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Lattice Formulation of the N=4 D=3 Twisted Super Yang-Mills

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A lattice formulation of a three dimensional super Yang-Mills model with a twisted $N = 4$ supersymmetry is proposed. The extended supersymmetry algebra of all eight supercharges is fully and exactly realized on the lattice with a modified "Leibniz rule". The formulation we employ here is a three dimensional extension of the manifestly gauge covariant method which was developed in our previous proposal of Dirac-Kähler twisted $N = 2$ super Yang-Mills on a two dimensional lattice. The twisted $N = 4$ supersymmetry algebra is geometrically realized on a three dimensional lattice with link supercharges and the use of "shifted" (anti-)commutators. A possible solution to the recent critiques on the link formulation will be discussed.

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1. Introduction

Formulating an exact supersymmetric model on a lattice is one of the most challenging subjects in lattice field theory. There has been already a number of works addressing this topic. Recently, it has been recognized that the so-called twisted version of supersymmetry (SUSY) plays a particularly important role in formulating supersymmetric models on a lattice (See Ref. [1, 2, 3] and references therein.) The crucial importance of twisted SUSY on the lattice could be traced back to the intrinsic relation between twisted fermions and Dirac-Kähler fermions. Based on this recognition, we proposed lattice formulations of the $D = N = 2$ super BF and Wess-Zumino models [1] as well as the $D = N = 2$ twisted super Yang-Mills (SYM) [2] by explicitly constructing the Dirac-Kähler twisted $N = 2$ SUSY algebra on a two dimensional lattice. The main feature of our formulation is that the "Leibniz rule" on the lattice can be exactly maintained throughout the formulation, and as a result, the lattice action is invariant w.r.t. all the supercharges associated with the twisted SUSY algebra. It has been also recognized in [2] that, besides the twisted $D = N = 2$ algebra, also the Dirac-Kähler twisted $D = N = 4$ SUSY algebra could be realized on the lattice with the lattice Leibniz rule. Recently, we pointed out that the $D = 3$ $N = 4$ twisted SUSY algebra, which has eight supercharges, can also be consistent with the lattice Leibniz rule conditions and then we proposed an explicit construction of the corresponding SYM action on the lattice [3], which is the main topic of this proceeding.

In recent papers the authors of [4, 5] posed some critiques on our formulations of the noncommutative approach [1] and of the link approach [2]. A possible answer to the critique on the noncommutative approach[1] will be given by the analysis of a matrix formulation of superfields[6]. Along a similar line of arguments, we propose a possible answer to the critiques in the link approach case.

2. Discretization of $N = 4$ twisted SUSY algebra in three dimensions

We first introduce the following $N = 4$ SUSY algebra in a Euclidean three dimensional continuum spacetime.

$$\{Q_{\alpha i}, \overline{Q}_{j\beta}\} = 2\delta_{ij}(\gamma_\mu)_{\alpha\beta}P_\mu \tag{2.1}$$

where the gamma matrices, γ_μ, can be taken as Pauli matrices, $\gamma^\mu(\mu = 1,2,3) \equiv (\sigma^1, \sigma^2, \sigma^3)$. $\overline{Q}_{i\alpha}$ can be taken as the complex conjugation of $Q_{\alpha i}$, $\overline{Q}_{i\alpha} = Q^*_{\alpha i} = Q^\dagger_{i\alpha}$ in the continuum spacetime.

The twisting procedure can be performed by introducing the twisted Lorentz generator as a diagonal sum of the original Lorentz and the internal rotation generators. The resulting algebra is most naturally expressed in terms of the following Dirac-Kähler expansion of the supercharges on the basis of the gamma matrices,

$$Q_{\alpha i} = (\mathbf{1}Q + \gamma_\mu Q_\mu)_{\alpha i}, \qquad \overline{Q}_{i\alpha} = (\mathbf{1}\overline{Q} + \gamma_\mu \overline{Q}_\mu)_{i\alpha}, \tag{2.2}$$

where $\mathbf{1}$ represents a two-by-two unit matrix. The coefficients of the above expansions, $(Q, \overline{Q}_\mu, Q_\mu, \overline{Q})$, are called twisted supercharges of $N = 4$ in a three dimensional continuum spacetime. After the twisting and the expansions, the original SUSY algebra (2.1) can be expressed as,

$$\{Q, \overline{Q}_\mu\} = P_\mu, \qquad \{Q_\mu, \overline{Q}_\nu\} = -i\epsilon_{\mu\nu\rho}P_\rho, \qquad \{\overline{Q}, Q_\mu\} = P_\mu, \tag{2.3}$$

where $\varepsilon_{\mu\nu\rho}$ is the three dimensional totally anti-symmetric tensor with $\varepsilon_{123} = +1$.

Since we have only finite lattice spacings on a lattice, infinitesimal translations should be replaced by finite difference operators, $P_\mu = i\partial_\mu \to i\Delta_{\pm\mu}$, where $\Delta_{\pm\mu}$ denote forward and backward difference operators, respectively. We locate $\Delta_{\pm\mu}$ on links from x to $x \pm n_\mu$, respectively, and define their operations as link commutators with shifts of $\pm n_\mu$. Correspondingly, we locate the supercharges Q_A on a link from x to $x + a_A$ and define its operation as a link (anti-)commutator with a shift a_A. Through these procedures, one could construct the lattice counterpart of the SUSY algebra provided a certain type of Leibniz rule conditions hold. It has been shown that the Dirac-Käher twisted type of the $N = D = 2$ and the $N = D = 4$ SUSY algebra could be consistently realized on the lattice [1, 2]. Furthermore, we recently pointed out that the $N = 4$ $D = 3$ Dirac-Käher twisted algebra could also be formulated consistently on the lattice and be expressed as [3],

$$\{Q, \overline{Q}_\mu\} = +i\Delta_{+\mu}, \qquad \{Q_\mu, \overline{Q}_\nu\} = +\varepsilon_{\mu\nu\rho}\Delta_{-\rho}, \qquad \{\overline{Q}, Q_\mu\} = +i\Delta_{+\mu}, \tag{2.4}$$

where the anti-commutators of the l.h.s are understood as link anti-commutators, for example,

$$\{Q, \overline{Q}_\mu\}_{x+a+\overline{a}_\mu,x} = Q_{x+a+\overline{a}_\mu,x+\overline{a}_\mu}\overline{Q}_{x+\overline{a}_\mu,x} + \overline{Q}_{x+\overline{a}_\mu+a,x+a}Q_{x+a,x}. \tag{2.5}$$

The corresponding Leibniz rule conditions,

$$a + \overline{a}_\mu = +n_\mu, \qquad a_\mu + \overline{a}_\nu = -|\varepsilon_{\mu\nu\rho}|n_\rho, \qquad \overline{a} + a_\mu = +n_\mu, \tag{2.6}$$

could be consistently satisfied by the following generic solutions,

$$a = (arbitrary), \qquad \overline{a}_\mu = +n_\mu - a, \qquad a_\mu = -\sum_{\lambda\neq\mu} n_\lambda + a, \qquad \overline{a} = +\sum_{\lambda=1}^{3} n_\lambda - a. \tag{2.7}$$

Notice that there is one vector arbitrariness in the choice of a_A, which governs the possible configurations of the three dimensional lattice. The typical examples are the symmetric choice (Fig.1) and the asymmetric choice (Fig.2). Notice that the summation of all the shift parameters $(a, \overline{a}_\mu, a_\mu, \overline{a})$ vanish,

$$\sum a_A = a + \overline{a}_1 + \overline{a}_2 + \overline{a}_3 + a_1 + a_2 + a_3 + \overline{a} = 0, \tag{2.8}$$

regardless of any particular choice of a_A.

3. Lattice formulation of the twisted $N = 4$ SYM in three dimensions

Based on the arguments in the previous section, We now proceed to construct the $D = 3$ $N = 4$ twisted SYM action on a Euclidean lattice. We first introduce fermionic and bosonic gauge link variables, ∇_A and $\mathscr{U}_{\pm\mu}$ which are located on links $(x + a_A, x)$ and $(x \pm n_\mu, x)$, respectively, just like Q_A and $\Delta_{\pm\mu}$. The gauge transformations of those link operators are given by,

$$(\nabla_A)_{x+a_A,x} \to G_{x+a_A}(\nabla_A)_{x+a_A,x}G_x^{-1}, \qquad (\mathscr{U}_{\pm\mu})_{x\pm n_\mu,x} \to G_{x\pm n_\mu}(\mathscr{U}_{\pm\mu})_{x\pm n_\mu,x}G_x^{-1}, \tag{3.1}$$

where G_x denotes the finite gauge transformation at the site x. Next we impose the following $D = 3$ $N = 4$ twisted SYM constraints on the lattice,

$$\{\nabla, \overline{\nabla}_\mu\}_{x+a+\overline{a}_\mu,x} = +i(\mathscr{U}_{+\mu})_{x+n_\mu,x}, \qquad \{\nabla_\mu, \overline{\nabla}_\nu\}_{x+a_\mu+\overline{a}_\nu,x} = -\varepsilon_{\mu\nu\rho}(\mathscr{U}_{-\rho})_{x-n_\rho,x}, \tag{3.2}$$

$$\{\overline{\nabla}, \nabla_\mu\}_{x+\overline{a}+a_\mu,x} = +i(\mathscr{U}_{+\mu})_{x+n_\mu,x}, \qquad \{others\} = 0, \tag{3.3}$$

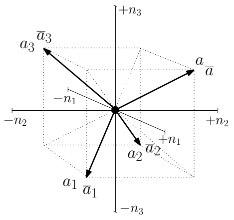

Figure 1: Symmetric choice of a_A

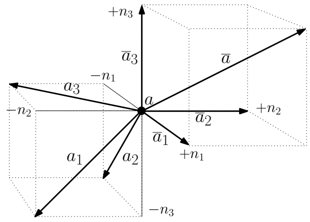

Figure 2: Asymmetric choice of a_A

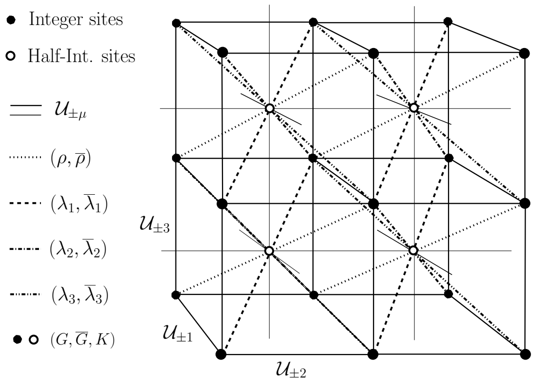

Figure 3: All the configurations in the $N=4$ $D=3$ twisted SYM action for symmetric a_A

	∇	$\overline{\nabla}_\mu$	∇_μ	$\overline{\nabla}$	$\mathscr{U}_{\pm\mu}$
shift	a	$\overline{a}_\mu$	a_μ	$\overline{a}$	$\pm n_\mu$

	ρ	$\overline{\lambda}_\mu$	λ_μ	$\overline{\rho}$	G	$\overline{G}$	K
shift	$-a$	$-\overline{a}_\mu$	$-a_\mu$	$-\overline{a}$	$a-\overline{a}$	$\overline{a}-a$	0

Table 1: Shifts carried by the link variables and the fields

where the left-hand sides should be understood as the link anti-commutators such as in (2.5).

Since the multiplet of the $D=3$ $N=4$ twisted SYM should contain three components of gauge fields as well as three components of scalar fields, we require that the above bosonic gauge link variables are to be defined in such a way to include the scalar contributions,

$$(\mathscr{U}_{\pm\mu})_{x\pm n_\mu,x} \equiv (e^{\pm i(A_\mu \pm \phi^{(\mu)})})_{x\pm n_\mu,x}, \tag{3.4}$$

where A_μ and $\phi^{(\mu)}(\mu=1,2,3)$ represent the hermitian three dimensional gauge field and the three components of the scalar field, respectively. Notice that the product of oppositely oriented bosonic gauge link variables does not give unity, $\mathscr{U}_{+\mu}\mathscr{U}_{-\mu} \neq 1$. It rather gives the contribution of the scalar fields. Once imposing the SYM constraints (3.2)-(3.3), we automatically obtain the entire information of lattice SYM multiplet through the analysis of Jacobi identities. It turns out that we have $N=4$ $D=3$ twisted fermions $(\rho,\overline{\lambda}_\mu,\lambda_\mu,\overline{\rho})$ and auxiliary fields $(G,\overline{G},K)$ besides the bosonic gauge link variables $\mathscr{U}_{\pm\mu}$. All the shift properties of the component fields are summarized in Table 1. The SUSY transformation of the twisted $N=4$ lattice gauge multiplet can be determined from the above Jacobi identity relations via

$$(s_A\varphi)_{x+a_A+a_\varphi,x} = (s_A)\varphi_{x+a_\varphi,x} \equiv [\nabla_A,\varphi\}_{x+a_A+a_\varphi,x}, \tag{3.5}$$

where $(\varphi)_{x+a_\varphi,x}$ denotes one of the component fields $(\mathscr{U}_{\pm\mu},\rho,\overline{\lambda}_\mu,\lambda_\mu,\overline{\rho},G,\overline{G},K)$. The results are summarized in Table 2. As a natural consequence of the constraints (3.2)-(3.3), one can see that the resulting $N=4$ $D=3$ twisted SUSY algebra for the component fields closes off-shell (modulo gauge transformations) on the lattice.

The construction of the twisted $D=3$ $N=4$ SUSY invariant action can be found by noticing the “chiral" and “anti-chiral" conditions of $\mathscr{U}_{\pm\mu}$,

$$S = +\sum_x \frac{1}{2}\bar{s}_1\bar{s}_2 s_1 s_2 \ \mathrm{tr}\ \mathscr{U}_{+3}\ \mathscr{U}_{+3} = -\sum_x \frac{1}{2} s\bar{s}_3\bar{s}s_3 \ \mathrm{tr}\ \mathscr{U}_{-3}\ \mathscr{U}_{-3} \tag{3.6}$$

	s	$\bar{s}_\mu$	s_μ	$\bar{s}$
$\mathcal{U}_{+\nu}$	0	$+i\epsilon_{\mu\nu\rho}\lambda_\rho$	$-i\epsilon_{\mu\nu\rho}\overline{\lambda}_\rho$	0
$\mathcal{U}_{-\nu}$	$+\overline{\lambda}_\nu$	$+\delta_{\mu\nu}\rho$	$+\delta_{\mu\nu}\overline{\rho}$	$+\lambda_\nu$
ρ	$-K+\frac{i}{2}[\mathcal{U}_{+\rho},\mathcal{U}_{-\rho}]$	0	$+\frac{1}{2}\epsilon_{\mu\rho\sigma}[\mathcal{U}_{-\rho},\mathcal{U}_{-\sigma}]$	$+\overline{G}$
$\overline{\lambda}_\nu$	0	$+i[\mathcal{U}_{+\mu},\mathcal{U}_{-\nu}]$ $+\delta_{\mu\nu}(K-\frac{i}{2}[\mathcal{U}_{+\rho},\mathcal{U}_{-\rho}])$	$-\delta_{\mu\nu}G$	$-\frac{1}{2}\epsilon_{\nu\rho\sigma}[\mathcal{U}_{+\rho},\mathcal{U}_{+\sigma}]$
λ_ν	$+\frac{1}{2}\epsilon_{\nu\rho\sigma}[\mathcal{U}_{+\rho},\mathcal{U}_{+\sigma}]$	$-\delta_{\mu\nu}\overline{G}$	$+i[\mathcal{U}_{+\mu},\mathcal{U}_{-\nu}]$ $-\delta_{\mu\nu}(K+\frac{i}{2}[\mathcal{U}_{+\rho},\mathcal{U}_{-\rho}])$	0
$\overline{\rho}$	$+G$	$-\frac{1}{2}\epsilon_{\mu\rho\sigma}[\mathcal{U}_{-\rho},\mathcal{U}_{-\sigma}]$	0	$+K+\frac{i}{2}[\mathcal{U}_{+\rho},\mathcal{U}_{-\rho}]$
G	0	$+\epsilon_{\mu\rho\sigma}[\mathcal{U}_{-\rho},\overline{\lambda}_\sigma]$ $+i[\mathcal{U}_{+\mu},\overline{\rho}]$	0	$-i[\mathcal{U}_{+\rho},\overline{\lambda}_\rho]$
$\overline{G}$	$-i[\mathcal{U}_{+\rho},\lambda_\rho]$	0	$-\epsilon_{\mu\rho\sigma}[\mathcal{U}_{-\rho},\lambda_\sigma]$ $+i[\mathcal{U}_{+\mu},\rho]$	0
K	$+\frac{i}{2}[\mathcal{U}_{+\rho},\overline{\lambda}_\rho]$	$+\frac{1}{2}\epsilon_{\mu\rho\sigma}[\mathcal{U}_{-\rho},\lambda_\sigma]$ $-\frac{i}{2}[\mathcal{U}_{+\mu},\rho]$	$+\frac{1}{2}\epsilon_{\mu\rho\sigma}[\mathcal{U}_{-\rho},\overline{\lambda}_\sigma]$ $+\frac{i}{2}[\mathcal{U}_{+\mu},\overline{\rho}]$	$-\frac{i}{2}[\mathcal{U}_{+\rho},\lambda_\rho]$

Table 2: SUSY trans. laws for the twisted $N=4$ $D=3$ lattice SYM multiplet $(\mathcal{U}_{\pm\mu},\rho,\overline{\lambda}_\mu,\lambda_\mu,\overline{\rho},G,\overline{G},K)$

$$\begin{aligned}
&= \sum_x \mathrm{tr}\Big[\ \frac{1}{4}[\mathcal{U}_{+\mu},\mathcal{U}_{-\mu}]_{x,x}[\mathcal{U}_{+\nu},\mathcal{U}_{-\nu}]_{x,x}+K^2_{x,x} \\
&\quad -\frac{1}{2}[\mathcal{U}_{+\mu},\mathcal{U}_{+\nu}]_{x,x-n_\mu-n_\nu}[\mathcal{U}_{-\mu},\mathcal{U}_{-\nu}]_{x-n_\mu-n_\nu,x}+G_{x,x+\overline{a}-a}\overline{G}_{x+\overline{a}-a,x} \\
&\quad +i(\overline{\lambda}_\mu)_{x,x+\overline{a}_\mu}[\mathcal{U}_{+\mu},\rho]_{x+\overline{a}_\mu,x}+i(\lambda_\mu)_{x,x+a_\mu}[\mathcal{U}_{+\mu},\overline{\rho}]_{x+a_\mu,x}+\epsilon_{\mu\nu\rho}(\lambda_\mu)_{x,x+a_\mu}[\mathcal{U}_{-\nu},\overline{\lambda}_\rho]_{x+a_\mu,x}\Big],
\end{aligned} \tag{3.7}$$

where the summation over x should cover integer sites as well as half-integer sites if one takes the symmetric choice of a_A (Fig.1), while for the asymmetric choice (Fig.2) it needs to cover only the integer sites. Due to this summation property, the order in the product of the supercharges is shown to be irrelevant up to total difference terms. Notice that the exact form w.r.t. all the supercharges and the nilpotency of each supercharge manifestly ensure the twisted $N=4$ SUSY invariance of the action. It is also important to note that each term in the action forms a closed loop, which ensures the manifest gauge invariance of the action. This property is originated from the vanishing sum of the shifts associated with the action,

$$\overline{a}_1+\overline{a}_2+a_1+a_2+n_3+n_3 = a+\overline{a}_3+\overline{a}+a_3-n_3-n_3 = 0, \tag{3.8}$$

which holds for any particular choice of a_A. The gauge invariance is thus maintained regardless of any particular choice of a_A. Fig.3 depicts all the field configurations in the action (3.7) in the case of the symmetric choice of a_A.

In proving the lattice SUSY invariance of the action by explicitly operating with the supercharge, we need to take care about the ordering of a product of component fields. This is related to the critique that the authors of Ref.[4] pointed out that a SUSY transformation on differently ordered products of the same component fields gives different expressions and thus is inconsistent. We claim that a particular ordering of component fields is chosen to be the proper ordering which leads to a correct lattice SUSY transformation. The proper ordering of a product of component fields inherits the ordering of a product of the original superfields [6]. An alternative ordering of a product of component fields which leads to a correct lattice SUSY transformation can be obtained from the proper ordering by shifting the coordinates of the interchanged component fields as if they were noncommutative:

$$(\phi_A)_{x+a_A+a_B,x+a_B}(\phi_B)_{x+a_B,x} = (-1)^{|\phi_A||\phi_B|}(\phi_B)_{x+a_A+a_B,x+a_A}(\phi_A)_{x+a_A,x}, \tag{3.9}$$

where the fields ϕ_A and ϕ_B carry a shift a_A and a_B, respectively. As far as the proper ordering of component fields is kept with the interchanging rule (3.9), a lattice SUSY transformation on a product of component fields gives a consistent transformation.

Another "inconsistency" posed in [5] is related to the link nature of the supercharge s_A and of the supercovariant derivative ∇_A. A SUSY transformation s_A on the action generates a link hole $(x+a_A,x)$ since all the terms in the action have a vanishing shift and thus are composed of closed loops. At first look a naive supercharge operation to the action leads to gauge variant terms since such terms have link holes. We claim that we need to introduce a covariantly constant fermionic parameter η_B which anti-commutes with all supercovariant derivatives in the shifted anti-commutator sense,

$$\{\nabla_A,\eta_B\}_{x+a_A-a_B,x} = (\nabla_A)_{x+a_A-a_B,x-a_B}(\eta_B)_{x-a_B,x}+(\eta_B)_{x+a_A-a_B,x+a_A}(\nabla_A)_{x+a_A,x} = 0, \tag{3.10}$$

where η_B has a shift $-a_B$ and thus can fill up the link holes to generate gauge invariant terms. We define the gauge transformation of the superparameter,

$$(\eta_A)_{x-a_A,x} \to G_{x-a_A}(\eta_A)_{x-a_A,x}G_x^{-1}. \tag{3.11}$$

We can then prove the exact SUSY invariance of the action by applying a shiftless combination of SUSY transformation $\eta_A s_A$ (no sum) to the action. The SUSY transformation of the component fields including this fermionic parameter is given by

$$(\eta_A s_A\varphi)_{x+a_\varphi,x} = (\eta_A)_{x+a_\varphi,x+a_\varphi+a_A}(s_A\varphi)_{x+a_\varphi+a_A,x}, \tag{3.12}$$

where the SUSY transformation $(s_A\varphi)_{x+a_\varphi+a_A,x}$ is defined by (3.5) and is given in Table 2.

The naïve continuum limit of the action (3.7) can be taken through the expansion of the gauge link variables (3.4) around unity. After using trace properties, one obtains the following continuum action,

$$\begin{aligned} S \to S_{cont} = \int d^3x \,\mathrm{tr}\Big[&\frac{1}{2}F_{\mu\nu}F_{\mu\nu}+K^2+G\overline{G} \\ &-[\mathscr{D}_\mu,\phi^{(\nu)}][\mathscr{D}_\mu,\phi^{(\nu)}]-\frac{1}{2}[\phi^{(\mu)},\phi^{(\nu)}][\phi^{(\mu)},\phi^{(\nu)}] \\ &-i\overline{\lambda}_\mu[\mathscr{D}_\mu,\rho]-i\lambda_\mu[\mathscr{D}_\mu,\overline{\rho}]+\varepsilon_{\mu\nu\rho}\lambda_\mu[\mathscr{D}_\nu,\overline{\lambda}_\rho] \\ &-\overline{\lambda}_\mu[\phi^{(\mu)},\rho]-\lambda_\mu[\phi^{(\mu)},\overline{\rho}]+i\varepsilon_{\mu\nu\rho}\lambda_\mu[\phi^{(\nu)},\overline{\lambda}_\rho]\Big], \end{aligned} \tag{3.13}$$

where $F_{\mu\nu} \equiv i[\mathscr{D}_\mu,\mathscr{D}_\nu]$ represents the field strength with $\mathscr{D}_\mu \equiv \partial_\mu - iA_\mu$, while $\phi^{(\mu)}(\mu=1,2,3)$ denote the three independent hermitian scalar fields in the twisted $N=4$ $D=3$ SYM multiplet in the continuum spacetime. One could see that the kinetic term and the potential term as well as the Yukawa coupling terms for the scalar fields naturally come up from the contributions of zero-area loops in the lattice action. The above action (3.13) is in complete agreement with the continuum construction of the $N=4$ twisted SYM in three dimensions.

4. Discussions

A fully exact SUSY invariant formulation of the twisted $N = 4$ SYM action on a three dimensional lattice is presented. Twisted $N = 4$ SUSY invariance is a natural consequence of the exact form of the action with respect to all the twisted supercharges up to surface terms. Possible answers to the critiques on the formulation of the link approach are given. It is pointed out that there is a proper ordering of a product of component fields which leads to the correct lattice SUSY transformation. We need to introduce superparameters which anti-commute with all the supercovariant derivatives. It would be important to find an explicit representation of such superparameters. We further have to accept that the structure behind the nature of the component fields which carry a shift and satisfy the relation (3.9) still remains to be better clarified. We consider that the lattice SUSY transformation can be defined only semilocally due to the next neighboring ambiguity of the difference operation and thus affects the ordering of component fields. Superfields may be able to take care of this semilocal nature of SUSY transformation faithfully[6].

Although we have not addressed the issue of hermiticity in detail, it is possible to understand hermiticity properties and Majorana nature of fermions in the two dimensional formulation[7]. We recognize that hermiticity properties of lattice SYM should be clarified through a better geometrical understandings of chirality on the lattice. It should also be mentioned that a dimensional reduction of the three dimensional $N = 4$ twisted SYM could give us a formulation of the $N = 4$ twisted SYM on a two dimensional lattice, which corresponds to a double charged system of the $N = D = 2$ twisted SYM[7]. It is also important to proceed to perform a possible lattice formulation of the $N = D = 4$ Dirac-Kähler twisted SYM which should be carried out basically in the same manner as presented here. The results of these analyses will be given elsewhere.

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