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Glueballs and mesons in the superfluid phase of two color QCD

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QCD with two colors undergoes a transition to a superfluid phase with diquark condensate when the quark chemical potential equals half the pion mass. We investigate the gluonic aspects of the transition by inspecting the behavior of the glueball correlators evaluated via a multi-step smearing procedure for several values of chemical potential ranging between zero and the saturation threshold. The results are based on an analysis of 0^{++} glueball correlators, on a sample of 40000 independent configurations on each parameter set. The amplitudes of the correlators peak for $\mu = m_\pi/2$, indicating that the superfluid phase transition affects the gluonic sector as well. The mass of the fundamental state decreases in the superfluid phase, and the amplitude of the propagators drops, suggesting a reduction of the gluon condensate, in agreement with model calculations. The analysis of the smearing dependence of the results helps disentangling the role of long and short distance phenomena at the superfluid transition.

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1. Introduction, and simulation setup

QCD-like models whose determinant remains real at nonzero chemical potential afford the possibility of standard Monte Carlo simulations. Two-color QCD is one such model, which has been extensively studied over the past few years. Many results have been obtained in the fermionic sector, see e.g. [1] for a concise review, while gluodynamics is comparably much less known. However, gluodynamics is arguably the sector where results from two-color QCD have most direct relevance to real QCD. The few studies of finite density gluodynamics performed so far concentrated on the Polyakov loop [2], topological susceptibility [3] and gluon propagator [4]. In this note - which continues a work initiated in ref. [5] – we study glueball correlators at finite density. A more complete account of our results, with a complete set of references, will appear soon [6]. In the same paper we will present, together with the glueball spectrum, a high statistics update of the results on the meson spectrum presented in [5]. Since the new results in the fermionic sector merely confirm our previous findings, we will concentrate here on the glueball results.

The simulations were performed using a standard hybrid molecular dynamics algorithm for eight continuum flavors of staggered fermions [7] on a $6^3 \times 12$ lattice. We accumulated 40000 MC trajectories of unit length for each value of the chemical potential and two quark masses, m=0.05 and m=0.07.

We have measured the fermionic spectrum at $\mu = 0.0$, obtaining $m_\pi = 0.56(2)$ and $m_\rho = 1.4(1)$ for $m = 0.05$, and $m_\rho = 1.5(2)$ and m_π = 0,64 for m=0.07. In both cases m_ρ and m_π are well resolved, with the superfluid transition at $\mu = m_\pi/$ well below the threshold of the predicted vector condensation , $\mu = m_\rho/2$[8]. Moreover, $m_\pi/m_\rho < 1.$ implies that we are still in the region where chiral perturbation theory can be safely applied. If, just to get an idea of the lattice spacing, we extrapolate linearly m_ρ according to $m_\rho(m) = m_\rho(0) - km$ we arrive at $m_\rho(.0) = 1.1$ in lattice units.

2. Glueball measurements: operators and smearing

The operators commonly used for measuring scalar gluonic correlators exciting glueball mass are Wilson loops. For simplicity we restricted ourselves to plaquette-like operators that can be built from four links.

Simple glueball wave functions such as the plaquette have small overlaps with the lowest-lying glueball states. Moreover, the overlaps become rapidly smaller as the lattice spacing is decreased. Furthermore the plaquette couples strongly to ultraviolet fluctuations, increasing the noise in the correlators. To have reliable glueball correlation functions at different distances, it is mandatory to reduce statistical fluctuations.

Different smoothing procedure and methods to remove the unphysical short-distance fluctuations have been introduced. A short review can be found in [9].

One of the most successful approach is the smearing method. This procedure, originally proposed for pure gauge SU(3) in [10], consists in the construction of correlation functions of operators which are a functional of the field smeared in space and not in time.

This procedure reduces the noise associated with lattice artifacts and can be iterated. Only spatial links participate on the averaging. Thus the transfer matrix is not affected by the smearing procedure and remains positive definite. The values of the smearing coefficient and the iterations are tuned in order to optimize the performance of the method [11, 12, 13].

We analyze glueball states with zero momentum in the $A1^{++}$ (one-dimensional) irreducible representations of the relevant cubic point group (on a lattice the full rotational symmetry is broken down to only cubic symmetry). The E^{++} (2-dimensional) turns out to be, as expected, extremely noisy and in the present note we do not discuss it.

The A_1^{++} and E^{++} representations correspond to the continuum $O(3) \otimes Z(2)$, $J^{PC} = 0^{++}$ and 2^{++} respectively. We can then label the associated glueballs as 0^{++} and 2^{++} states. Of course this correspondence is not one to one but infinite to one. Therefore what we can measure is the lowest excitation in the corresponding representation of the cubic group. With dynamical fermions mixing is possible with fermionic states, so strictly speaking we should always use the expression 'lowest excitation in the corresponding representation of the cubic group' rather then 'gluebal mass'.

The glueball operators are defined by means of the plaquettes $P_{ij}(\vec{x},t)$ on ij plane as in the following:

$$\phi^{0^{++}}(t) = \mathrm{tr}\sum_{\vec{x}}\left[P_{12}(\vec{x},t) + P_{23}(\vec{x},t) + P_{13}(\vec{x},t)\right] \tag{2.1}$$

which transforms according to the A_1^{++} representation and couples to the scalar glueball 0^{++}, and

$$\phi_a^{2^{++}}(t) = \mathrm{tr}\sum_{\vec{x}}\left[P_{12}(\vec{x},t) - P_{13}(\vec{x},t)\right] \tag{2.2}$$

$$\phi_b^{2^{++}}(t) = \mathrm{tr}\sum_{\vec{x}}\left[P_{12}(\vec{x},t) + P_{23}(\vec{x},t) - 2P_{13}(\vec{x},t)\right] \tag{2.3}$$

which transform both according to the E^{++} representations and couples to the tensor glueball 2^{++}.

Higher levels of smearing are obtained by varying the weight w and by iterating the procedure N_s times.

Glueball masses are calculated from the behavior of the correlation functions. We have analyzed glueball correlation as function of smearing parameters. In effect, in ref. [12], it has been shown that to a a good approximation the two-dimensional parameters space of the number of sweeps N_s and the smearing weight w may be reduced to a single dimension via the parameter $T_s = N_s \times w$.

To assess the optimal parameter choice in our study, we have analyzed in some more detail the deviations from the unidimensional parametrization. The results of Figs. 1 below show that the unidimensional parametrization remains true till $w \leq 0.3$, irrespective of the number of smearing steps, at least within the allowed range of smearing steps $N_s \leq S/2$, where S is the spatial size of the lattice.

To be on the safe side, we will base our discussions on results within the range of validity of the unidimensional parametrization,taking into account the limitations imposed by the spatial size of the lattice: in conclusion, $w \leq 0.3, N_s \leq 3$.

3. Results

3.1 Amplitudes

If we consider the cluster properties of the correlator themselves, we immediately associate

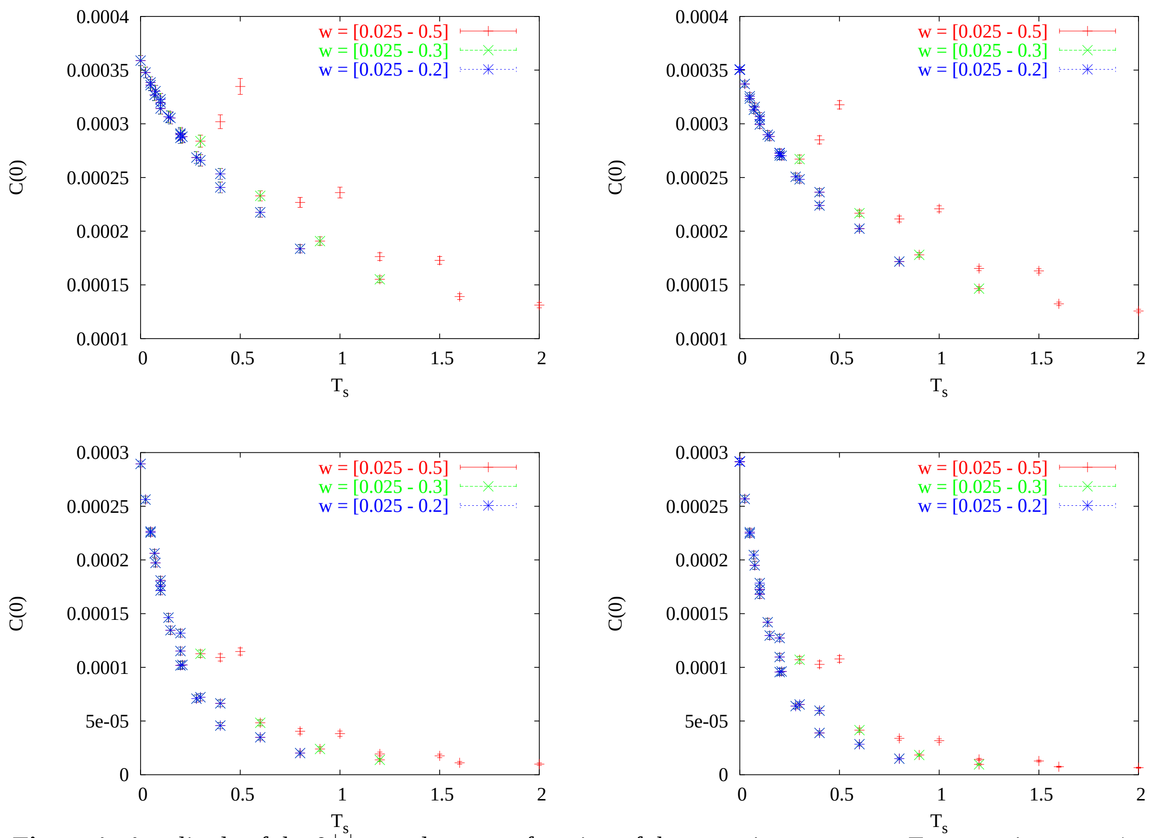

Figure 1: Amplitude of the 0^{++} correlator as a function of the smearing parameter T_s, smeraring steps 1 to 4, and smearing weights as indicated. 'Universality' holds till $w \leq 0.3$. $\mu = 0, m = 0.05$ and 0.07 (upper); $\mu = 0.6$ and m = 0.05 and 0.07, lower

the amplitudes with plaquette susceptibilities. A second intepretation, albeit indirect, associate the amplitude with the gluon condensate.

Interestingly, and not surprisingly! the amplitudes peak at the critical point $\mu_c = m_\pi/$, providing a very clean estimate of the position of the critical point itself, and a clear-cut evidence that the critical behavious seen in the fermionic sector shows up in purely gluonic observables as well.

At larger μ, close to the saturation region, the amplitudes increase again, catching up with the quenched results as they should (see e.g. [3]).

Note that the amplitudes are ultraviolet divergent. However, the smearing procedure[10]-which we have reviewd above- should remove these divergencies, togheter with other short distance artifacts. In the bona fide superfluid region - $m_\pi/2 < \mu < m_\rho/2$ our results show that the effect of smearing is more significant that in the normal phase: this further indicates the reduction of the soft, non-perturbative component of the propagators in this phase. Note that the gluonic condensate contributes to the amplitude, hence the reduction of the amplitudes in the superfluid phase is consistent with the decrease of the gluon condensate predicted by model studies [14].

3.2 The superfluid phase

The glueball propagators in the superfluid phase are amenable to a standard analysis based on hyperbolic cosine fits, supporting the view that glueballs still exist as bound states in this phase. The main observation - supported either by the results of the fits – see Figure 3 – by the effective

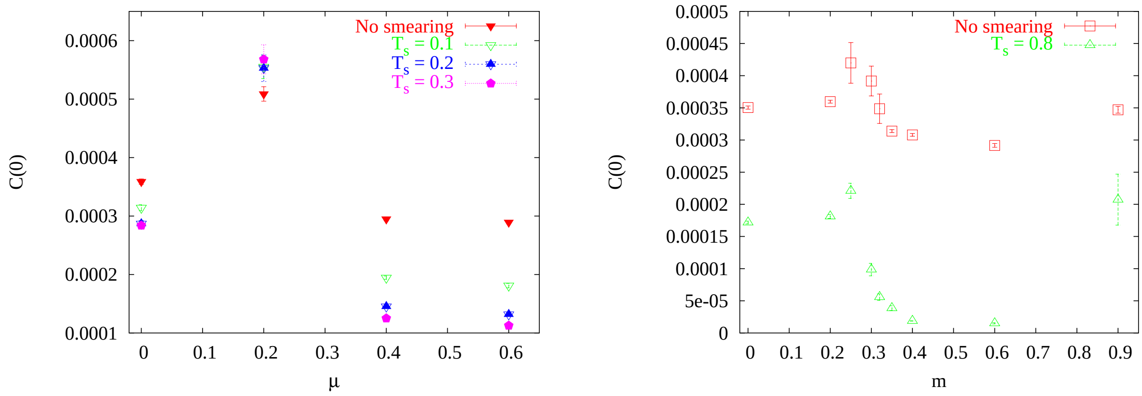

Figure 2: Zero dinstance correlations as a function of μ for two values of T_s in the safe region, and quark mass = 0.05(left) and 0.07

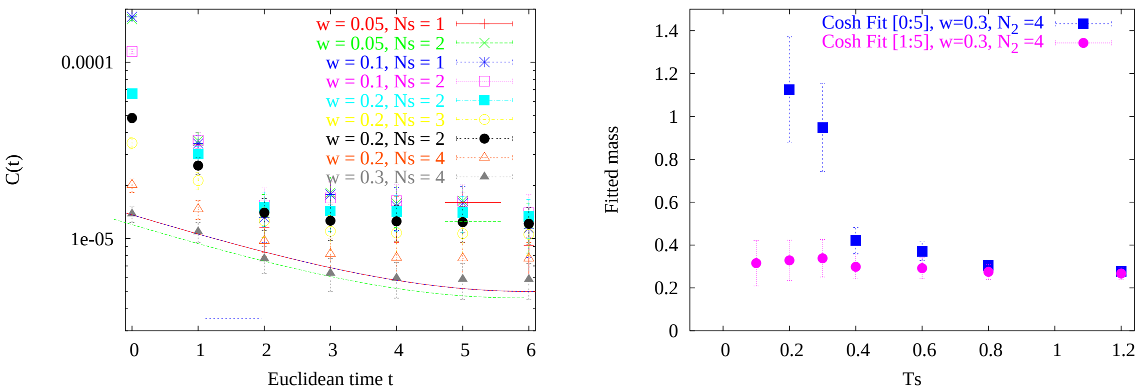

Figure 3: Sample of fits(left) and fit results for two different time intervals as a function of the parameter $T_s = N_s \times w$ for $\mu = 0.6$ and $m = 0.05$.

mass analysis and by a direct comparisons with the normal phase (see next Section) is that the lighest excitations in the gluonic channel in the superluid phase is ligher than in the normal phase.

The fits show that the scalar glueball looks a reasonable bound state in the superfluid phase since it can be easily fitted to a simple $\cosh(mt)$ form. There is no direct indication of a modification of the associated spectral function, and this holds true for both masses. Aside, it is interesting to compare our observations with the results of a study of the glueball spectrum at nonzero temperature [15, 16].

It is then meaningful to directly compare the glueball correlators in the two phases, which is done in Figure 4. These results show that the lowest excitation with 0^{++} quantum numbers becomes ligher in the superfluid phase.

3.3 The critical region

In the critical region we found an unexpected oscillatory behaviour of the glueball propagator. To a much lesser extent, this can still be perceived in the superfluid phase, although, as discussed above, at $\mu = 0.6$ the results are again well represented by a conventional hyperbolic cosine behaviour.

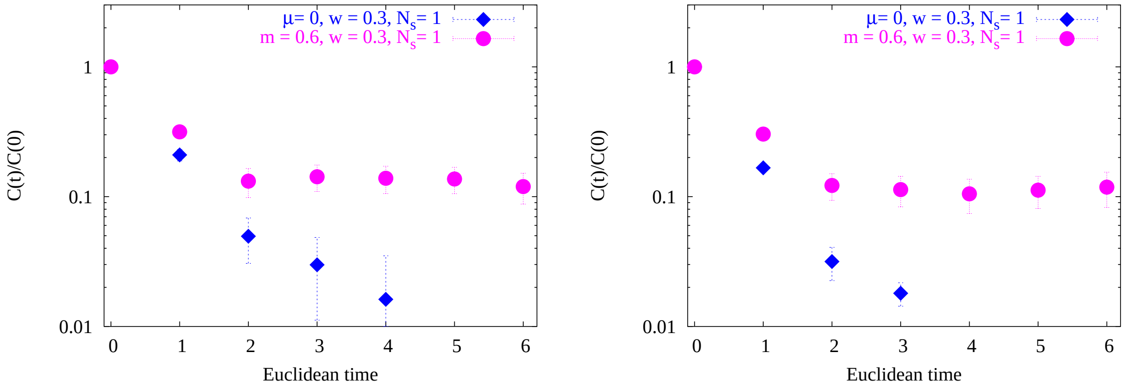

Figure 4: Propagators of the scalar glueball at mass = 0.05(0.07) , left(right) in the normal and superfluid phase, normalized to one at zero distance

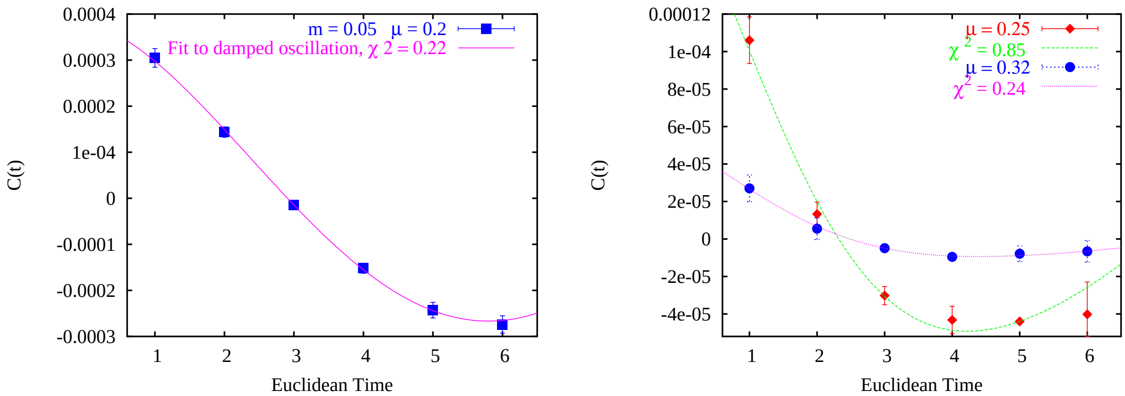

Figure 5: The oscillatory behaviour at $m = .0.05$, $\mu = 0.2$ with superimposed the fit described in the text (left). two representative results at $m = 0.07$ (right)

The behaviour was observed first at $m = 0.05, \mu = 0.2$. Subsequently we did a fine μ scan of the critical region at $m = 0.07$ and found evidence for the same behaviour.

The conclusion from our numerical study is that the propagators in the critical region are well described by

$$C(t) = Ae^{-mt}cos(\alpha t) \tag{3.1}$$

In words, the propagators develop complex poles in the critical region, which come in complex conjugate pairs.

Clearly, because of the lattice periodicity in time, the possible periods are strictly quantized, and, as soon as a very small imaginary component of the pole of the propagator develops, it manifest itself in a period of $N_t = 12$ lattice units. Heavier poles might be observables and give rise to smaller periods, which is probably what we observe at large μ. They should eventually decouple or disappear. Only a scaling analysis either with N_t and with the coupling can answer this question.

3.4 Summary

We have observed a gluonic transition coincident with the superfluid transition at $\mu = m_\pi/2$, for $m = 0.05$ and $m = 0.07$, associated with a peak in the amplitude of the glueball propagator.

In the superfluid region the (smeared) amplitude of the propagator is suppressed, signalling a reduction of the non–perturbative, large distance contributions to the gauge dynamics - i.e. a reduction of the gluon condensates.

Glueballs propagators in the superfluid phase, away from the critical point, can be fitted by a conventional $\cosh(mt)$ behaviour. This, together with the observation that the Polyakov loop apparently is not sensitive to this transition [6] confirms that the superfluid phase is confining [4, 14]. This should be contrasted with observations at finite density, and a nonzero temperatures where the transition is indeed deconfining[17, 2, 3]

A direct comparison of the propagators in the two phases shows that the lowest mode in the 0^{++} channel is much lighter in the superfluid phase than in the normal phase. This confirms and extends our previous results on the meson and glueball level ordering [5].

In the critical region the glueball propagators are dominated by a complex pole. This might indicate some structure in the 4d space-time possibly amplified by lattice artifacts. A study of the N_t scaling, closer to the continuum limit, should set this issue.

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