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Progress on Perturbative Matching Calculations for the Charm Quark Mass using the HISQ Action

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The highly-improved staggered quark (HISQ) action is the most accurate discretization scheme to date for the charm quark. Here we report on the progress of perturbative matching for the quark mass using the HISQ action. The matching is done through $O(\alpha_s^2)$ using a combination of Monte Carlo simulations at weak coupling and diagrammatic perturbation theory. When combined with on-going simulation efforts using the HISQ action, a determination of the charm quark mass to a few percent accuracy can be achieved. Of particular interest will be a comparison with the recent sum rule determination of the charm mass due to Kühn et al. [1].

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1. Introduction

Quark masses are fundamental parameters that go in the standard model and it is important to determine them precisely to constrain models beyond the standard model and to use them as inputs for phenomenological calculations. As we do not find free quarks in nature, their masses cannot be directly measured. One needs to instead do a comparison between lattice and experiment to extract quark masses. Quarks interact via the strong force, therefore such a calculation would be nonperturbative. Lattice QCD methods are well suited for this problem. This work builds on recent developments in staggered quarks [2].

Previously, the light quark masses $m_{u,d}^{\overline{MS}}$ and $m_s^{\overline{MS}}$ have been determined by several collaborations. We list some $m_s^{\overline{MS}}$ determinations. The HPQCD collaboration used the AsqTad action with 3 dynamical quarks to determine $m_s^{\overline{MS}} = 87 \pm 4 \pm 4$ MeV [3]. The CP-PACS and JLQCD collaborations used the Wilson action with 3 dynamical quarks to calculate $m_s^{\overline{MS}} = 91.1_{6.2}^{14.6} MeV$[4]. The QCDSF-UKQCD collaborations used clover fermions with 2 flavors of sea quarks to get $m_s^{\overline{MS}} = 110 - 130$ MeV [5]. The ALPHA collaboration used Wilson quarks with 2 dynamical quarks and obtained $m_s^{\overline{MS}} = 97(22)$ MeV [6]. The *SPQcdR* collaboration used Wilson Quarks with 2 dynamical flavors to get $m_s^{\overline{MS}} = 101(8)(^{25}_{0})$ MeV [7]. The Particle Data Group reports the value $m_s^{\overline{MS}} = 95 \pm 25$ MeV [8].

Our goal is to also determine m_c with the HISQ (Highly Improved Staggered) quark action to a few percent accuracy, which will be determined by taste-changing effects and discretization. The taste changing effects for the HISQ action are up to 3-4 times smaller than for the Asqtad action as detailed in [2], and a comparison of the two actions is shown in Fig. 1. It will be interesting to compare our results for m_c with the sum rules calculation by Kühn et al. [1], where an error of 1% is quoted.

Our aim is to do a perturbative matching calculation and obtain the renormalization factors for m_c in the $\overline{MS}$ scheme. Below we list the perturbative expansions of the pole mass in terms of the bare mass, its relation to the $\overline{MS}$ mass, and the relevant mass renormalization factors:

$$\begin{aligned}
m^{Pole} &= m_0[1+\alpha_{lat}(A_{11} log(m_0 a) + A_{10}), \\
&+ \alpha_{lat}^2 log^2(m_0 a) + A_{21} log(m_0 a) + A_{20} + ...], \\
m^{\overline{MS}}(\mu) &= m^{Pole}(1 + Z_1(\frac{\mu}{m^{Pole}})\frac{\alpha_{\overline{MS}}}{\pi} + Z_2(\frac{\mu}{m^{Pole}})\frac{\alpha_{\overline{MS}}^2}{\pi^2}), \\
m^{\overline{MS}}(\mu) &= \frac{am_0}{a} Z_m(\mu a, m_0 a), \\
Z_m(\mu a, m_0 a) &= 1 + Z_{m,1}(\mu a)\alpha_V(q^*) + Z_{m,2}(\mu a)\alpha_V^2 + ...
\end{aligned}$$

Note that the correct expansion parameter to use is the renormalized coupling α_V, the perturbative series in α_{lat} is not well behaved.

2. Diagrammatic Method

One way to do the matching is to use perturbation theory and compute all the diagrams up to the order at which we work. Since matching corrects for the short distance effects brought about by

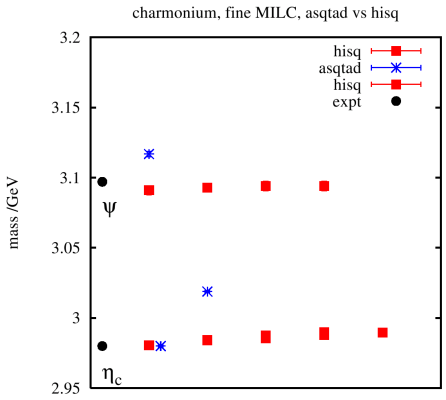

Figure 1: Comparison of results for the HISQ vs Asqtad actions [9]. The HISQ results correspond (from left to right) to 0-link, 1-link, 2-link and 3-link mesons. For Asqtad η_c we show results for the 0- and 1-link pseudoscalars.

the finite lattice spacing, asymptotic freedom allows us to use perturbation theory. This approach is very involved, as it requires calculating many diagrams, and the Feynman rules for the HISQ action are extremely complicated. The relevant diagrams are shown in Fig. 2 .

3. Another Method: Weak Coupling Monte Carlo

An alternative to diagrammatic perturbation theory is to use Monte Carlo simulations at weak couplings, where the theory enters the perturbative phase. Simulations involving a particular operator, in this calculation the pole mass, are done at several values of the strong coupling, and the resulting data are then fitted to an expansion in α_V to yield the perturbative coefficients. The expectation value of an observable can be calculated on the lattice using

$$< M > = \int [DU][d\psi D\overline{\psi}] M(U,\psi) e^{-\beta S[U,\psi]},$$

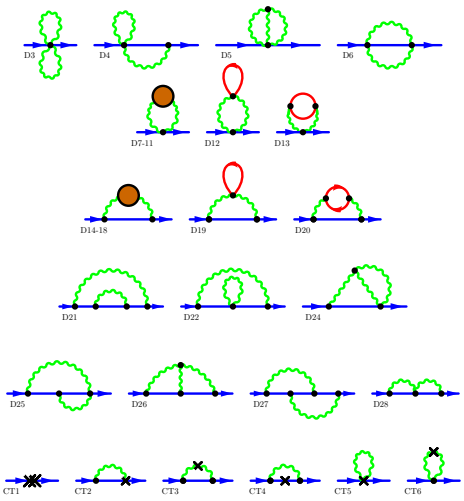

Figure 2: Relevant diagrams for the diagrammatic method.

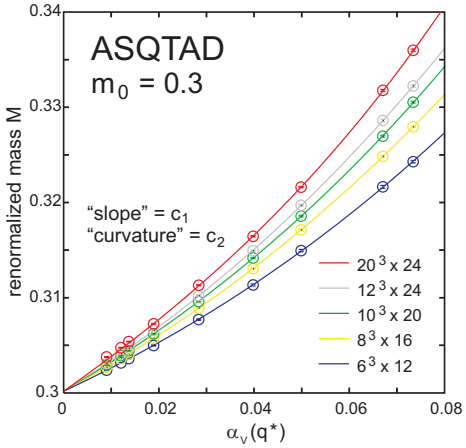

Figure 3: The renormalized mass M measured at different couplings and lattice volumes for the ASQTAD action. The data points are fitted to an expansion in $\alpha_V(q^*)$ (solid lines). The "slopes" and "curvatures" are the first and second order coefficients respectively.

where $\beta = 10/g^2$. It is worth reiterating that in practice it is crucial to use the renormalized coupling instead of the bare lattice coupling $\alpha_{lat} \equiv g^2/4\pi$, for which perturbation theory is very poorly convergent. A good choice is $\alpha_V(q^*)$ defined by the static potential, along with an estimate of the optimal scale q^* for the quantity of interest. The coupling is then converted back to α_{lat} using the known third order relation between α_V and α_{lat} [10].

As an example, Fig. 3 shows the renormalized mass M measured at different couplings and lattice volumes for the ASQTAD action in the quenched approximation. The data points are fitted to an expansion of the form

$$M = m_{tree} + c_1\alpha_V + c_2^V\alpha_V^2 + \ldots,$$

where m_{tree} is the tree-level mass. The "slope" of the curve gives the first order coefficient c_1 (independent of the scheme) and the "curvature" is equal to c_2^V. One can use the c_1 value calculated from one loop perturbation theory to determine c_2 more accurately. Fig. 4a shows the infinite volume extrapolation of c_1. For comparison, results from diagrammatic perturbation theory [11], both at finite volume and in the infinite volume limit, are also plotted. Numerical values of c_2 can be calculated from Fig. 3 also by calculating the curvatures. To improve the accuracy, however, we re-fit the data with c_1 fixed to the analytic values at finite volume. Our results are shown in Fig. 4b. The overall agreement with our diagrammatic perturbation theory calculations is remarkable.

Fig. 5 shows the first order coefficients for the HISQ action, extrapolated to the infinite volume limit. For comparison, we also plot the same coefficients obtained using the diagrammatical method, and observe that the agreement is again outstanding. The 1-loop perturbative calculations at finite volume, which could be used to extract the second order coefficients from the data, are in progress.

Table 1 shows the perturbative coefficient A_{20} calculated with the AsqTad action, neglecting sea quarks, compared to diagrammatic perturbation theory results. We find good agreement between the two sets of results.

We have demonstrated that perturbative coefficients for mass renormalization can be obtained with high accuracy from Monte Carlo simulations at weak couplings. This numerical method

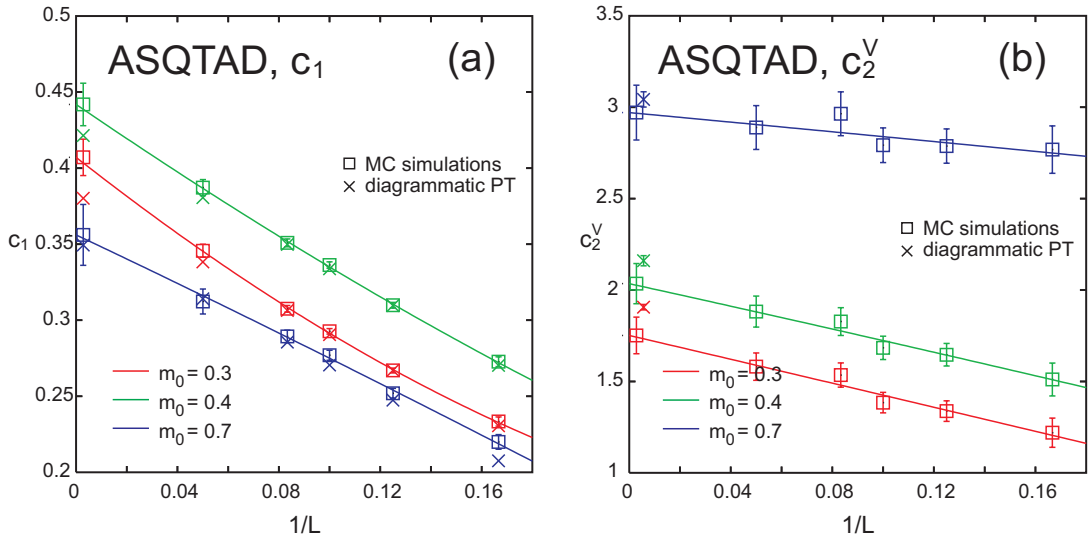

Figure 4: a) Infinite volume extrapolation of the first order coefficients. The error bars are invisible at this scale for the analytic results. b) Infinite volume extrapolation of the second order coefficients. Results are obtained by fixing c_1 to the analytic values at finite volume.

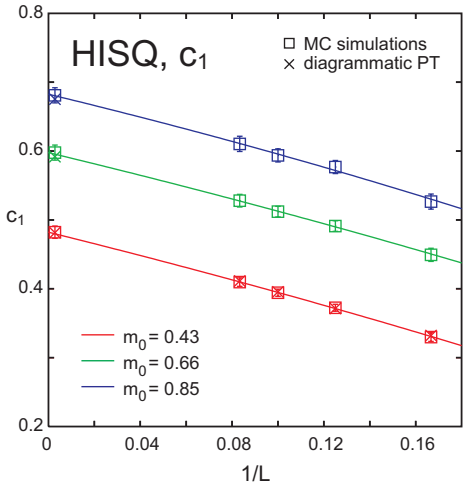

Figure 5: Infinite volume extrapolation of the first order coefficients for the HISQ action. The error bars are invisible at this scale for the analytic results.

am	Diagrammatic PT	Weak Coupling MC
0.3	5.78(5)	5.26(33)
0.4	5.61(7)	5.30(28)
0.5	5.47(6)	5.25(24)
0.6	5.23(6)	5.18(22)
0.7	5.15(6)	5.05(21)

Table 1: Asqtad quenched A_{20}, comparison between diagrammatic perturbation theory and high-beta Monte Carlo simulations.

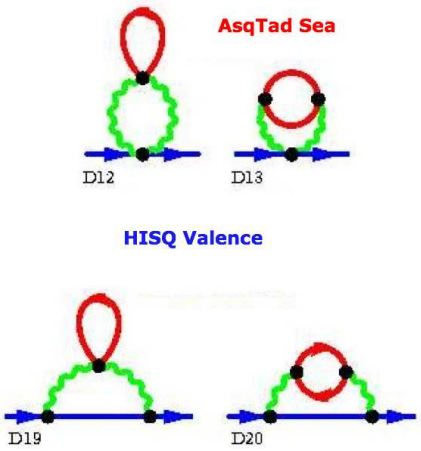

Figure 6: There are only 4 diagrams to compute in the fermionic part.

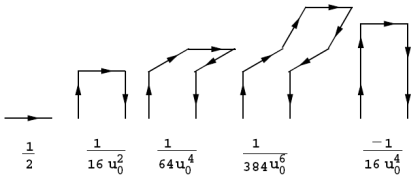

Figure 7: Fattening the link. This needs to be done twice, once with the thin link, and once more with the already fattened link.

provides a valuable alternative to diagrammatic perturbation theory.

4. Strategy

Our short term goal is to obtain the gluonic part of our calculation by the weak coupling Monte Carlo method, and get the fermionic part using diagrammatical perturbation theory. The diagrams needed are shown in Fig. 6.

In the long term, it would be desirable to do the entire calculation diagrammatically and also to compute the gluonic part using the Monte Carlo weak coupling approach in order to compare the two methods.

5. Handling HISQ Perturbation Theory

The HISQ action is obtained by fattening a simple link variable, reunitarizing the result, and fattening the resulting link again. Fattening is necessary for suppressing taste changing interactions, and fattening twice suppresses taste changing further. Reunitarization is necessary to suppress the unphysical tadpole diagrams which only exist in the lattice discretization and not the continuum limit. Fattening of a link can be visualised as in Fig. 7. Pieces of the action (in this case the links shown above) can be combined by doing Fourier convolutions, which means one assigns the gluons

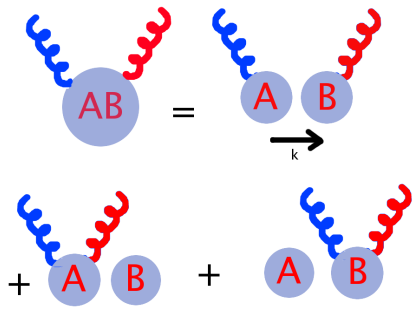

Figure 8: Convolution of two operators for two gluons.

to the pieces in all possible ways. Fig. 8 is a simple illustration with two operators and two gluons. Once we perform the convolutions and reunitarize, we obtain an improved link variable. We need to fatten this yet again. We therefore repeat the same process of convoluting, this time with the already fattened and reunitarized link, to obtain the full HISQ vertices.

6. Conclusions

Our project involves performing perturbative matching calculations to find m_c through $O(\alpha_s^2)$ using the HISQ quark action. To achieve this, work is underway to do the fermionic part of the calculation diagrammatically, while the gluonic part is computed using the weak coupling Monte Carlo method. When both calculations are complete, the results will be combined to obtain perturbative coefficients for the m_C mass renormalization. Our initial tests show that perturbative coefficients can be obtained accurately using the weak coupling Monte Carlo method. For the longer term goal of doing the entire calculation diagrammatically, the vertex functions necessary to achieve this aim have been prepared.

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