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η' mass across the chiral phase transition

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We present a report on progress about our project aimed at determining the behavior of the anomaly term around the chiral phase transition for two light flavors, which is known to be strictly related to the order of the phase transition itself. In particular we discuss our method for determining the η' mass by measuring topological charge correlators and present preliminary results.

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1. Introduction

The order of the phase transition for QCD with two light flavors ($N_f = 2$) is an important, still open issue. A second order transition would imply a crossover at $m_q \neq 0$, hence a possible critical point in the $T - \mu$ plane, where μ is the baryon chemical potential. Instead a first order transition would likely persist also for small quark masses, with no apriori need for such critical point.

Predictions about the order of the transition can be based on a renormalization group analysis of the effective chiral model [1] and turn out to be strictly related to the realization of the axial $U_A(1)$ at the phase transition point. If the $U_A(1)$ anomaly is effective at the transition, *i.e.* if there is no light pseudoscalar meson in the singlet channel, the effective model has a infrared stable fixed point in the $O(4)$ ($O(2)$ in the case of staggered discretization) universality class, hence the phase transition can be second order in that universality class or first order. If instead the $U_A(1)$ anomaly is not effective, *i.e.* if the η' meson becomes light at the phase transition, then the prediction [2] for the effective model is that of second order in the $U(2)_L \otimes U(2)_R / U(2)_V$ universality class or first order (see also Ref. [3] for a numerical analysis of this issue within strongly coupled QED).

In a direct finite size scaling analysis of the chiral phase transition [4] we have found evidence against $O(2) - O(4)$ scaling and hints in favour of a first order transition, which are currently being checked for all possible systematic effects [5]. As a due complement to our analysis, we have also planned to perform a study of the behavior of $m_{\eta'}$ across the chiral phase transition.

Determinations of the η' mass on the lattice are notoriously difficult, because of the disconnected diagrams entering the η' propagator, whose numerical determination turns out to be very noisy [6, 7, 8, 9]. For that reason we have decided to follow a different approach, which is based on the determination of $m_{\eta'}$ through the measurement of topological charge correlators. We present here an explorative study and discuss some preliminary results.

2. The method

The two point function of the topological charge density operator

$$Q(x) = \frac{g^2}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x) \tag{2.1}$$

is dominated at large distances by the lightest physical state coupled to Q, *i.e.* the pseudoscalar singlet meson η' in presence of dynamical fermions. In particular we can write, for the temporal correlator at zero momentum

$$\lim_{t\to\infty} \int d^3x \langle Q(\vec{x},t) Q(0) \rangle \sim A e^{-m_{\eta'} t}\,, \tag{2.2}$$

where the constant A is negative by reflection positivity. Therefore $m_{\eta'}$ can be determined by studying topological charge correlators.

Any definition of the discretized topological charge density $Q_L(x)$ can be adopted, with proper care about renormalizations and contact terms. In particular the lattice correlator $\langle Q_L(x) Q_L(0) \rangle$ is related to $\langle Q(x) Q(0) \rangle$ by

$$\langle Q_L(x) Q_L(0) \rangle = Z^2 \langle Q(x) Q(0) \rangle + c_L(x) \tag{2.3}$$

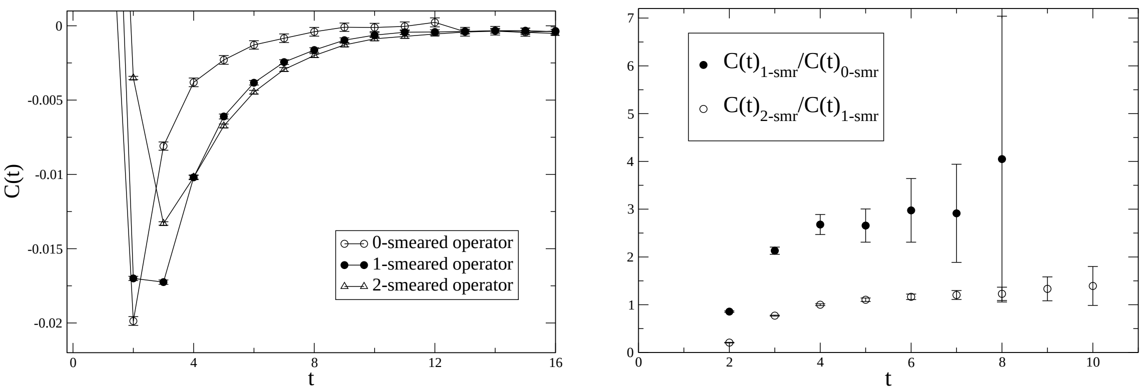

Figure 1: Topological charge correlators and their ratios up to the second smearing level in the pure gauge theory on anisotropic lattices.

where Z is a multiplicative renormalization and $c_L(x)$ a delta-like positive contact term: a similar term is present also in the continuum definition, ensuring $\chi = \langle Q^2 \rangle / V > 0$. Actually, in presence of dynamical fermions, mixings with other pseudoscalar fermion operators are present, which however do not change the asymptotic behavior of $\langle Q_L(x) Q_L(0) \rangle$, since they all couple to the pseudoscalar singlet channel.

On the lattice the contact term $c_L(x)$ is non zero over a finite region of size S_{O_L} around $x = 0$, where reflection positivity $\langle Q_L(x) Q_L(0) \rangle < 0$ is violated. S_{O_L} depends on the extension of the lattice operator $Q_L(x)$. Therefore the relation

$$C(t) \equiv \sum_{\vec{x}} \langle Q_L(\vec{x}, t) Q_L(0) \rangle \sim Z^2 A e^{-m_{\eta'} t} \tag{2.4}$$

holds for large enough t, provided also that $t > S_{O_L}$. It clearly appears that the multiplicative renormalization constant Z is not relevant for determining $m_{\eta'}$.

Our choice for $Q_L(x)$ is that of a simple discretization of $Q(x)$ given in terms of gauge fields. We consider for instance the sequence of smeared operators [10]

$$Q_L^{(i)}(x) = \frac{-1}{2^9 \pi^2} \sum_{\mu\nu\rho\sigma=\pm 1}^{\pm 4} \tilde{\varepsilon}_{\mu\nu\rho\sigma} \mathrm{Tr} \left(\Pi^{(i)}_{\mu\nu}(x) \Pi^{(i)}_{\rho\sigma}(x) \right) , \tag{2.5}$$

where $\Pi^{(i)}_{\mu\nu}(x)$ is the plaquette operator constructed with i–times smeared links $U^{(i)}_\mu(x)$, which are defined as

$$\begin{aligned} U^{(0)}_\mu(x) &= U_\mu(x) \, , \\ \overline{U}^{(i)}_\mu(x) &= (1-c) U^{(i-1)}_\mu(x) + \frac{c}{6} \sum_{\substack{\alpha=\pm 1 \\ |\alpha| \neq \mu}}^{\pm 4} U^{(i-1)}_\alpha(x) U^{(i-1)}_\mu(x+\hat{\alpha}) U^{(i-1)}_\alpha(x+\hat{\mu})^\dagger , \\ U^{(i)}_\mu(x) &= \overline{U}^{(i)}_\mu(x) / (\frac{1}{3} \mathrm{Tr} \overline{U}^{(i)}_\mu(x)^\dagger \overline{U}^{(i)}_\mu(x))^{1/2} \end{aligned} \tag{2.6}$$

($c = 0.9$ is usually taken as an optimal choice for the $SU(3)$ gauge group). The asymptotic behavior of the correlator is independent of the operator and solely related to the η' mass. The effect of

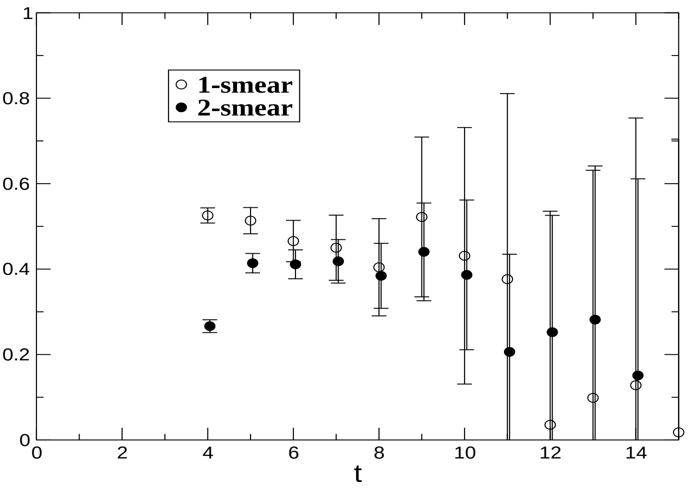

Figure 2: Effective mass plot obtained from the 1-smeared and 2-smeared topological charge correlator.

smearing is that of damping the UV fluctuations: noise is reduced and the multiplicative renormalization Z increases, with a great improvement in the signal/noise ratio [11]. However smearing also increases the size (in lattice units) of the operator $O_L(x)$, hence the size S_{O_L} of the region where the correlator $C(t)$ is still affected by contact terms ($c_L(x) \neq 0$). Therefore we have to look for an optimal balance between the two opposite effects: that can be difficult because of the large value of $m_{\eta'}$ and/or of the limited number of lattice sites available at finite temperature.

In order to make the problem less critical we have decided to work on anisotropic lattices. That choice has indeed various benefits. First, a smaller temporal lattice spacing a_t leads to a larger number of temporal lattice sites L_t for a fixed temperature $T = 1/(L_t a_t)$, therefore to an increased number of useful determinations of the correlator $C(t)$. Moreover, in the specific case of the topological charge correlator, the size S_{O_L} of the region where the correlator $C(t)$ is still affected by contact terms is reduced in physical units. Finally, the temperature can be fine tuned by simply changing L_t, *i.e.* without changing the physical scale and/or the spatial volume, thus isolating effects purely due to a change of T.

We illustrate as an example results obtained in the pure gauge theory, where we have used the anisotropic action defined as

$$S_G = \frac{\beta}{N_c}\frac{1}{\gamma}\sum_{x,i<j\leq 3} \mathrm{ReTr}\,(1-\Pi_{ij}(x)) + \frac{\beta}{N_c}\gamma \sum_{x,i\leq 3} \mathrm{ReTr}\,(1-\Pi_{i4}(x)) \; . \tag{2.7}$$

with $\beta = 6.25$, $\gamma = 3.2552$, leading to $\xi = 4$ and $a_t \approx 0.021$ fm [12, 13]. We show results obtained on a $24^3 \times 40$ lattice, corresponding to a spatial extent of about 2 fm and to a temperature $T \simeq 230$ MeV. In Fig. 1 [left] we show the correlators up to the second smearing level with a zoom in the region where reflection positivity is respected. Asymptotically we expect $\langle Q_L^{(i)}(x) Q_L^{(i)}(0)\rangle \simeq Z_i^2 \langle Q(x)Q(0)\rangle$ for every operator, hence ratios of different lattice correlators must reach a plateau at a value $\sim (Z_i/Z_j)^2$. That is indeed what happens, as can be appreciated from Fig. 1 [right]. If we look at the effective mass plot $-\ln(C(t+1)/C(t))$, which is shown in Fig. 2 for the 1-smeared and

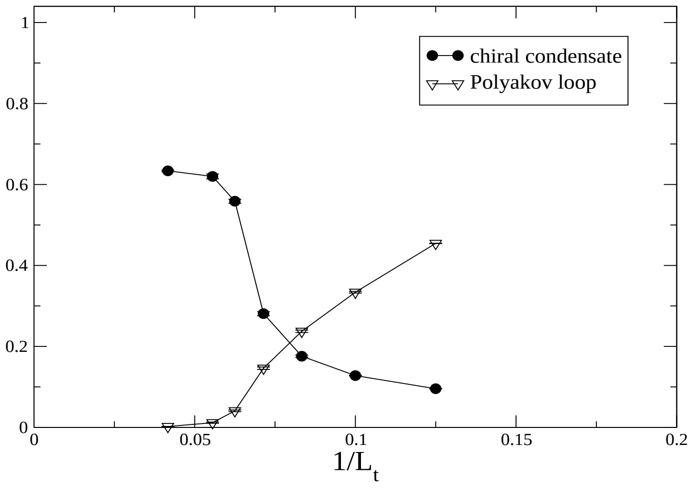

Figure 3: Chiral condensate and Polyakov loop as a function of $1/L_t$ on an anisotropic lattice in full QCD.

2-smeared operators, we notice that a plateau is reached at large distances, with compatible results obtained for operators at different smearing levels.

3. $N_f = 2$ simulations

For our exploratory full QCD simulations we have adopted the anisotropic gauge action proposed in Ref. [14] for two light staggered flavors. That is defined in terms of standard pure gauge and staggered actions, with $\beta = 5.3$, $am_q = 0.008$ and bare anisotropy $\xi_0 = 3.0$, corresponding to $m_\pi/m_\rho \sim 0.3$, $a_s \simeq 0.34$ fm and $a_t \simeq 0.085$ fm, hence to a renormalized anisotropy $\xi \equiv a_s/a_t \simeq 4$.

We are performing numerical simulations on the apeNEXT facility in Rome using lattices with $L_s = 16$ ($L_s a_s \sim 5$ fm) and variable L_t. In Fig. 3 we report results obtained for the chiral condensate and the Polyakov loop as a function of $1/L_t$, showing a phase transition taking place around $L_t = 16$. Results for the topological charge correlators have been collected on lattices with $L_t = 24$ ($T \sim 100$ MeV, ~ 30K molecular dynamics time units) and $L_t = 16$, which is right at the onset of the chiral transition ($T \sim 150$ MeV, ~ 20k molecular dynamics time units).

As can be appreciated from Fig. 4, in the full QCD case the signal turns out to be much noiser than in the quenched case, with the result that only a rough estimate of $m_{\eta'}$ can be performed on the low temperature ($L_t = 24$) lattice, by fitting numerical data obtained for the 2-smeared operator starting from $t = 5a$ on and leading to $m_{\eta'} = 1.3(3)$ GeV, while no sensible determination is still possible on the $L_t = 16$ lattice.

Even if these results are still preliminary, they clearly show that noise problems, which are encountered in standard determinations of the η' mass which exploit correlators of fermionic operator, are met also in the case of pure gluonic operators. As a matter of fact, we are still not able to make reliable statements about the behavior of $m_{\eta'}$ at the phase transition. We estimate that an increase in statistics of at least a factor 10 is necessary to get any signal around the critical point

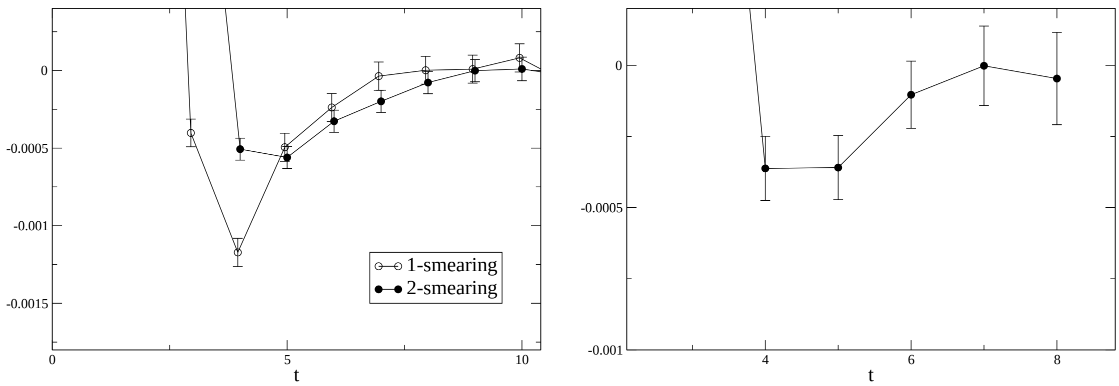

Figure 4: Topological charge correlators in full QCD on $L_t = 24$ (left, 1 and 2-smeared operators) and $L_t = 16$ (right, 2-smeared operator).

($L_t = 16$), however we are also looking for strategies alternative to brute force, like using different topological charge density operators and mixed correlators in order to optimize the signal, or performing simulations on lattices with increased anisotropies.

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References

[1] R. D. Pisarski and F. Wilczek, Phys. Rev. D **29** (1984) 338.

[2] F. Basile, A. Pelissetto and E. Vicari, PoS **LAT2005** (2006) 199 [arXiv:hep-lat/0509018].

[3] S. Chandrasekharan and A. C. Mehta, arXiv:0705.0617 [hep-lat]; PoS **LAT2006** (2006) 128 [arXiv:hep-lat/0611025].

[4] M. D'Elia, A. Di Giacomo and C. Pica, Phys. Rev. D **72** (2005) 114510 [arXiv:hep-lat/0503030].

[5] G. Cossu, M. D'Elia, A. Di Giacomo and C. Pica, arXiv:0706.4470 [hep-lat].

[6] K. Schilling, H. Neff and T. Lippert, Lect. Notes Phys. **663** (2005) 147 [arXiv:hep-lat/0401005].

[7] C. R. Allton *et al.* [UKQCD Collaboration], Phys. Rev. D **70** (2004) 014501 [arXiv:hep-lat/0403007].

[8] V. I. Lesk *et al.* [CP-PACS Collaboration], Phys. Rev. D **67** (2003) 074503 [arXiv:hep-lat/0211040]; S. Aoki *et al.* [JLQCD Collaborations], PoS **LAT2006** (2006) 204 [arXiv:hep-lat/0610021].

[9] E. B. Gregory, A. C. Irving, C. M. Richards and C. McNeile, PoS **LAT2006** (2006) 176 [arXiv:hep-lat/0610044].

[10] C. Christou, A. Di Giacomo, H. Panagopoulos and E. Vicari, Phys. Rev. D **53** (1996) 2619 [arXiv:hep-lat/9510023].

[11] B. Alles, M. D'Elia and A. Di Giacomo, Nucl. Phys. B **494** (1997) 281 [Erratum-ibid. B **679** (2004) 397] [arXiv:hep-lat/9605013].

[12] T. R. Klassen, Nucl. Phys. B **533** (1998) 557 [arXiv:hep-lat/9803010].

[13] N. Ishii, H. Suganuma and H. Matsufuru, Phys. Rev. D **66** (2002) 014507 [arXiv:hep-lat/0109011].

[14] L. Levkova, T. Manke and R. Mawhinney, Phys. Rev. D **73** (2006) 074504 [arXiv:hep-lat/0603031].