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The Hadronic Spectrum of 2-Colour QCD at Non-zero Chemical Potential

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The hadron spectrum of SU(2) lattice gauge theory with two flavours of Wilson quark is studied on an $8^3 \times 16$ lattice using all-to-all propagators, with particular emphasis on the dependence on quark chemical potential μ. As μ is increased from zero the diquark states with non-zero baryon number B respond as expected, while states with $B = 0$ remain unaffected until the onset of non-zero baryon density at $\mu = M_\pi/2$. Post onset the pi-meson becomes heavier in accordance with chiral perturbation theory while the rho becomes lighter. In the diquark sector a Goldstone state associated with a superfluid ground state can be identified. A further consequence of superfluidity is an approximate degeneracy between mesons and baryons with the same spacetime and isospin quantum numbers.

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1. Introduction

At large baryon chemical potential μ_B the properties of QCD are expected to change as the system moves from a confined nuclear matter phase to a deconfined quark matter phase where the relevant degrees of freedom are quarks and gluons. Weak-coupling techniques can be used at asymptotic densities and have revealed a superconducting colour-flavour locked phase. However as density is reduced towards phenomeologically reasonable values, the precise nature of the ground state appears very sensitive to both the input parmeters and the nature of the non-perturbative assumptions. It seems natural to use Lattice QCD to investigate these issues, but unfortunately whilst the lattice has been used very successfully to investigate QCD with large T, the well known "Sign Problem" has made progress for $\mu_B/T \gg 1$ impossible.

Orthodox simulation techniques can be applied, however, to the case of two colour QCD ($\mathrm{QC_2D}$) with gauge group SU(2). Whilst this theory differs in important ways from QCD, for instance in having bosonic baryons in the spectrum, and in having a superfluid, rather than superconducting, ground state at large μ_B, it remains the simplest gauge theory in which a systematic non-perturbative treatment of a baryonic medium is possible. Recent simulations [1] have provided evidence for two distinct forms of two color matter; the superfluid dilute Bose gas formed from diquark bound states which form at onset (ie. for $\mu_B > \mu_{Bo} = M_\pi$), and a deconfined "quark matter" phase at larger densities resulting from BCS condensation at the Fermi surface. It is possible that studies in this regime may have qualitative or even quantitative relevance for QCD quark matter; for instance Schäfer [2] has highlighted how the impact of instantons on the excitation spectrum at high baryon density could be elucidated by lattice simulations.

In this preceeding we study the μ_B-dependence of the hadron spectrum in both meson and baryon sectors of $\mathrm{QC_2D}$ with $N_f = 2$ flavours of Wilson quark. We also study the nature of the Goldstone mode associated with superfluidity, as done for $\mathrm{QC_2D}$ with staggered lattice fermions in [4], and expose the specifically two colour phenomenon of "meson-baryon" mixing in the superfluid state.

2. Formulation

The gauge-invariant lattice action with $N_f = 2$ degenerate fermion flavours is [1]

$$S = \bar\psi_1 M(\mu)\psi_1 + \bar\psi_2 M(\mu)\psi_2 - \kappa j(\bar\psi_1 K \bar\psi_2^T - \psi_2^T K \psi_1), \tag{2.1}$$

with M the conventional Wilson fermion matrix (with lattice spacing $a = 1$)

$$M_{xy}(\mu) = \delta_{xy} - \kappa \sum_\nu \left[(1-\gamma_\nu)e^{\mu\delta_{\nu 0}} U_\nu(x)\delta_{y,x+\hat\nu} + (1+\gamma_\nu)e^{-\mu\delta_{\nu 0}} U_\nu^\dagger(y)\delta_{y,x-\hat\nu}\right], \tag{2.2}$$

κ the hopping parameter, μ the quark chemical potential, and j the strength of an $\mathrm{SU(2)}_L \otimes \mathrm{SU(2)}_R$-invariant diquark source term needed to regularise IR fluctuations in the superfluid phase, which should be extrapolated to zero to reach the physical limit. The subscript on the fermion fields is a flavour index, the anti-unitary operator $K = K^T \equiv C\gamma_5\tau_2$, where $C\gamma_\mu C^{-1} = -\gamma_\mu^T = -\gamma_\mu^*$ and the Pauli matrix τ_2 acts on colour indices. The hadronic states examined are $q\bar q$ mesons and qq, $\bar q\bar q$ diquark baryons and anti-baryons. In all cases we use local interpolating operators of the form

$\bar{\psi}(x)\Gamma\psi(x)$, $\psi^T(x)K\Gamma\psi(x)$, $\bar{\psi}(x)K\bar{\Gamma}\bar{\psi}^T(x)$. The matrix $\Gamma = \gamma_0\bar{\Gamma}^\dagger\gamma_0$ determines the spacetime quantum numbers of the hadron, with inclusion of the K factor ensuring that mesons and baryons with the same Γ have the same J^P. In this study we will focus on states with $\Gamma \in \{\mathbf{1}, \gamma_5, \gamma_j, i\gamma_5\gamma_j\}$ with $j = 1, \ldots, 3$ corresponding to $J^P \in \{0^+, 0^-, 1^-, 1^+\}$.

3. Numerical Method

We studied an ensemble of gauge configurations generated on a $8^3 \times 16$ lattice at various values of μ using a Hybrid Monte Carlo algorithm with the fermion action (2.1) supplemented by a standard Wilson gauge action [1]. The parameters were $\beta = 1.7$, $\kappa = 0.178$; studies of the string tension suggest a "physical" lattice spacing $a = 0.26(1)$fm,[1] and studies of the $\mu = 0$ meson spectrum yield $M_\pi a = 0.79(1)$ and $M_\pi/M_\rho = 0.80(1)$ [6]. For the most part the diquark source $ja = 0.04$, though for a few values of μ the series $ja = 0.06, 0.04, 0.02$ was studied in order to permit a $j \to 0$ extrapolation.

Quark propagators on each configuration were calculated by all-to-all techniques [7] using time, spin and flavour dilution. Whilst this equates to constructing $16 \times 4 \times 2 = 128$ dilution vectors per propagator and thus performing 256 inversions per configuration, at $\mu \neq 0$ this was required for acceptable statistical precision. Disconnected diagrams relevant for each state were calculated and saved seperately; with the current level of statistics these contributions are both noisy and compatible with zero.

To extract masses the meson correlation functions were fitted to a cosh function. The time range was adjusted to achieve a stable fit while minimising the obtained $\chi^2/d.o.f.$ The $\mu \neq 0$ states with baryon number $B \neq 0$, such as the diquarks and the kaons, are no longer degenerate with their anti-particles. This results in correlators which are no longer time-symmetric and must be fitted by a sum of two independent exponentials. Such a 4-parameter fit is clearly more susceptible to noise than the standard cosh form. Naïvely in the vacuum phase below onset, the states' masses receive an additive contribution $\pm\mu B N_c$. Hence as μ increases the correlation function is increasingly dominated by the lighter of the particle-antiparticle pair and the heavier state rapidly becomes very difficult to fit.

4. Results

All results presented here come from analysis of hadron correlators formed from connected quark propagators. The result for the pion mass at $\mu = 0$ is $M_\pi a = 0.800(3)$. The prediction of chiral perturbation theory (χPT) [8], applicable strictly when there is a separation of scales between the pion and heavier hadrons, is that $\mu_o = M_\pi/2$. This suggests that given the measured pion mass of $M_\pi(\mu = 0)a = 0.800(3)$, the onset of the superfluid phase should take place at $\mu_o a \simeq 0.4$.

In principle all our results should be extrapolated to the "physical" limit $j \to 0$, but unfortunately available resources preclude a systematic study for all μ. Here we follow [1] by studying $ja = 0.02, \ldots, 0.06$ at three representative μ points: just below onset, just above onset, and well into the superfluid phase. The results for M_π and M_ρ are shown in the open symbols in Fig. 1.

Results for the meson spectrum as a function of μ are displayed in Fig. 1. At $\mu = 0$ the

[1]This value is based on string tension measurements on a $12^3 \times 24$ lattice and supplants those reported in [1, 6].

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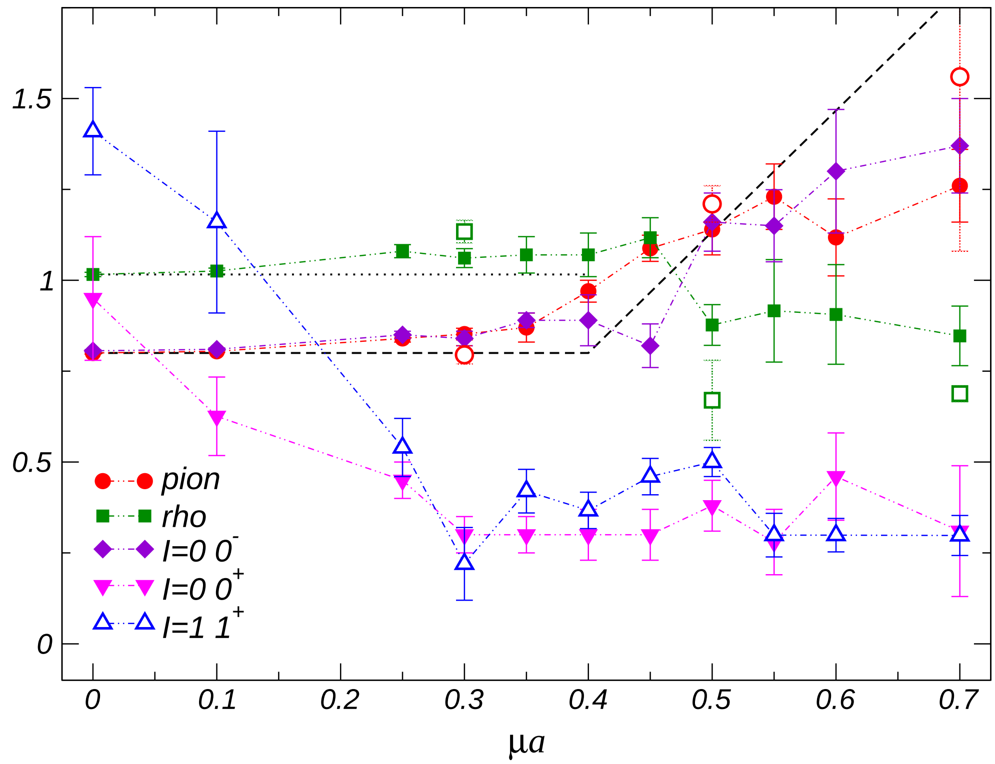

Figure 1: Graph showing meson masses as a function of μ for $ja = 0.04$. The dashed line is the χPT prediction for the pion. Open symbols denote extrapolations to $j \to 0$.

isovector 0^- and 1^- states, ie. the pion and rho, are consistent with the values found in [1, 6]. Throughout the vacuum phase (i.e. $\mu < \mu_o$) M_π and M_ρ are more or less constant as expected for states with $B = 0$, although they do show some increase for $\mu a > 0.2$. For the pion this appears to be a $j \neq 0$ artifact vanishing in the $j \to 0$ limit, but things are not so clear for the noisier rho. At and beyond onset at $\mu_o a \simeq 0.4$ the pion and rho signals become much noiser as reflected in the error bars, but it is still possible to identify trends. The pion starts to become heavier at onset and appears to increase in mass monotonically with μ in the limit $j \to 0$. The $j = 0$ χPT prediction $M_\pi = M_\pi(\mu = 0)\theta(\mu_0 - \mu) + 2\mu\theta(\mu - \mu_o)$ [8] (dotted line) is followed in a qualitative sense. The increase of the pion mass post-onset is characteristic of a state formed from q and $\bar{q}$ with a symmetric combination of quantum numbers under the residual global symmetries (i.e. the P_S state in the notation of [8]) in a theory with Dyson index $\beta_D = 1$.

Post-onset the rho becomes significantly lighter, in agreement with the result found in simulations on $4^3 \times 8$ with significantly heavier quarks by the Hiroshima group [3]. This effect becomes stronger as $j \to 0$. Reduction of M_ρ in a nuclear medium has been proposed to explain the low mass lepton pair enhancement observed in heavy ion collisions [9].

Results for isovector 0^+ and isoscalar 1^+ states are omitted from Fig. 1 for the sake of clarity. The former shows no signifcant signal and the latter follows the rho almost exactly until $\mu a > 0.5$ at which point it becomes too noisy to measure. We do, however, include the similar results for the isoscalar 0^-. By ignoring the disconnecting pieces the only difference between these two states and the pion and rho is the sign of the term proportional to $\langle q(x)q^T(y)\rangle$ i.e. the anomalous term.

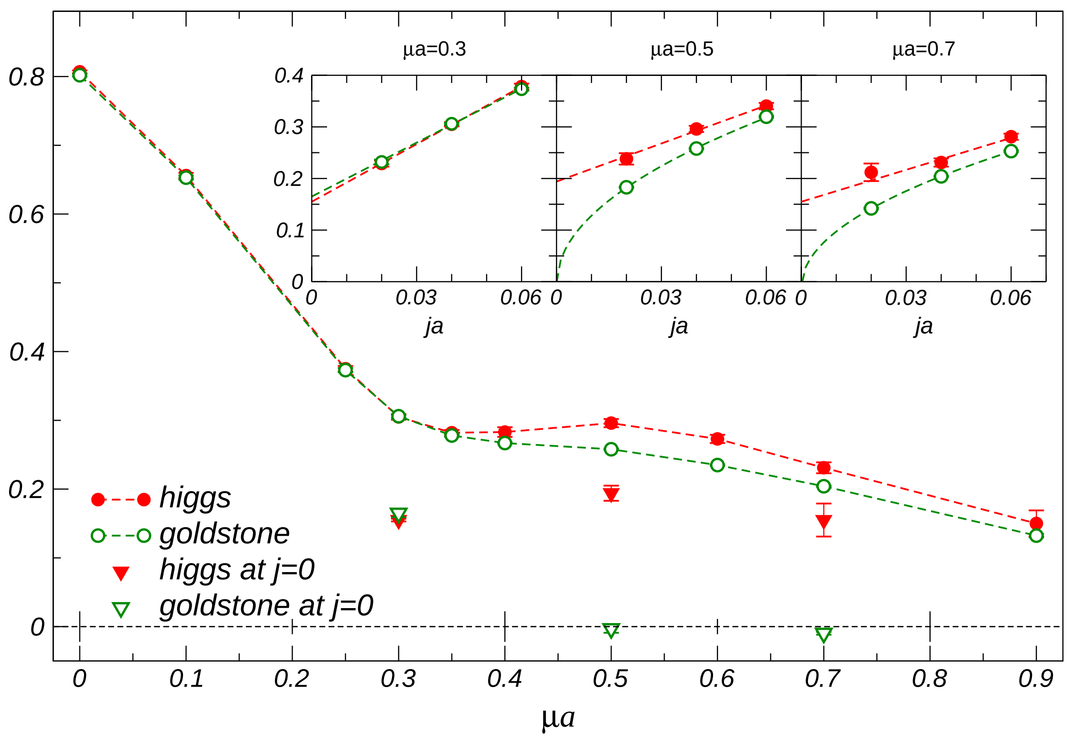

Figure 2: Higgs and Goldstone masses as a function of μ. The three insets show results as j is varied at fixed $\mu a = 0.3, 0.5, 0.7$. Extrapolations to $j = 0$ are displayed on the main graph with triangular symbols.

That they are degenerate shows that this is negligible pre-onset. Beyond onset, there is a small window in which the 0^- in the isoscalar channel is significantly lighter than the isovector, and is indeed roughly degenerate with the $I = 0$ 0^- diquark of Fig. 3, to be discussed below.

The two remaining mesons shown in Fig. 1, the isovector 1^+ and isoscalar 0^+, show a similar behaviour, both starting off relatively heavy (and noisy) and then rapidly dropping as μ increases. By $\mu a = 0.3$ they have reached a minimum and stay more or less constant as μ increases further. We shall argue below that the low mass of the 0^+ state is due to its overlap with the Goldstone boson in the superfluid phase, when baryon number ceases to be a good quantum number; the low mass of the 1^+ is more unexpected.

To understand the physics of the diquark sector, it is helpful to begin with the Higgs and Goldstone states with a varying diquark source strength j. Fig. 2 shows Higgs and Goldstone masses as functions of μ at $ja = 0.04$. The insets show how the two states scale with j at three selected values of μ. Below onset Higgs and Goldstone are degenerate, both scaling approximately linearly with j [2]. Post onset the degeneracy is broken, and the relation $M_{Goldstone} \propto \sqrt{j}$ predicted in χPT [8] appears to hold.

The two states remain degenerate until onset at which point the Goldstone becomes lighter than the Higgs, and appears to become massless as $j \to 0$. This is a clear manifestation of spontaneous breaking of $U(1)_B$ symmetry breaking for $\mu > \mu_o$, implying a superfluid ground state in which

[2] The pre-onset behaviour $M(j) = M(0)(1 + bj^2)^{\frac{1}{4}}$ predicted by χPT [8] may be difficult to distingush from linear behaviour in this regime.

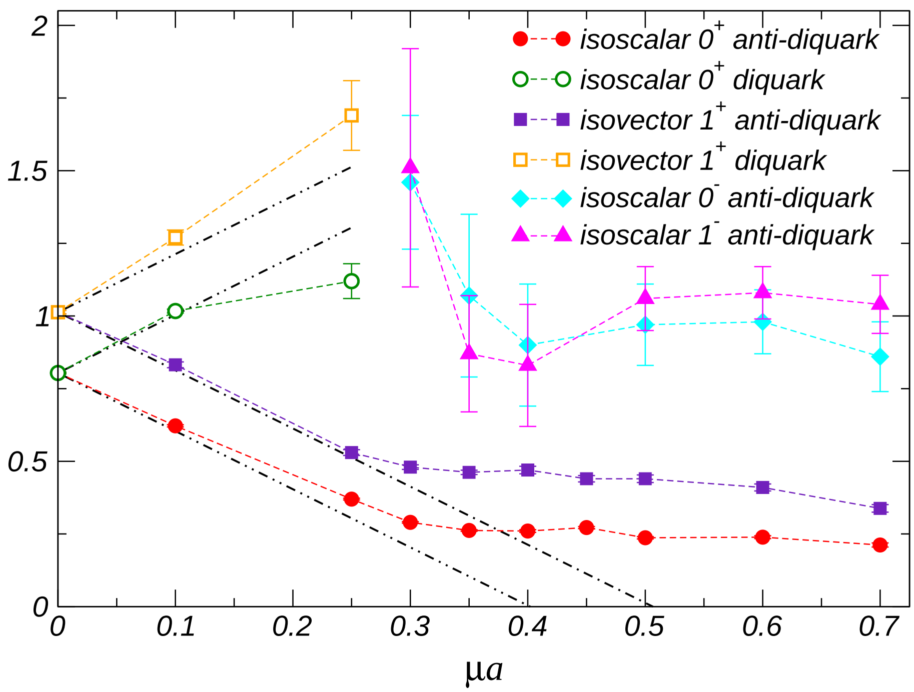

Figure 3: Graph showing diquark masses as a function of μ. Only states corresponding to operators with $B < 0$ are displayed for $\mu a > 0.25$. The dot-dashed lines have intercepts at $M_{\pi,\rho}$, and gradients ± 2.

baryon number is no longer a good quantum number, and therefore meson and diquark states are in principle indistinguishable.

The diquark spectrum in the remaining spin-0 and spin-1 channels is shown in Fig. 3. It is striking that the signal-noise ratio is much higher for some diquarks than for the mesons, also seen in simulations with staggered fermions [4]. The two cleanest signals are for the isoscalar 0^+ and the isovector 1^+. The first observation is that there is a relation between the meson and diquark spectra which holds for $\mu = j = 0$ if disconnected diagrams are neglected:

$$M_D(J^P) = M_M(J^{-P}). \tag{4.1}$$

For $0 < \mu < \mu_o$, during which the physical ground state remains the vacuum, we thus predict $M_D(0^+) = M_\pi \pm 2\mu$, $M_D(1^+) = M_\rho \pm 2\mu$, shown as dot-dashed lines in Fig. 3. Indeed both diquark particle-antiparticle pairs behave as expected up to $\mu a \approx 0.3$. Diquark masses are not shown beyond $\mu a = 0.25$ as they become unfittable, as explained in Sec. 3. After this both 0^+ and 1^+ anti-diquark states flatten off and slowly decrease with μ. The other two isoscalar diquarks constructed from local operators, namely the 0^- and 1^-, are extremely heavy and hard to fit below onset, but above onset have a sufficiently good signal for us to deduce masses comparable with $M_\pi(\mu = 0), M_\rho(\mu = 0)$. Although the noise in the meson sector is admittedly large, the approximate degeneracy between meson and baryon sectors in the 0^+ and 1^+ channels seen in Figs. 1 and 3 is consistent with the meson-baryon degeneracy in the superfluid state discussed above. Meson-diquark degeneracy has also been observed in quenched studies at $\mu \neq 0$ with staggered fermions [10].

The isoscalar 0^- diquark is a particularly interesting state in QC$_2$D because of meson-baryon mixing in the superfluid phase. It has the same quantum numbers as the η' meson [2], and hence its mass acts as a probe of instanton effects and/or possible restoration of $U(1)_A$ symmetry in a baryonic medium. Unfortunately the current simulations are not close enough to the chiral limit to settle this issue via observation of a π-η' mass splitting.

5. Discussion

The main achievements of this study have been observations of:

- The reversal of the pion and rho levels on crossing from vacuum into a baryonic medium. In the vacuum $\mu < \mu_o$, $M_{\pi,\rho}$ is approximately constant, probably because there is no diquark state with the same quantum numbers with which to mix.
- The breaking of the degeneracy between Higgs and Goldstone diquark states for $\mu > \mu_o$, and the Goldstone mass scaling as $\sqrt{j}$ in accordance with general theoretical properties of spontaneous symmetry breaking by condensation of fermion pairs.
- Further evidence for meson-baryon mixing in the degeneracy of $I = 0\ 0^+$ and $I = 1\ 1^+$ states for $\mu > \mu_o$. Post onset the 1^+ appears to be the next lightest state after the Goldstone and Higgs. The fact that the mesons with these quantum numbers appear not to have constant mass even pre-onset (see Fig. 1) can also be ascribed to meson-baryon mixing, since for $j \neq 0$ there is a non-zero amplitude for $\bar{\psi}\psi$ to project onto a baryon.

Lttle has been uncovered about the effects of a second deconfining phase transition suspected to occur at $\mu_d a \approx 0.65$ on this system [1]. The only possible discernable trend is a levelling off of the already massive pion state for $\mu a \gtrsim 0.5$ seen in Fig. 1. However the increasing statistical noise makes this observation provisional at best. In a deconfined phase we might expect mesons and baryons to be formed from particle-hole and particle-particle pairs in the neighbourhood of a Fermi surface, and it is possible that the local operators used in this study have a poor projection onto the true quasiparticle excitations. We hope that studies of meson and diquark wavefunctions currently in progress will clarify the situation.

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