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The $N_t = 8$ QCD thermal transition with DWF

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The 2 and 2+1 flavor QCD thermal phase transition is studied using domain wall fermions at $N_t = 8$ and fifth dimension with 32 sites. The location of the transition and the effects of the residual chiral symmetry breaking are investigated.

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1. Overview

Domain wall fermions (DWF) are very appealing for the study of QCD near the thermal phase transition. For a review see [1] and references theirin. DWF preserve exactly the flavor symmetry of QCD and as a result for two light flavors produce exactly three degenerate pions. Furthermore, they provide an extra parameter, the number of sites along the fifth dimension L_s. By increasing L_s one can reduce the artificial breaking of chiral symmetry by the lattice. The computer time increases only linearly with L_s and therefore one can, in principle, achieve high reduction of the lattice-induced chiral symmetry breaking. The main effect of this residual breaking is the induction of an addition to the input quark mass m_f. This addition is called the residual mass, m_{res}.

The dependence of m_{res} on L_s has been the subject of much investigation (for example see [2, 3, 4] and references theirin). In short, for weak couplings corresponding to inverse lattice spacings $1/a$ at least as large as $a^{-1} \approx 2$ GeV m_{res} decreases exponentially with increasing L_s. The decay rate becomes faster as the lattice spacing becomes smaller. For illustration, if we assume that the critical temperature is around 180 MeV then one would need at least $N_t = 12$ where N_t are the sites along the time direction. This is currently out of reach. Present day supercomputers available for lattice QCD have just reached the $N_t = 8$ landmark.

At lattice spacings in the range of 0.5 GeV to 2 GeV the circumstances are more difficult. It has been found that m_{res} still decreases with L_s but the decrease is slow and mostly of the form $1/L_s$ with a coefficient that becomes smaller with decreasing lattice spacing [2]. Again, assuming a critical temperature of about 180 MeV , an $N_t = 4$ lattice study corresponds to $a^{-1} = 0.7$ GeV . It was found that at that lattice spacing one would need L_s of a few hundred in order to achieve an m_{eff} that produces pions near their physical values. The first DWF studies of QCD thermodynamics were done for $N_t = 4$ several years ago when supercomputers had speed in the GFlops range. These studies were limited because for $L_s \approx 100$ the required computational resources were high.

Today supercomputer speeds are in the several TFlops range. Furthermore, new algorithmic methods have provided additional simulation speedup. As a result QCD thermodynamics with $N_t = 8$ is now possible. This corresponds to $a^{-1} \approx 1.4$ GeV . Still, DWF are not in the preferred domain but the size of m_{res} is now smaller and because of the increased computational speeds an $L_s = 32$ or larger is not prohibitive.

In addition, renewed interest in DWF thermodynamics has come because of the RHIC and the upcoming LHC experiments. To fully understand these experiments the QCD equation of state is needed in order to guide the hydrodynamics codes that make contact with the experimental results. The HotQCD collaboration is currently in the process of calculating the equation of state using lattice QCD with p4 and asqtad fermions [5, 6]. Because the critical temperature plays such a crucial role in the equation of state it will also be calculated using DWF. It is therefore now time to revisit DWF QCD thermodynamics and understand their behavior and challenges at $N_t = 8$. First results to help guide these efforts will appear elsewhere. A summary is presented here.

2. Two flavors

The QCD thermal phase transition is easier to study for two degenerate light quark flavors. In this spirit the first $N_t = 8$ DWF simulations were done for two flavors. The $N_t = 8$ results for the

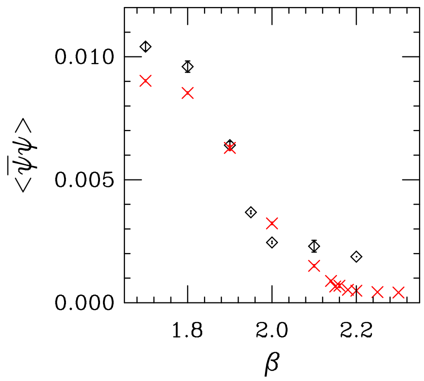

Figure 1: $<\overline{\psi}\psi>$ versus β for two flavors. Diamonds are for: $N_t = 4$, $V = 8^3$, $L_s = 24$, $m_f = 0.02$, $m_0 = 1.9$. Crosses are for: $N_t = 8$, $V = 16^3$, $L_s = 32$, $m_f = 0.004$, $m_0 = 1.9$.

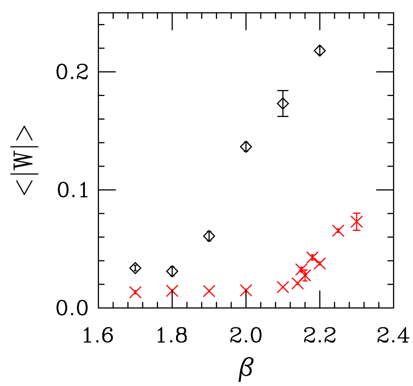

Figure 2: $|W|$ versus β. Diamonds are for: $N_t = 4$, $V = 8^3$, $L_s = 24$, $m_f = 0.02$, $m_0 = 1.9$. Crosses are for: $N_t = 8$, $V = 16^3$, $L_s = 32$, $m_f = 0.004$, $m_0 = 1.9$.

chiral condensate $<\overline{\psi}\psi>$ are shown in figure 1. In that figure the $N_t = 4$ results from [7] are also shown for comparison. As expected a rapid change is observed indicating the chirally broken and chirally symmetric phases of QCD. However, this rapid change occurs at approximately the same value of $\beta = \beta_c$ for both $N_t = 4$ and $N_t = 8$. This is clearly wrong since the lattice spacing should decrease by a factor of two and therefore β_c for $N_t = 8$ should be larger than for $N_t = 4$.

In figure 2 the magnitude of the Wilson line $|W|$ is plotted versus β. Here, one can again see the rapid change in $|W|$ indicating the crossover between the confined and de-confined phases of QCD. However, unlike $<\overline{\psi}\psi>$ the Wilson line for $N_t = 8$ shows a transition at a larger value of β than that for $N_t = 4$. This is as expected.

Furthermore, it is believed that $<\overline{\psi}\psi>$ and $|W|$ should show a transition at similar values of β. From figures 1 and 2 it appears that they do not, which further indicates that the $<\overline{\psi}\psi>$

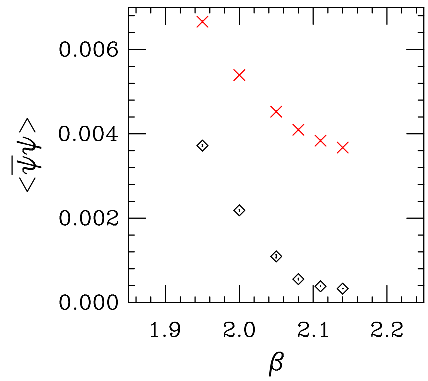

Figure 3: $<\overline{\psi}\psi>$ versus β at $N_t = 8$ for two light flavors with $m_f = m_{light} = 0.003$ (diamonds) and one strange with $m_f = m_{strange} = 0.037$ (crosses). Here $V = 16^3$, $L_s = 32$ and $m_0 = 1.8$.

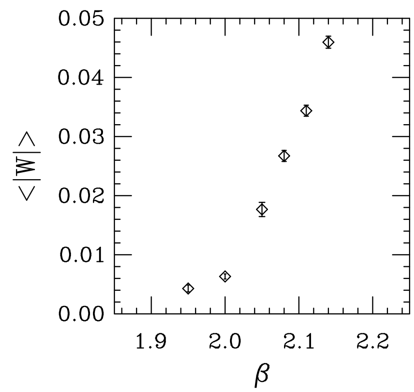

Figure 4: $|W|$ versus β at $N_t = 8$ for $m_{light} = 0.003$, $m_{strange} = 0.037$, $V = 16^3$, $L_s = 32$, and $m_0 = 1.8$.

transition occurs at a smaller value of β than it should.

This small mystery unravels by realizing that the simulations are performed at fixed m_f. However, as β is varied the residual mass m_{res} is changing and as a result the total quark mass is changing. This would induce a corresponding change to $<\overline{\psi}\psi>$ obscuring its dependence on temperature and causing a misidentifying of the crossover region. As discussed earlier this effect should be more prominent at smaller β values. Then instead of focusing in the middle of the crossover region one can locate the point where $<\overline{\psi}\psi>$ first departs from the small fixed value in the symmetric phase as β is lowered. That point occurs in a larger value of β at $N_t = 8$ than in $N_t = 4$. Furthermore, viewed this way, $<\overline{\psi}\psi>$ and $|W|$ show a transition at similar values of β (notice that one would expect $|W|$ to be less affected by this since it depends on the quark mass only through vacuum effects). This will be explored in more detail in the following sections.

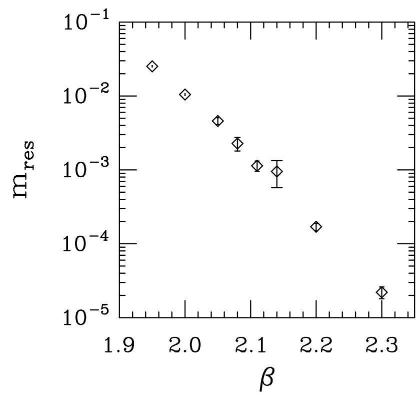

Figure 5: The residual mass m_{res} versus β at $N_t = 8$ with $L_s = 32$, $V = 16^3$, $m_{light} = 0.003$, $m_{strange} = 0.037$, and $m_0 = 1.8$. The two points at the largest values of β are from the zero temperature simulations in [2]

3. Two plus one flavors

The results from 2+1 flavor simulations with masses $m_{light} = 0.003$ and $m_{strange} = 0.037$ are shown in figure 3 for $< \overline{\psi}\psi >$ and in figure 4 for $|W|$. Again, one can see the rapid rise in both quantities. However, one again sees the disparity between the region of rapid rise for $< \overline{\psi}\psi >$ and $|W|$. Nevertheless, this too could be resolved if one considers the point where $< \overline{\psi}\psi >$ first departs from the small fixed value in the symmetric phase as β is lowered instead of the middle of the region of rapid rise of $< \overline{\psi}\psi >$.

4. The residual mass

The above discussion can be made much more precise by directly measuring m_{res} across the crossover region. The method used here is as in [2]. The results are shown in figure 5. This is a large effect. The dependence on β is exponential.

This can be understood from the following simple argument. It has been established that the reason for the $1/L_s$ dependence of m_{res} is the presence of gauge field dislocations [2, 3, 4]. Gauge field Dislocations, are instanton type objects that have size about one lattice spacing. They cause the net topology to change and as a result they induce a large value into m_{res}. The gauge action of these objects manifests itself in the path integral with a weight $exp(-\beta S_d)$ where S_d is the amount they contribute to the action. Therefore their effect becomes exponentially large with decreasing β. Fortunately, this is true the other way around too. Their effect becomes exponentially suppressed with increasing β.

One can now explain the behavior of $< \overline{\psi}\psi >$ in figures 1 and 3. The rapid increase of $< \overline{\psi}\psi >$ as β is decreased is partly a lattice artifact. The chiral condensate has a linear dependence to the quark mass. Therefore, as m_{res} increases with decreasing β so does $< \overline{\psi}\psi >$. This effect "contaminates" the results for $< \overline{\psi}\psi >$. However, one can still distinguish the point where $< \overline{\psi}\psi >$

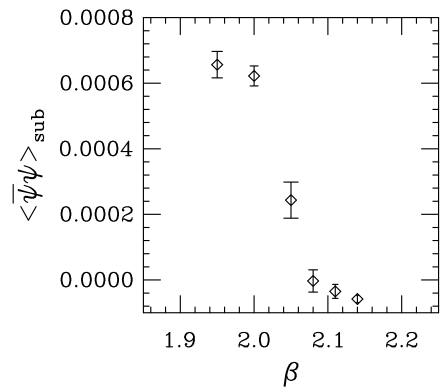

Figure 6: As in figure 3 but for $<\overline{\psi}\psi>$ versus β "subtracted" to correspond to zero quark mass.

first departs from the small fixed value in the symmetric phase as β is lowered. This point can be used as a "rough" indicator of the edge of the crossover region.

Nevertheless, the crossover region should be located using more precise methods such as measurements of the relevant susceptibilities (see for example [6]). These methods are sensitive to the longer correlation lengths and confinement/de-confinement physics present in the crossover region and therefore should be less sensitive to the varying quark mass. This is under investigation.

5. The subtracted $<\overline{\psi}\psi>$

We can try to remove this large dependence of $<\overline{\psi}\psi>$ on the residual chiral symmetry breaking by subtracting $<\overline{\psi}\psi>$ measured for the strange quark scaled by the ratio of the sum of residual and explicit quark masses: $<\overline{\psi}\psi>_{\rm sub}=<\overline{\psi}\psi>|_{m_{light}} - \frac{m_{light}+m_{\rm res(light)}}{m_{strange}+m_{\rm res(strange)}} <\overline{\psi}\psi>|_{m_{strange}}$.

Recall that the chiral condensate contains a large, $1/a^2$-divergent contribution which is linear in the input quark mass, m_f. To the extent that the effects of residual chiral symmetry breaking on the chiral condensate can be represented by simply replacing the input quark mass by the sum of the residual and input quark masses and the strange quark condensate is dominated by this mass-dependent term, then the quantity $<\overline{\psi}\psi>_{\rm sub}$ defined above will be free of residual chiral symmetry breaking effects. This is shown in figure 6.

However, it should be emphasized that for a divergent quantity such as $<\overline{\psi}\psi>$, we expect that the residual chiral symmetry breaking effects will not be accurately described by the replacement $m_f \to m_f + m_{\rm res}$. For example, the quenched chiral condensate studied in [8] showed that such a naive estimate of the effects of residual chiral symmetry breaking on $<\overline{\psi}\psi>$ were accurate on only the factor-of-two level.

Thus, the attractive result for $<\overline{\psi}\psi>_{\rm sub}$ shown in Figure 6 is somewhat surprising. At the smallest value of $\beta = 1.95$, this subtraction has reduced the chiral condensate by a factor of six, suggesting a considerably greater accuracy for this naive subtraction than was expected.

6. Future directions

From the above discussion it is clear that in order to be able to locate the crossover region with some accuracy one would have to: 1) Measure the relevant susceptibilities and 2) keep the effective physical quark mass fixed as β is varied across the crossover region. This can be achieved in different ways. For example, one can vary m_f as β is varied so that the mass $m_f + m_{res}$ relevant to the low energy theory remains fixed. At $L_s = 32$ $m_{res} \approx 0.01$ at $\beta = 2.0$ and therefore one would have to keep a rather large quark mass fixed. In this case a larger value of L_s is possible and would be desirable. Another possibility is to take advantage of the exponential decrease of m_{res} vs. β and instead simulate at $N_t = 10$. Finally, one should further explore the methods of gap domain wall fermions [9] since they may offer substantial advantage.

Acknowledgments

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