PROCEEDINGS OF SCIENCE

High Energy Scattering in the AdS dual to "QCD"

Richard C. Brower*
Physics Department, Boston University, Boston MA 02215
E-mail: brower@bu.edu

AdS string duals to QCD-like theories are beginning to shed new light on high energy scattering of hadrons. Here we discuss the beginning of the unitarization program for high energy scattering based on String/Gauge duality. The eikonal expansion for the strong coupling Pomeron is presented, which when applied to a confining background metric respects and saturates the Froissart bound. On a technical level, we expose the central role of SL(2,C) symmetry for the strong coupling Pomeron kernel in the conformal limit.

The XXV International Symposium on Lattice Field Theory
July 30-4 August 2007
Regensburg, Germany

*Speaker.

PoS(LATTICE 2007)260

1. Introduction

The subject of high energy scattering for hadrons has a long history, predating both QCD and string theory. Here I wish to report on progress toward a fuller understanding of this limit. However the reader should be warned that this short presentation will simplify a rather technical and complex subject and she or he is referred to references in the bibliography of recent papers [1, 2, 3, 4] for much more careful and cautious assessment. In the context of QCD, there is a fundamental question that should have a definite answer. QCD is apparently the correct theory of the strong nuclear force with confinement and a mass gap. It is a self-consistent unitary theory with a well-defined S-matrix, which is UV complete for any number of colors $N > 1$ and a limited number of fundamental quark flavors. (The flavor constraint requires $n_f < 11N/2$ to maintain asymptotic freedom and a more stringent upper bound on n_f to avoid the Banks-Zaks conformal IR fixed point.) Consequently, in the absence of all other interactions, one can in principle determine properties of QCD at arbitrarily high energies. For example, in three flavor QCD, we may ask what is the exact asymptotic form for the high energy limit for the total cross section for scattering any of the stable hadrons (pion, kaon, nucleon etc). Specifically, the celebrated Froissart theorem from 1961 gives a rigorous bound,

$$\sigma_{Tot}(p+p \to X) \leq m_p^2 C_{pp}(m_\pi/m_p) log^2(s/s_0) \ , \tag{1.1}$$

when applied to the total pp cross section, as the center of mass energy $E = \sqrt{s}$ goes to infinity. Even in the pure glue sector ($n_f = 0$), a similar theoretical bound must hold for glueball scattering, $\sigma_{Tot} \leq \Lambda_{qcd}^2 C_0 \log(s/s_0)$, where C_0 is a dimensionless constant.

Surprisingly after almost 50 years since the proof of the Froissart bound, we still are not certain that this bound is saturated and if so how to compute the coefficient $C_{pp}(m_\pi/m_p)$! What is the dependence of this coefficient in the chiral ($m_\pi \to 0$) limit? Questions like this not only pose a sharp theoretical challenge, they have significant phenomenological consequences. At very high energies such as in cosmic rays or even the LHC, the lack of a prediction of the QCD cross sections, makes it difficult to determine if new physics is responsible or not for the observed increase in the cross section. More generally one would like to know what distribution of multi-particles configurations dominate high energy hadronic scattering and the rate for the diffractive production of new particles such as the Higgs or new TeV spectra. We are still far from a clear picture, let alone quantitative control of these phenomena. Here let me report on the recent developments in this subject based on Maldacena's weak/strong duality relating Yang Mills theories to string theories in (deformed) Anti-de Sitter space.

2. Geometry of Pomeron Exchange

In the Regge theory the traditional approach to the leading high energy behavior is related to a rather mysterious object in the complex J-plane referred to as the **Pomeron**, whatever that is! In spite of the difficulty in computing the properties of the Pomeron exchange process, in QCD there is in principle a clean, albeit indirect, definition. By expanding the elastic amplitude for SU(N) QCD ,

$$A(s,t) = g_0^2 A_1(s,t,\lambda) + g_0^4 A_2(s,t,\lambda) + \cdots \tag{2.1}$$

in $g_0 \sim 1/N$ at fixed 'tHooft coupling $\lambda = g_{YM}^2 N$, we may adopt the definition:

$$\boxed{\text{Pomeron} \equiv \text{leading contribution at large } N \text{ to the vacuum exchange at large s and fixed t}}$$

While this may appear to be circular, there are some definite consequences. Both weak coupling SU(N) QCD and strong coupling dual string theory identify this leading $1/N_c^2$ term with the topology of a t-channel exchange flux tube corresponding to a single color trace operator in Yang Mills theory or a closed string in the dual description.

In weak coupling perturbation theory to first order in the 'tHooft coupling λ and all of order $(\lambda \log(s))^n$, the summation of QCD diagrams leads to the BFKL Pomeron kernel. This kernel is the solution to a t-channel Bethe Salpeter equation for exchanging two "Reggeized" gluons. Also to this order, the beta function is zero and so QCD maybe viewed as as a conformal theory. Consequently it is relevant to compare this result with recently identified strong coupling kernel using the AdS dual to $\mathcal{N} = 4$ super conformal Yang Mills theory [2]. Indeed the strong coupling result does exhibit a remarkable similarity to the BFKL kernel that can begin to shed light on this Pomeron kernel in the conformal limit for general 'tHooft coupling. Let us give a geometrical interpretation of this similarity.

Consider the Regge limit for a general n-particle scattering amplitude: $A(p_1, p_2, \cdots p_n)$. The rapidity gaps, $\ln(p_r^+ p_\ell^-)$, between any right- and left-moving particles are all $O(\log s)$, i.e., a large Lorentz boost, $\exp[yM_{+-}]$, with $y \sim \log s$, is required to switch from the frame co-moving with the left movers to the frame co-moving with the right movers. The J-plane is conjugate to rapidity, and as such is identified with the eigenvalue of the Lorentz boost generator M_{+-}. In the context of the AdS/CFT correspondence, consider the boost operator relative to the full $O(4,2)$ conformal group. In terms of transformations on light-cone variables, there are two interesting 6 parameter subgroups: The first is the well known collinear group $SL_L(2,R) \times SL_R(2,R)$ used in DGLAP for deep inelastic scattering, with generators,

$$SL_L(2,R),\ SL_R(2,R) \quad \text{generators:} \quad D \pm M_{+-}\ ,\ P_\pm\ ,\ K_\mp\ , \tag{2.2}$$

which corresponds in the dual AdS_5 bulk to isometries of the Minkowski AdS_3 light-cone sub-manifold. The second is $SL(2,C)$ (or Möbius invariance used in solving the weak coupling BFKL equations) with generators,

$$SL(2,C) \quad \text{generators:} \quad iD \pm M_{12}\ ,\ P_1 \pm iP_2\ ,\ K_1 \mp iK_2\ , \tag{2.3}$$

corresponding to the isometries of the Euclidean (transverse) AdS_3 subspace of AdS_5; Euclidean AdS_3 is the hyperbolic space H_3. Indeed $SL(2,C)$ is the subgroup generated by all elements of the conformal group that commute with the boost operator, M_{+-} and as such plays the same role as the little group which commutes with the energy operator P_0.

To understanding the origin of the SL(2,C) algebra, let us discuss the isometries of the Euclidean AdS_3 metric, $ds^2 = R^2[dz^2 + dwd\overline{w}]/z^2$, where the transverse subspace is $(w = x_1 + ix_2, z)$. The generators of the $SL(2,C)$ isometries of AdS_3 are

$$\begin{aligned} J_0 &= w\partial_w + \frac{1}{2} z\partial_z \quad , \quad J_- = -\partial_w \quad , \quad J_+ = w^2\partial_w + wz\partial_z - z^2\partial_{\overline{w}} \\ \overline{J}_0 &= \overline{w}\partial_{\overline{w}} + \frac{1}{2} z\partial_z \quad , \quad \overline{J}_- = -\partial_{\overline{w}} \quad , \quad \overline{J}_+ = \overline{w}^2\partial_{\overline{w}} + \overline{w}z\partial_z - z^2\partial_w\ . \end{aligned} \tag{2.4}$$

The singularities in the J-plane must be determined by the eigenvalues of the boost operator, which for our *AdS* Pomeron[1] is approximated by $M_{+-} = 2 - H_{+-}/(2\sqrt{\lambda}) + O(1/\lambda)$ to leading order in strong coupling. Indeed we note that the strong coupling Pomeron kernel, $\mathscr{K}(j, x^{\perp} - x'^{\perp}, z, z') = (zz'/R^4) G_3(j, v)$, is directly written in terms of the AdS_3 Green's function ,

$$G_3(j,v) = \frac{1}{4\pi} \frac{\left[1 + v + \sqrt{v(2+v)}\right]^{(2-\Delta_+(j))}}{\sqrt{v(2+v)}} \;, \tag{2.5}$$

which is the solution to the boost equation at strong coupling,

$$[H_{+-} + 2\sqrt{\lambda}(j-2)] G_3(j,v) = z^3 \delta(z - z') \delta^2(x_\perp - x'_\perp) \;. \tag{2.6}$$

As as a consequence of SL(2,C) invariance $G_3(j,v)$ depends only on the AdS_3 chordal distance, $v = ((x_\perp - x'_\perp)^2 + (z - z')^2)/2zz'$ and the AdS_3 conformal dimension, $\Delta_+(j) - 1$,

$$\Delta_+(j) = 2 + \sqrt{4 + 2\sqrt{\lambda}(j-2)} = 2 + \sqrt{2\sqrt{\lambda}(j - j_0)} \;. \tag{2.7}$$

The analytic continuation from DGLAP to BFKL operators has been discussed at weak coupling for some time. The demonstration of this relationship in all large-λ conformal theories, and the derivation of the formula (2.7), is given in section 3 of [2], where $\Delta_+(j) = 2$ at $j = j_0$ (the BFKL exponent) and $\Delta_+(j) = 4$ at $j = 2$ (for the energy-momentum tensor, the first DGLAP operator) was demonstrated. For clarity, we reproduce Fig. 1 from [2] showing the essential form of this function for large and small λ.

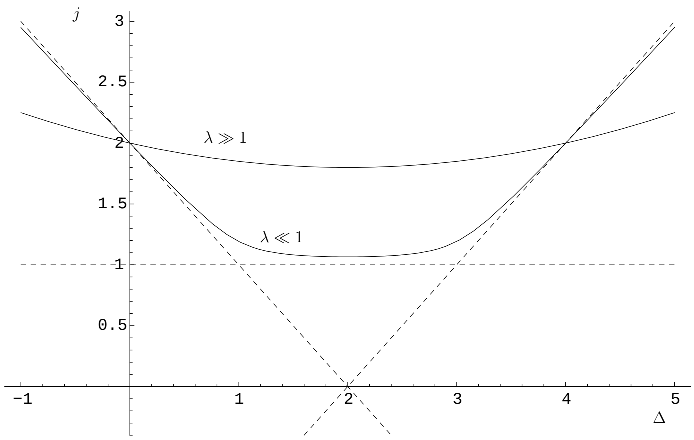

Figure 1: Schematic form of the $\Delta - j$ relation for $\lambda \ll 1$ and $\lambda \gg 1$. The dashed lines show the $\lambda = 0$ DGLAP branch (slope 1), BFKL branch (slope 0), and inverted DGLAP branch (slope -1). Note that the curves pass through the points (4,2) and (0,2) where the anomalous dimension must vanish. This curve is often plotted in terms of $\Delta - j$ instead of Δ, but this obscures the inversion symmetry $\Delta \to 4 - \Delta$.

[1] In Ref. [2] the eigenvalue condition $M_{+-} = j$ was also identified with the on-shell condition for the world sheet dilatation: $L_0 + \bar{L}_0 - 2 = 0$. Here we are concerned with the target space isometries.

With $H_{+-} = 3 - 2J^2 - 2\bar{J}^2$ expressed in terms of $SL(2,C)$ Casimirs, we are led directly to the J-plane spectrum, $j(\nu) = j_0 - \mathscr{D}\nu^2 + 0(\nu^4)$, and as first pointed out in Ref. [2] the strong coupling BFKL intercept is $j_0 = 2 - 2/\sqrt{\lambda}$ and the diffusion constant is $\mathscr{D} = 2/\sqrt{\lambda}$.

It is interesting to note that this structure is similar to the weak coupling one-loop n_g gluon BFKL spin chain operator in the large N limit. Here the boost operator is approximated by $M_{+-} = 1 - (\alpha N/\pi)H_{\rm BFKL}$, where $H_{\rm BFKL} = \frac{1}{4}\sum_{i=1}^{n_g}[\mathscr{H}(J^2_{i,i+1}) + \mathscr{H}(\bar{J}^2_{i,i+1})]$ is a sum over two-body operator with holomorphic and anti-holomorphic functions of the Casimir. The Yang Mills coupling is defined as $\alpha = g^2_{YM}/4\pi$. Even numbers of gluons (n_g) contribute to the BFKL Pomeron with charge conjugations $C = +1$ and the odd number of gluons to the so called "odderon" with charge conjugations $C = -1$. The consequence for the leading J-plane singularity in the two gluon channel is now,

$$j(\nu) = j_0 - \mathscr{D}\nu^2 + 0(\nu^4) \,, \tag{2.8}$$

with $j_0 = 1 + 4\ln 2\alpha N/\pi$ and $\mathscr{D} = 14\zeta(3)\alpha N/\pi$.

Let us note some differences between the strong-coupling and weakbrained-coupling limits. First, j_0 moves from 1 to 2 as λ moves from small to large. Also, the formulas for $j(\nu)$ given above have different regimes of validity; at strong coupling the energy-momentum tensor at $j = 2$ (along with the nearby $j \sim 2$ DGLAP operators) lies within the region of validity of the strong-coupling expression, while the explicit factor of λ in M_{+-} means the weak coupling BFKL result breaks down before $j = 2$. In strong coupling perhaps one should visualize the Pomeron as the exchange of single trace planar diagram with an infinite number of t-channel gluons whose interactions are approximated via a mean field approximation.

3. The eikonal approximation

We now turn to the problem of the eikonal summation of multiple Regge exchange graphs for the AdS_5 strong coupling Pomeron. The standard eikonal formula takes the classic form,

$$A(s,t) = -2is \int d^2b e^{-ib^\perp q_\perp} \left[e^{i\chi(s,b^\perp)} - 1\right] \,, \tag{3.1}$$

where $t = -q^2_\perp$. For the Regge pole model of the Pomeron exchange, $\chi(s,b^\perp)$ is the Fourier transform to impact parameter space of the elastic amplitude in the one-Reggeon exchange approximation,

$$\chi(s,b^\perp) = \frac{1}{2s}\int \frac{d^2q_\perp}{(2\pi)^2} e^{ib^\perp q_\perp} A^{(1)}(s,t) \,, \tag{3.2}$$

with $A^{(1)}(s,t) = -[(e^{-i\pi\alpha(t)} \pm 1)/\sin\pi\alpha(t)]\beta(t)s^{\alpha(t)}$. This is the leading contribution to the sum of graphs depicted in Fig. 2 below. Let us compare this with our result for the eikonalization of the AdS_5 graviton of Ref. [4]

$$A_{2\to 2}(s,t) \simeq -2is \int d^2b\, e^{-ib^\perp q_\perp} \int dz dz' P_{13}(z) P_{24}(z') \left[e^{i\chi(s,b^\perp,z,z')} - 1\right] \tag{3.3}$$

where $b = x^\perp - x'^\perp$ due to translational invariance. The salient new features relative to the above four-dimensional expressions are the new transverse co-ordinate for the fifth dimension in AdS_5 and the product of wave functions for left-moving $(1 \to 3)$ and right-moving $(2 \to 4)$ states,

$$P_{13}(z) = (z/R)^2\sqrt{g(z)}\Phi_1(z)\Phi_3(z) \qquad \text{and} \qquad P_{24}(z) = (z'/R)^2\sqrt{g(z')}\Phi_2(z')\Phi_4(z') \tag{3.4}$$

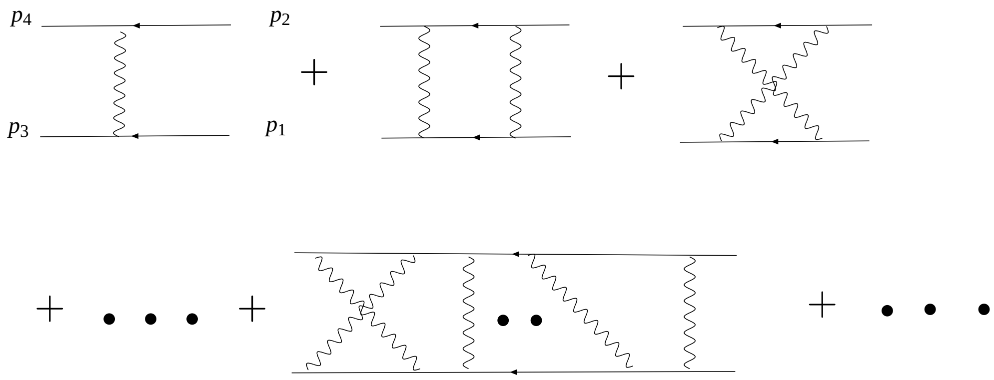

Figure 2: Ladder and crossed ladder diagrams contributing to the eikonal approximation in the high energy limit.

The obvious (and correct) guess for the eikonalization of AdS_5 Pomeron is to simply use the appropriate AdS_3 kernel given by inverse Mellin transform of the J-plane kernel presented above,

$$\chi(s, x^{\perp} - x'^{\perp}, z, z') = \frac{g_0^2 R^4}{2(zz')^2 s} \mathscr{K}(s, x^{\perp} - x'^{\perp}, z, z') \;, \tag{3.5}$$

where $g_0^2 = \kappa_5^2 / R^3$. This is a natural generalization of our earlier result for AdS graviton exchange [4], whose kernel can be obtained by taking the limit $\lambda \to \infty$.

3.1 Frozen String Bits in Flat Space

It is also interesting to compare our strong coupling results in AdS space with the eikonal formula of Amati, Ciafaloni and Veneziano [6] for the superstring in flat space. The flat space solution does not require a truncation of the infinite number of normal modes of a full string world sheet description, so similarities with the general mechanism for eikonalization in string theory found in our strong coupling AdS example suggest further generalization beyond strong coupling. In flat space the superstring eikonal phase $\widehat{\chi}$ is a matrix for all 2 to 2 particle scattering amplitudes in the planar approximation. Similar to our AdS_5 eikonal amplitude, this matrix can be re-expressed geometrically, this time by a change of basis to an infinite dimensional "impact parameter" space for the transverse positions of individual string "bits" $x_\perp(\sigma)$ of the colliding strings:

$$T_4 \sim -2is \int \mathscr{D}x_\perp \mathscr{D}x'_\perp d^{D-2} b_\perp P_{13}[x_\perp(\sigma)] P_{24}[x'_\perp(\sigma')] e^{ib_\perp q^\perp} \left[e^{i\chi(s, b_\perp; x_\perp, x'_\perp)} - 1 \right] . \tag{3.6}$$

$P_{31}[x_\perp(\sigma)] = |\Phi[x_\perp(\sigma)]|^2$ and $P_{42}[x'_\perp(\sigma')] = |\Phi[x'_\perp(\sigma')]|^2$ are then expressed as the square of Gaussian wavefunctionals [2],

$$\Phi[x_\perp(\sigma)] = \langle x_\perp(\sigma) | 0; 0 \rangle = \exp\Big[-\frac{1}{16\pi^2 \alpha'} \oint d\sigma_1 \oint d\sigma_2 \frac{x_\perp(\sigma_1) x_\perp(\sigma_2)}{\sin^2(\frac{\sigma_1 - \sigma_2}{2}) + \varepsilon^2} \Big] \;, \tag{3.7}$$

for the overlap of the string vacuum state, $|0;0\rangle$, and the string bit distribution at the time of impact $x^+ = 0$.

Thus we see that the geometrical extension of the transverse dimensions that we saw above, where the KK radial mode z allowed us to rewrite a multi-channel problem in four dimensions using a transverse AdS_3, has an analogue here. For the string, the exact flat space eikonal amplitude, a multi-channel problem involving a tower of massive string states, is diagonalized using an infinite dimensional space which is a product of transverse impact-parameter spaces, one for each string bit. During the collision, each string bit interacts instantaneously in light-cone time $X^+ = \tau$ undergoing *zero* deflection. The string bits are frozen.

4. Future directions

While our discussion above has emphasized results for the conformal theory, the eikonal expression still holds for confining backgrounds with the appropriate kernel. For example the hardwall model with a cut-off at $z_{IR} = 1/\Lambda_{qcd}$ in the IR region is again an AdS_3 Green function with appropriate boundary condition at z_{IR}. However the consequences are important. We now have a confining QCD-like dual with a discrete spectrum. This allows one to argue that the eikonal contribution to the total cross section respects and saturates the Froissart bound. One future goals is to show that the linearity approximation for the eikonal sum holds for a sufficiently large region in impact parameter space to prove saturation of the Froissart bound in strong coupling confining gauge theories. After this, we plan to identify the specific non-linear contributions due to Pomeron splitting as for example in the triple Pomeron coupling. In extreme strong coupling limit ($\sqrt{\lambda}/\log(s) \to \infty$), the dual theory is the Einstein-Hilbert action in a curved (AdS like) background. By isolating the leading contribution in the gravity limit first, we can proceed systematically to introduce the $1/\sqrt{\lambda}$ contributions to guide the development of a dual Reggeon effective field theory analogous to the earlier Gribov calculus. These are ambitious goals but ones that have real promise to bring new clarity to high energy hadronic physics.

Acknowlegements: I wish to acknowledge my collaborators Joe Polchinski, Matt Strassler and Chung-I Tan, to whom I am much indebted.

References

[1] J. Polchinski and M.Strassler, “Deep inelastic scattering and gauge/string duality” JHEP 05 (2003) 012.

[2] R.C. Brower,J. Polchinski, M. Strassler and C-I Tan, “The Pomeron and Gauge / String Duality” hep-th/0603115 (2006).

[3] L. Cornalba, M. S. Costa and J. Penedones, “Eikonal Approximation in AdS/CFT: Resuming the Gravitational Loop Expansion” arXiv:0707.0120 (2007).

[4] R.C. Brower, M. Strassler and C-I Tan, “On the Eikonal Approximation in AdS Space” arXiv:0707.2408 (2007)

[5] M. Froissart, “Asymptotic Behavior and Subtractions in the Mandelstam Representation” Phys. Rev. 123 (1961)1053.

[6] D. Amati, M. Ciafaloni and G. Veneziano “Classical and Quantum Gravity Effects from Planckian Energy Superstring Collisions” Int. J. Mod. Phys. A4 (1988) 1615-1661.

PoS(LATTICE 2007)260