Screening masses of mesons in 2+1 flavour QCD

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We present results for screening masses of light and strange mesons in $2+1$ flavour QCD using improved (*p4fat3*) staggered fermions on 6×24^3 lattices. We have studied the screening masses of scalar, pseudo-scalar, vector and axial-vector mesons along the line of constant physics, determined by a pion mass ≈ 220 MeV and a kaon mass ≈ 500 MeV. In order to investigate the cut-off and volume dependencies we have also performed studies of the meson screening correlators in the non-interacting theory using the *p4* and the standard staggered discretizations.

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1. Introduction

To acquire a detailed knowledge about the nature of the Quark Gluon Plasma (QGP) it is essential to study the in-medium properties of hadronic excitations. These studies provide information about the important length-scales in the QGP. This in turn gives an idea about the relevant degrees of freedom in the QGP and their possible physical effects. Furthermore, these studies also illuminate crucial aspects of the chiral and the $U_A(1)$ axial symmetry restorations in QCD. Since in the finite temperature lattice QCD simulations the temporal extent of the lattice is limited by the inverse of the temperature most of the finite temperature lattice studies have been concentrated on spatial correlation function of hadrons. The exponential decay of these spatial correlators defines the so called *screening masses* [1]. Physical interpretation of this quantity is as follows— if one puts a test hadron in the QGP medium then the spatial distance beyond which its effects are effectively screened is given by the inverse of its screening mass. For some recent lattice results on the screening masses of mesons see Ref. [2] and for a review of the earlier results see Ref. [3]. Here we present the first ever lattice results for the meson screening masses in $2+1$ flavour QCD with realistic quark masses.

2. Operators

The lattice staggered meson operators are defined as $\mathscr{M}(\boldsymbol{x}) = \bar{\psi}(\boldsymbol{x})\left(\Gamma^D \otimes \Gamma^F\right)\psi(\boldsymbol{x})$, where $\psi(\boldsymbol{x})$ is the fermion field at Euclidean space-time $\boldsymbol{x} = (x,y,z,\tau)$. The matrices Γ^D and Γ^F are products of γ-matrices and generate the spin-flavour structure of the corresponding meson. Here we are interested in the local meson operators, for which $\Gamma^D = \Gamma^F \equiv \Gamma$. For the staggered fermions the local meson operators can be written as $\mathscr{M}(\boldsymbol{x}) = \tilde{\phi}(\boldsymbol{x})\bar{\chi}(\boldsymbol{x})\chi(\boldsymbol{x})$, where $\tilde{\phi}(\boldsymbol{x})$ is a phase factor depending on the choice of Γ.

We consider only the connected part of the screening correlators, *i.e.* we consider only the flavour non-singlet states. The connected part of the staggered meson screening correlator, projected to zero momentum, can be obtained as

$$C(z) = \sum_{\tilde{\boldsymbol{x}}} \phi(\boldsymbol{x}) \left\langle \left(M_{\boldsymbol{x0}}^{-1}\right)^{\dagger} M_{\boldsymbol{0x}}^{-1} \right\rangle, \tag{2.1}$$

where $M_{\boldsymbol{0x}}^{-1}$ is the full staggered propagator (*i.e.* for $N_F = 4$) from $\boldsymbol{0}$ to $\boldsymbol{x}$, $\phi(\boldsymbol{x}) = (-1)^{x+y+\tau}\tilde{\phi}(\boldsymbol{x})$ and $\tilde{\boldsymbol{x}} = (x,y,\tau)$. Since a staggered fermion meson correlator, in general, contains two different mesons with opposite parity [4, 5] we parametrize this correlator as

$$C(z) = A_{NO}\cosh\left[M_-^{scr}\left(z - \frac{N_s}{2}\right)\right] + (-1)^z A_O \cosh\left[M_+^{scr}\left(z - \frac{N_s}{2}\right)\right]. \tag{2.2}$$

The parameters M_-^{scr} and M_+^{scr} are the screening masses of the corresponding mesons. According to our convention M_-^{scr} (M_+^{scr}) corresponds to the screening masses of the lightest negative (positive) parity states and $A_{NO} \geq 0$, $A_O \leq 0$ [1].

For the staggered fermion there are 8 possible local meson operators [4, 5]. In this work we have studied all 8 of them. The corresponding phase factors $\phi(\boldsymbol{x})$ are listed them in Table 1.

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[1]We have chosen our convention such that the Goldstone pion comes as the non-oscillating part of the screening correlator and with positive amplitude.

Operators	$\phi(\boldsymbol{x})$	Γ		J^{PC}		Physical states	
		NO	O	NO	O	NO	O
$\mathscr{M}1$	$(-1)^{x+y+\tau}$	$\gamma_3\gamma_5$	$\mathbb{1}$	0^{-+}	0^{++}	π_2	a_0
$\mathscr{M}2$	1	γ_5	γ_3	0^{-+}	0^{+-}	$\pi^{\pm}$	$-$
$\mathscr{M}3$	$(-1)^{y+\tau}$	$\gamma_1\gamma_3$	$\gamma_1\gamma_5$	1^{--}	1^{++}	ρ_2^T	a_1^T
$\mathscr{M}4$	$(-1)^{x+\tau}$	$\gamma_2\gamma_3$	$\gamma_2\gamma_5$	1^{--}	1^{++}	ρ_2^T	a_1^T
$\mathscr{M}5$	$(-1)^{x+y}$	$\gamma_4\gamma_3$	$\gamma_4\gamma_5$	1^{--}	1^{++}	ρ_2^L	a_1^L
$\mathscr{M}6$	$(-1)^{x}$	γ_1	$\gamma_2\gamma_4$	1^{--}	1^{+-}	ρ_1^T	b_1^T
$\mathscr{M}7$	$(-1)^{y}$	γ_2	$\gamma_1\gamma_4$	1^{--}	1^{+-}	ρ_1^T	b_1^T
$\mathscr{M}8$	$(-1)^{\tau}$	γ_4	$\gamma_1\gamma_2$	1^{--}	1^{+-}	ρ_1^L	b_1^L

Table 1: A complete list of the meson operators studied in this work (see *e.g.* Table 1 of Ref. [5], with $z \leftrightarrow \tau$ interchanged). Possible assignments of the corresponding physical states are given only for the *u-d* flavours.

3. Results

For the present study we have used the gauge configurations generated by the RBC-Bielefeld collaboration using the *p4fat3* staggered action on lattices of size $N_\tau \times N_s^3 = 6 \times 24^3$. We have studied the meson screening correlators along the Line of Constant Physics (LoCP) defined by the zero temperature pion mass $m_\pi \approx 220$ MeV and the zero temperature kaon mass $m_K \approx 500$ MeV. (For more details about the simulation parameters and procedure see Ref. [6]). For each meson channel we have investigated the screening masses in three different sectors corresponding to three different combination of quark masses, *viz.* the $\bar{u}d$, $\bar{u}s$ and $\bar{s}s$ sectors.

In a staggered fermion meson screening correlator a meson is always accompanied by an opposite parity meson (see Eq. [2.2]). In our present study, although we have seen the signatures of both these mesons at our lowest temperature (145 MeV) the amplitude of one of them died out very fast with the increase of temperature. Hence, in most of the cases we found the signature of only one of the mesons. We summarize these findings below— **(I)** In the $\mathscr{M}1$ channel we have seen signature of only the positive parity Scalar (SC), and not the negative parity non-Goldstone Pseudo-Scalar (PS). We will denote the screening masses of this SC channel as M_{SC}^{scr}. **(II)** In $\mathscr{M}2$ we have seen the signature of the Goldstone PS, the only state present in this channel. We will denote the screening masses of this channel by M_{PS}^{scr}. **(III)** In $\mathscr{M}3$ and $\mathscr{M}4$ we have found signatures of only the positive parity Axial-Vector (AV). We have also found that the screening masses of the AV (denoted by M_{AV}^{scr} later) coming from these two channels are degenerate for all temperatures. **(IV)** In $\mathscr{M}5$ and $\mathscr{M}6$ we have only found the negative parity Vector (V) states. The screening masses of these V states (M_V^{scr}) coming from these two channels are found to be degenerate for all temperatures. **(V)** We found that $\mathscr{M}5$ and $\mathscr{M}8$ are very noisy and reasonable signals were obtained only at our three highest temperatures. At these temperatures we found signatures of both the V and AV states in these two correlators. For these two channels, at our three highest temperatures (*viz.* $T = 321$, 363 and 413 MeV), we have found— (a) $M_V^{scr}(\mathscr{M}5) = M_{AV}^{scr}(\mathscr{M}8) = M_{AV}^{scr} = M_V^{scr}$, (b) $M_{AV}^{scr}(\mathscr{M}5) = M_V^{scr}(\mathscr{M}8) > M_{AV}^{scr} = M_V^{scr}$. In fact, at high temperatures all the V, AV states are not even expected to be degenerate. For the spatial correlators at $T = 0$ the rotational symmetry group of the lattice corresponds to the continuum $O(3)$. At non-zero temperature this breaks down
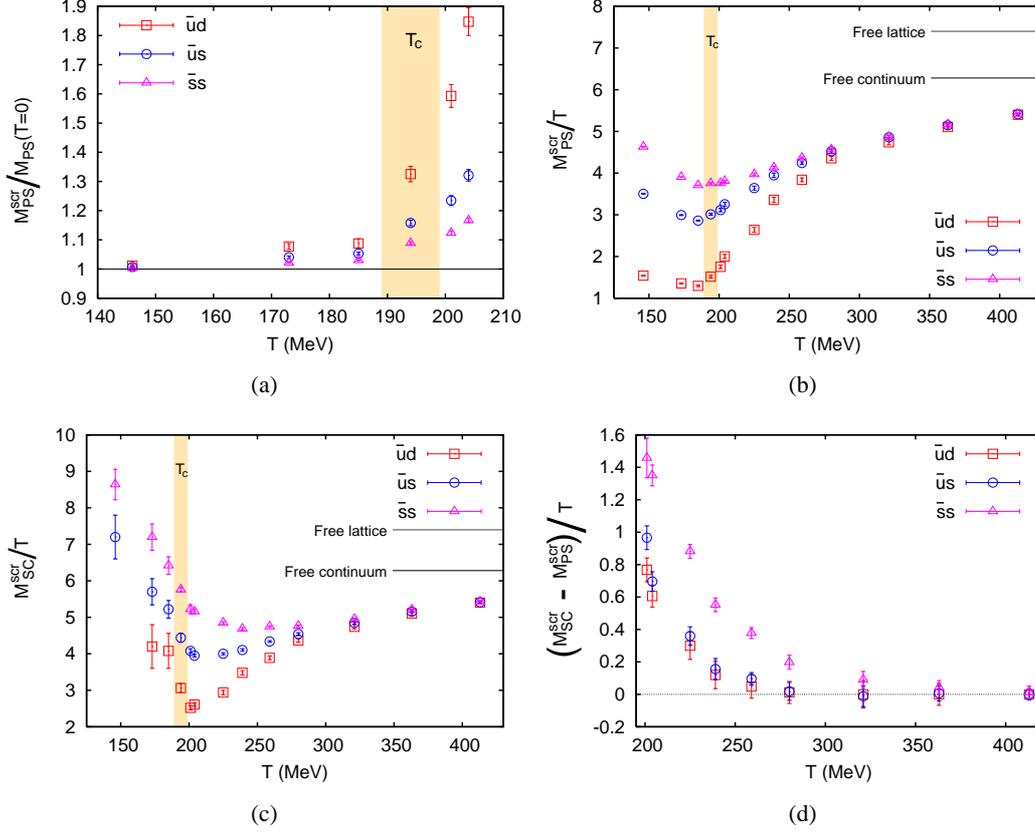
Figure 1: (a) Deviation of the PS screening masses from the corresponding zero temperature PS masses (determined at the same couplings along the LoCP). (b) Temperature dependence of the PS screening masses. (c) Temperature dependence of the SC screening masses. (d) Difference of the SC and PS screening masses as a function of temperature. Shaded regions approximately indicate the transition region for the *p4fat3* staggered action on 6×24^3 lattice with $m_\pi \approx 220$ MeV and $m_K \approx 500$ MeV.

to $O(2) \times Z(2)$. This, combined with the fact that $\mathcal{M}5$ and $\mathcal{M}8$ have longitudinal polarizations compared to the transverse polarizations of the other V, AV channels, explains the non-degeneracy of the V, AV states coming form $\mathcal{M}5$ and $\mathcal{M}8$ [7].

Since at zero temperature the screening masses are identical to the ordinary masses it is interesting to investigate at what temperature their values start to differ. As can be seen in Fig. 1(a) the ratio of M_{PS}^{scr} to $M_{PS}(T=0)$, the ordinary zero temperature PS masses determined at the same couplings along the LoCP, starts differing from one for temperatures $T \gtrsim 145$ MeV for all the three quark sectors $\bar{u}d$, $\bar{u}s$, $\bar{s}s$. In the range 145 MeV $\leq T < T_c$ ($\approx 194 \pm 5$ MeV), M_{PS}^{scr} distinctly differs from $M_{PS}(T=0)$ but only by $5-10\%$ depending on the quark mass.

In Fig. 1(b) and Fig. 1(c) we show the temperature dependencies of M_{PS}^{scr}/T and M_{SC}^{scr}/T respectively. Note that the minima of these two screening masses (normalized by temperature) occur at different temperatures. This probably indicates that the chiral symmetry (related to the minimum of M_{PS}^{scr}/T) and the $U_A(1)$ axial symmetry (related to the minimum of M_{SC}^{scr}/T) restorations are taking place at different temperatures. Even at out largest temperature $T \gtrsim 400$ MeV both M_{PS}^{scr} and M_{SC}^{scr} differ from their free continuum value of $2\pi T$ by $\sim 10\%$ and even more ($\sim 30\%$) from
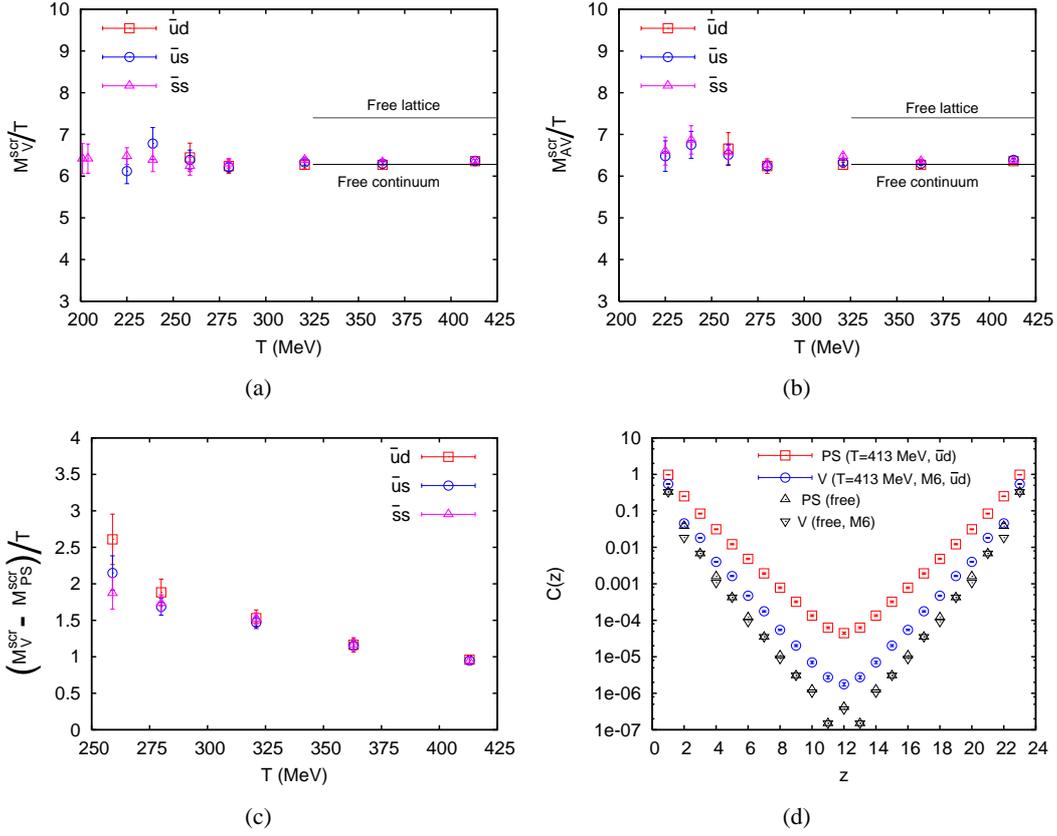
Figure 2: (a) Temperature dependence of the screening masses of the V channel. (b) Temperature dependence of the screening masses of the AV channel. (c) Difference between the screening masses in the V and PS channel. (d) Comparison between the V and PS meson screening correlators for the interacting case and also for the free theory on an identical lattice.

their "free lattice values" (see Section 4). It is expected, at least in the chiral limit, that M_{PS}^{scr} and M_{SC}^{scr} will be degenerate once the $U_A(1)$ symmetry is effectively restored. In Fig. 1(d) we plot the difference between M_{SC}^{scr} and M_{PS}^{scr} as a function of temperature. For the sectors containing the light quarks M_{PS}^{scr} and M_{SC}^{scr} become degenerate only at $T > 250$ MeV, which is significantly higher than the transition temperature T_c, and for the $\bar{s}s$ sector this happens at a even higher temperature. These observations indicate that the effective restoration of $U_A(1)$ probably does not take place at the transition temperature.

In Fig. 2(a) and Fig. 2(b) we show the temperature dependencies of M_V^{scr} and M_{AV}^{scr} respectively. We have found that above the transition temperature both M_V^{scr} and M_{AV}^{scr} are, within errors, compatible with their free continuum value of $2\pi T$ for all the three quark sectors. Within our errors they are also degenerate with each other for the whole temperature range $T > T_c$. However, they differ by $\sim 20\%$ from their free lattice values.

Via Fig. 2(c) we investigate the degeneracy of M_{PS}^{scr} and M_V^{scr}, which is expected to take place at least in the limit of infinite temperature. We have found that the difference between the M_V^{scr} and M_{PS}^{scr} is far from zero at temperatures $T > 400$ MeV ($> 2T_c$). Moreover, our analysis has shown that this difference is independent of the quark sectors for $T > 300$ MeV. In Fig. 2(d) we
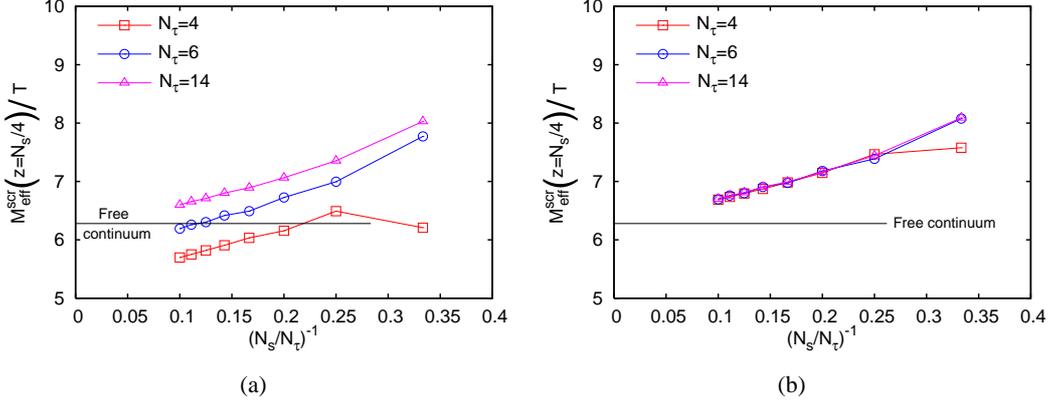
Figure 3: (a) Dependencies of the effective screening mass (see Eq. [4.1]) of the PS channel ($\mathscr{M}2$) on N_τ and on the aspect ratio (N_s/N_τ) for a free (non-interacting) theory with standard staggered fermions. (b) Same as (a) but with *p4* staggered action.

plot the screening correlators for the PS and V channels of the $\bar{u}d$ sector and compare it with the corresponding correlators for the free theory on an identical lattice. It is evident form this plot that the V and PS correlators are degenerate for the free case, but for the interacting theory it is clearly not so. This indicates that the non-degeneracy of M_V^{scr} and M_{PS}^{scr} is probably not a lattice artifact and possibly arises due to the presence interactions. However, one has to keep in mind that compared to the free case the cut-off and finite volume effects could, in principle, be very different for the interacting theory.

4. Free case

In order to have an idea about the cut-off and volume dependencies of our results we have studied these meson screening correlators for the free (non-interacting) theory using different lattice discretizations for the fermions, *viz.* the standard staggered and the *p4* fermions. Such studies can tell us about the lattice artifacts already present in the free case. For the free case we computed the meson screening correlators semi-analytically and looked at the effective mass of the PS ($\mathscr{M}2$) channel [2], defined as—

$$aM_{eff}^{scr}(z) = -\ln\left[\frac{C(z+1)}{C(z)}\right], \tag{4.1}$$

a being the lattice spacing. Note that in the previous section the term "free lattice values" means— the value of $M_{eff}^{scr}(z=N_s/4)$ computed for the non-interacting theory using *p4* staggered fermions on a 6×24^3 lattice.

In Fig. 3(a) and Fig. 3(b) we show $M_{eff}^{scr}(z=N_s/4)$ as a function of the inverse of the aspect ratio (N_s/N_τ) for the standard staggered and the *p4* fermions respectively. As can be seen, for the improved *p4* action there is almost no N_τ dependence for $N_s/N_\tau \geq 4$. On the other hand, for the unimproved standard staggered action the discretization errors, *i.e.* the N_τ dependence, are significantly large.

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[2]In the free case we found that the screening masses of all the channels are degenerate for all values of N_τ with $N_s/N_\tau \geq 4$.

However, for both the *p4* and the standard staggered action we have found that $M_{eff}^{scr}(z = N_s/4)$ is strongly dependent on the aspect ratio and the corresponding continuum result of $2\pi T$ is reached only in the limit of infinite volume. This suggests that the screening masses determined from lattice simulations may have significant volume dependence. Hence a study of the volume dependence of the screening masses is extremely important.

5. Summary

We have investigated the screening masses of mesons in $2+1$ flavour QCD from 8 different local meson operators (listed in Table 1) and for three different quark sectors $\bar{u}d$, $\bar{u}s$ and $\bar{s}s$. For this purpose we have used the gauge configurations, generated by the *RBC-Bielefeld* collaboration, using the improved *p4fat3* fermion action on 6×24^3 lattices and along the LoCP determined by $m_\pi \approx 220$ MeV and $m_K \approx 500$ MeV.

We have found that in the PS channel the screening masses are identical to the corresponding zero temperature (ordinary) masses for temperatures $T \lesssim 145$ MeV. In the high temperature regime both the PS and the SC screening masses differ from the free continuum of $2\pi T$ by $\sim 10\%$ even for $T > 400$ MeV, which is larger than twice the transition temperature. Moreover, for the $\bar{u}d$ and $\bar{u}s$ sectors the PS and SC screening masses become degenerate only at $T \gtrsim 250$ MeV and at an even higher temperature for the $\bar{s}s$ sector.

For the V and AV channels we have found that the screening masses are compatible, within our errors, with each other and with $2\pi T$ for the whole temperature range of $T > T_c$. However, they are $\sim 20\%$ below form our estimation of their free lattice values. In contrast to the free lattice results the screening masses of the V and PS channels do not become degenerate even for $T > 400$ MeV. All these features are true for all the three quark sectors.

We have also investigated the meson screening correlators for the non-interacting theory. Whereas the cut-off dependence of the screening masses are almost negligible for the improved *p4* fermions, for the standard staggered fermion formulation it is quite large. However, in both cases the screening masses show very strong volume dependence. This makes the study of finite volume effects in meson screening correlators extremely important. We hope to address this issue in future.

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