = \sum_{P\in\sigma} S_P + \sum_{P\notin\sigma} S_P\,. \tag{4.1}$$

The two contributions to the plaquette action can be conveniently written as

$$\langle S_P\rangle_{\mathrm{vort}} = \frac{1}{N_P}\langle \sum_{P\in\sigma} S_P\rangle = \frac{1}{2}\Big(\langle S_P\rangle - \langle \widetilde{S}_P\rangle\Big)\,, \qquad \langle S_P\rangle_{\mathrm{rest}} = \frac{1}{N_P}\langle \sum_{P\notin\sigma} S_P\rangle = \frac{1}{2}\Big(\langle S_P\rangle + \langle \widetilde{S}_P\rangle\Big)\,, \tag{4.2}$$

where $N_P = 6N_s^3 \times N_t$ is the total number of plaquettes on the lattice. The action

$$\widetilde{S}_P[U] = S_P[\widetilde{U}] = 1 - \frac{1}{2}\mathrm{Tr}\widetilde{U}_P = 1 - \frac{1}{2} Z_P \mathrm{Tr} U_P\,, \quad \text{where} \quad \widetilde{U}_l = Z_l U_l\,, \quad Z_l = \mathrm{sign}\mathrm{Tr} U_l\,, \tag{4.3}$$

can be interpreted as the action of the system with formally "removed" vortices. The above relations are valid in the Maximal Center gauge. The standard plaquette action $S_P[U]$ is given by Eq. (2.6).

In Figure 2(b) we show the both contributions to the trace anomaly calculated for $L_s = 18$ and $L_t = 4, 18$ lattices using from 100 to 800 configurations (depending on the value of β and the lattice geometry). The contribution from the vortex worldsheets is negative, in agreement with general theoretical expectations [14]. The maximum absolute value of the vortex contribution is about three times larger than the pure–gluon contribution calculated numerically in Ref. [9]. This property of the topological magnetic contribution agrees with the observation [15] that the magnetic gluon condensate provides a large negative contribution to the equation of state. Finally, the contribution to the anomaly originating from the rest of the space-time (outside the vortex worldsheets) is large and positive. The total sum of these contributions [also shown in Figure 2(b)] is in agreement with a known result in the $SU(2)$ lattice gauge theory [9].

It is worth noting that for the chosen lattice parameters the maximal contribution of the magnetic vortices to the trace anomaly is achieved when the vortices occupy on average only 5% of the space-time. The negative contribution from the vortices is almost canceled by the positive contribution from the rest (95%) of the space–time. This fact allows us to conclude that the local gluonic fields in the vortex worldsheets are much stronger than the fields outside the vortices.

5. Conclusion

We found that both the magnetic defects, the monopoles and the vortices, contribute significantly to the trace anomaly and, via Eq. (2.5), to the equation of state of the gluon plasma. The contribution of the monopoles is positive while the vortices provide a negative contribution. These results are particularly interesting in view of the fact [6, 7] – which is particularly important in the plasma regime [4] – that the monopoles and vortices are part of the generic object, which constitutes a monopole–vortex chain/net. The contribution from the monopoles is calculated through the determination of the monopole action, which takes into account local self-interaction as well as nonlocal interactions between separated monopoles. In contrast, the contribution of the magnetic vortices to the anomaly is calculated locally. Thus, the qualitative difference between the monopole and vortex contributions is most probably due to the effect of the nonlocal interactions.

In conclusion, we stress our main result: the monopole-vortex chains in $SU(2)$ gauge theory and the monopole–vortex nets in $SU(3)$ gauge theory are thermodynamically relevant objects in gluon plasma, because they contribute significantly to the equation of state of the plasma.

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