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Perturbative matching of heavy-light currents at one-loop

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We present results of a perturbative matching calculation performed at one-loop for heavy-light currents. We use the Fermilab action for the heavy quarks, the Asqtad action for the light quarks, and an improved gluon action. We also present results for heavy-heavy currents with Fermilab heavy quarks and improved glue.

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1. Introduction

The Fermilab Lattice and MILC collaboration's program includes calculations of the hadronic matrix elements for weak D and B meson decays, in particular, the decay constants f_D, f_{D_s}, f_B, and f_{B_s} and the semileptonic form factors for $B \to \pi\ell\nu$, $D \to \pi(K)\ell\nu$, and $B \to D^*\ell\nu$. In this work we present a perturbative matching calculation of the relevant current renormalizations to one-loop order. The numerical simulations for the above physics analyses use MILC ensembles with improved glue and $2+1$ Asqtad staggered sea quarks [1]. The light valence quarks are also generated from Asqtad staggered quarks and converted to naive quarks. The heavy (charm and beauty) quarks are treated with the Fermilab action. See Ref. [2] for more details on the actions and parameters used in the numerical simulations.

2. Definitions

In this work we follow the analysis of Ref. [3], where the one-loop corrections to heavy-light and heavy-heavy current renormalizations were calculated for Fermilab heavy and Clover light quarks with Wilson glue.

The heavy-light currents have the form

$$J_\mu^{hl\,\mathrm{lat}} = \overline{\psi}_h \Gamma_\mu \psi_l \,, \tag{2.1}$$

where $\Gamma_\mu = \gamma_\mu$ or $\gamma_\mu\gamma_5$ and ψ_l denotes a naive Asqtad Dirac spinor. The Fermilab Dirac spinor, ψ_h, is rotated by

$$\psi_h = \psi\,[1 + ad_1\gamma\cdot\mathbf{D}] \,, \tag{2.2}$$

with the tree-level coefficient

$$d_1 = \frac{1}{2+m_0a} - \frac{1}{2(1+m_0a)} \,. \tag{2.3}$$

The heavy-heavy currents have the form

$$J_\mu^{hh'\,\mathrm{lat}} = \overline{\psi}_h \Gamma_\mu \psi_{h'} \,, \tag{2.4}$$

where now both spinors are rotated Dirac spinors. Since the heavy quarks are rotated, the lattice currents of Eqns. (2.1) and (2.4) include the leading order tree-level discretization corrections.

The current renormalization is defined as

$$Z_{J_\Gamma}^{hl} = \frac{(Z_{2h}^{(1/2)}\Lambda_{J_\Gamma}Z_{2l}^{(1/2)})^{\mathrm{cont}}}{(Z_{2h}^{(1/2)}\Lambda_{J_\Gamma}Z_{2l}^{(1/2)})^{\mathrm{lat}}} \,, \tag{2.5}$$

where Λ_{J_Γ} are the vertex corrections and Z_{2h} (Z_{2l}) are the heavy (light) quark wave function renormalizations.

We factor out the dominant mass dependence due to the tree-level wave function renormalization of the heavy Fermilab quark by defining the perturbative expansion as

$$e^{-m_1^{[0]}a/2} Z_{J_\Gamma}^{hl} = 1 + g_0^2 Z_{J_\Gamma}^{hl\,[1]} + \dots \,, \tag{2.6}$$

where the heavy quark masses are defined as usual,

$$m_1^{[0]}a = \log(1+m_0a)\,,\quad m_0a = 1/(2\kappa_h) - 1/(2\kappa_{crit})\,. \tag{2.7}$$

Since Z_{V_4} for degenerate masses is easy to calculate nonperturbatively, it is useful to define

$$\rho^{hl}_{J_\Gamma} \equiv \frac{Z^{hl}_{J_\Gamma}}{\sqrt{Z^{hh}_{V_4} Z^{ll}_{V_4}}} = 1 + g_0^2 \rho^{[1]}_{J_\Gamma} + \dots\,. \tag{2.8}$$

In this case, the dominant mass dependence cancels by construction.

Analogously, for heavy-heavy currents we have:

$$Z^{hh'}_{J_\Gamma} = \frac{(Z^{(1/2)}_{2h} \Lambda_{J_\Gamma} Z^{(1/2)}_{2h'})^{\rm cont}}{(Z^{(1/2)}_{2h} \Lambda_{J_\Gamma} Z^{(1/2)}_{2h'})^{\rm lat}}\,. \tag{2.9}$$

Taking the leading mass dependence out again, the perturbative expansion is defined as

$$e^{-(m^{[0]}_{1h} + m^{[0]}_{1h'})a/2} Z^{hh'}_{J_\Gamma} = 1 + g_0^2 Z^{hh'\,[1]}_{J_\Gamma} + \dots\,. \tag{2.10}$$

Finally, the ρ factors for heavy-heavy currents are defined as

$$\rho^{hh'}_{J_\Gamma} \equiv \frac{Z^{hh'}_{J_\Gamma}}{\sqrt{Z^{hh}_{V_4} Z^{h'h'}_{V_4}}} = 1 + g_0^2 \rho^{hh'\,[1]}_{J_\Gamma} + \dots\,. \tag{2.11}$$

3. Procedure

In this work we use the automated perturbation theory techniques developed by Lüscher and Weisz [4] to generate the Feynman rules for the lattice actions. We then integrate the loop diagrams by “brute-force” using VEGAS [5]. The advantage of using automated perturbation theory is that it is relatively easy to switch actions [6]. Indeed, we have results for the current renormalizations for two gluon actions, two light quark actions and the heavy quark action.

The one-loop diagrams for the vertex corrections (including the rotations) are given in Ref. [3]. We have performed the following tests of our calculation:

- For the automated perturbation theory code, we have compared our vertices and propagators against known results.
- We have written two independent programs for calculating the current renormalizations based on the automated perturbation theory code.
- We have a third independent calculation of the current renormalizations using traditional semi-analytic methods.
- Our results for the heavy-heavy currents agree with those of Ref. [3] when we switch from the improved gluon propagator to the Wilson gluon propagator. We also reproduce the results of Ref. [3] for heavy-light currents with Clover light quarks and Wilson glue.
- Our result for the Asqtad naive wave function renormalization agrees with Ref. [7], and our result for the naive-naive vertex correction with Wilson glue agrees with Ref. [8].

4. Results

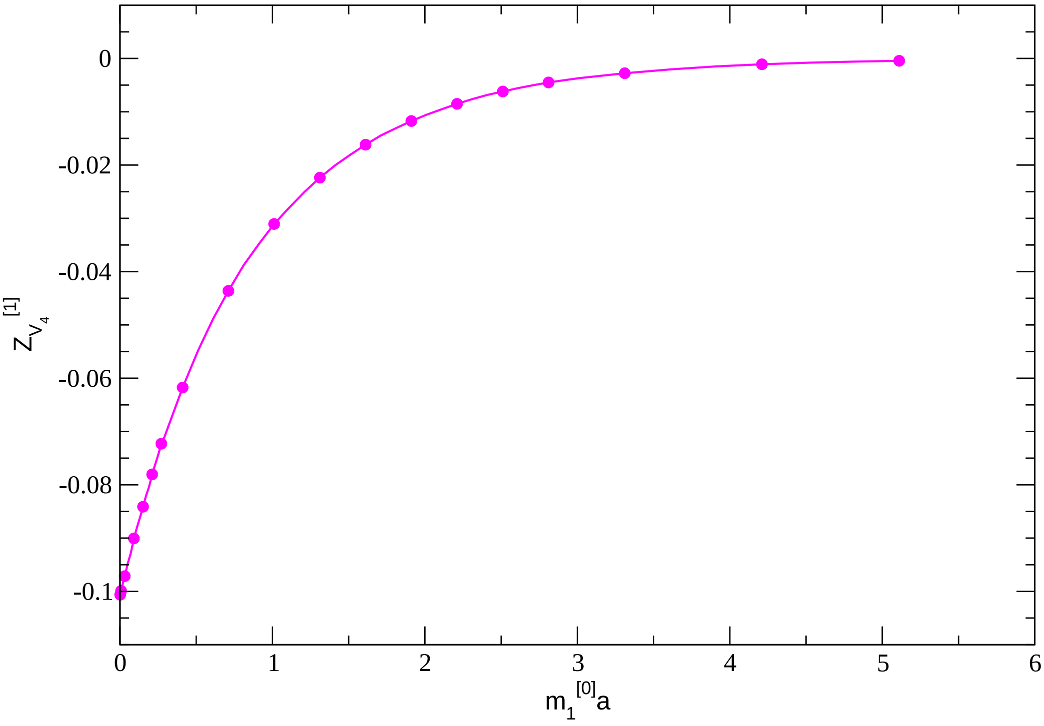

Figure 1: $Z_{V_4}^{hh\,[1]}$ for equal masses as a function of $m_1^{[0]}$.

Figures 1–4 illustrate our results for the Z's, ρ's, and q^*'s as functions of the heavy-quark mass, $m_1^{[0]}a$. The q^*'s are calculated from the log moments using Eqn. (13) of Ref. [9].

Our results for the heavy-heavy currents are very similar to those of Ref. [3], since they differ only in the gluon propagator. The main features of the mass dependence are the same. Figures 1–2 show results for the degenerate mass V_4 current. We also have results for the other currents (V_i, A_4, A_i) as well as results for currents with unequal masses. In the massless limit we find

$$Z_{V_4}^{hh\,[1]}(m_1^{[0]} = 0) = -0.10056\,(3)\,, \tag{4.1}$$

in good agreement with Ref. [10]. This is another test of our calculation.

Figure 3 shows a comparison of the current renormalization of the heavy-naive A_4 current with the corresponding ρ factor, and Figure 4 shows $\rho_{V_4}^{hl\,[1]}$ and $\rho_{V_i}^{hl\,[1]}$. First, the general features of the heavy-quark mass dependence are similar to the results of Ref. [3]. Second, $\rho_{A_4}^{hl\,[1]}$ is significantly smaller than $Z_{A_4}^{hl\,[1]}$ over the relevant mass range. Hence, the cancellation between the numerator and denominator of Eq. (2.8) already observed in Ref. [3] also takes place for heavy-naive currents.

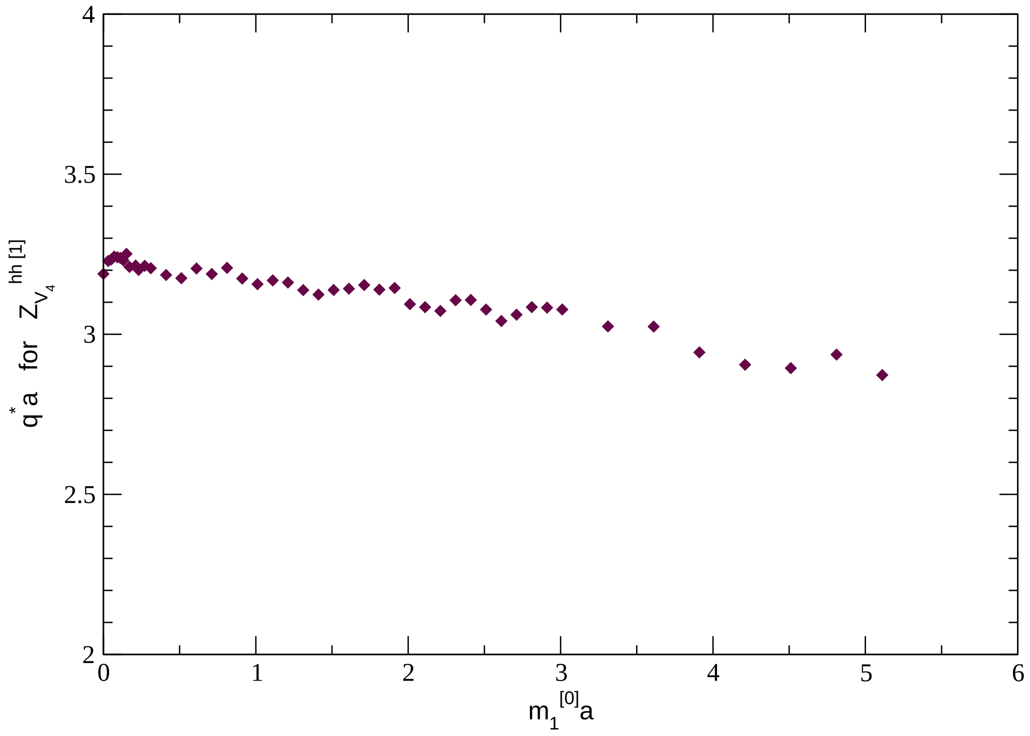

Figure 2: q^*a for $Z_{V_4}^{hh\,[1]}$ for equal masses as a function of $m_1^{[0]}$.

In the massless limit we find:

$$\rho_{V_4}^{hl\,[1]}(m_1^{[0]} = 0) \;=\; -3.038\,(2)\cdot 10^{-3} \qquad (4.2)$$
$$\rho_{V_i}^{hl\,[1]}(m_1^{[0]} = 0) \;=\; -3.05\,(5)\cdot 10^{-3}$$

We also have results for the naive V_4 current renormalization. In the massless limit we find

$$Z_{V_4}^{ll\,[1]}(m_0 = 0) = -0.10457\,(4). \qquad (4.3)$$

We have studied the mass dependence of $Z_{V_4}^{ll\,[1]}$ by varying m_0 between zero and the strange quark mass. We find that $Z_{V_4}^{ll\,[1]}$ is essentially independent of m_0.

In summary, we have calculated the current renormalizations relevant for the numerical analyses of heavy-light decay constants and semileptonic form factors performed by the Fermilab Lattice and MILC collaborations. We calculate the full mass dependence of the Z's and ρ's. The one-loop corrections to the ρ factors are small. They vary roughly between 0.4% and 4%, depending on lattice spacing.

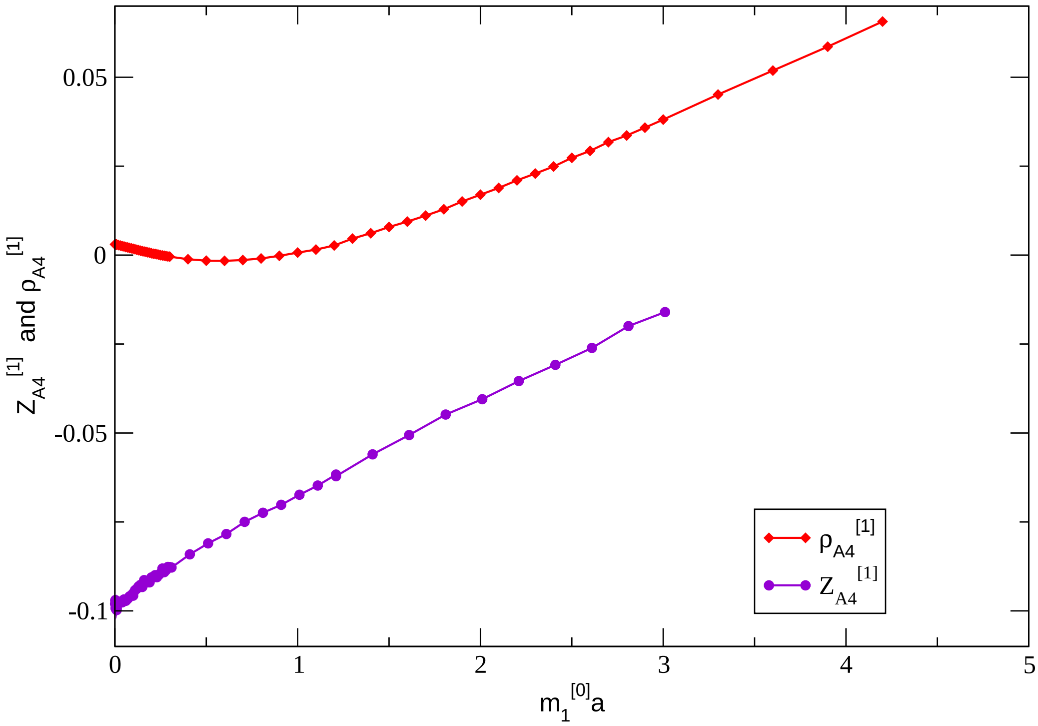

Figure 3: Comparison of $Z_{A_4}^{hl\,[1]}$ with $\rho_{A_4}^{hl}$

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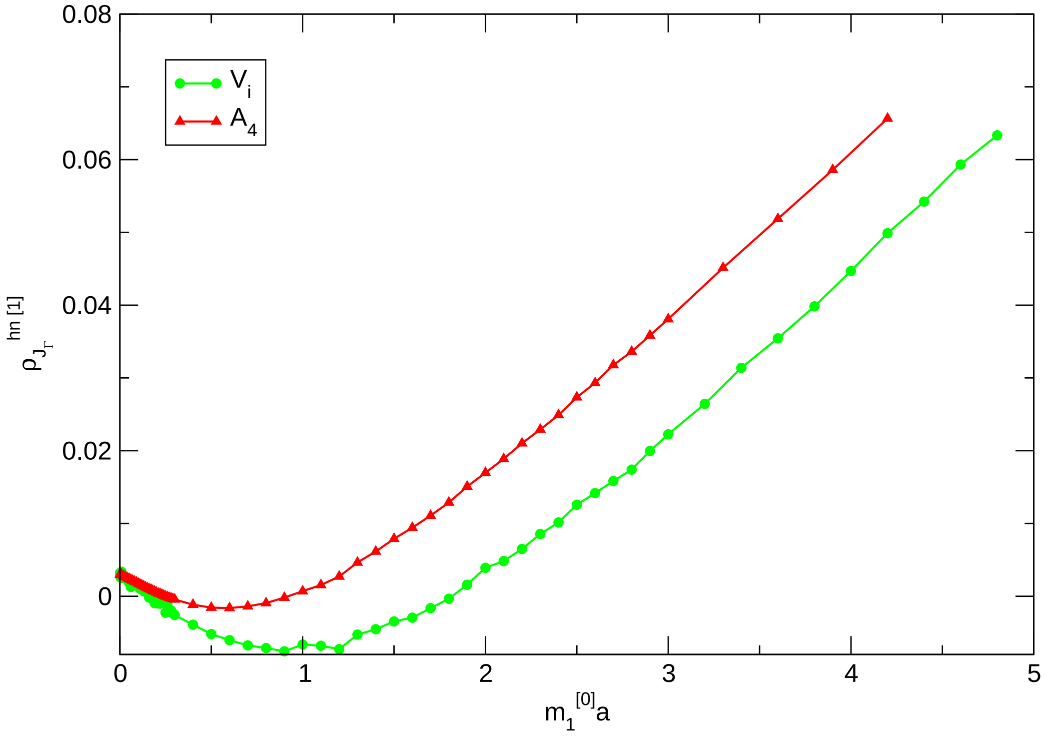

Figure 4: ρ factors for heavy-naive currents

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