PROCEEDINGS OF SCIENCE

Layered Phase Investigations

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The extra dimensional defects that are introduced to generate the lattice chiral zero modes are not simply a computational trick, but have interesting physical consequences. After reviewing what is known about the layered phase they can generate, I argue how it is possible to simulate Yang-Mills theories with reduced systematic errors and speculate on how it might be possible to study the fluctuations of the layers' topological charge.

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The layered phase was discovered in 1984[1] as a possible way for dimensional reduction. The authors found that an anisotropic 4+1-dimensional lattice gauge theory, with compact $U(1)$ gauge group with action (and standard notation)

$$S_{\text{gauge}} = \beta \sum_x \sum_{\mu<\nu} \left(1 - \text{Re}\left(U_\mu \nu(x)\right)\right) + \beta' \sum_x \sum_\mu \left(1 - \text{Re}\left(U_\mu 5(x)\right)\right) \tag{1}$$

possessed, in addition to the bulk confining (β and β' small) and Coulomb (β' and β large) phases, a "layered" phase, where the Wilson loops within a four-dimensional layer followed a perimeter law, whereas those along the extra dimension followed an area law. Of particular interest was that this new phase survived, when taking the one loop corrections into account, only for four-dimensional layers, but not for lower-dimensional ones. This invites speculation that it will subsist beyond the validity of the (classical) equations of motion.

A year later, within a totally different context, Callan and Harvey[2] noticed that fermions coupled to domain walls in $2n+1$ dimensions have zero modes localized on the domain wall, whose chirality depends on the sign of the gradient of the effective mass of the fermion along the extra dimension. Some years later Kaplan[3] proposed to use this mechanism on the lattice to evade the Nielsen-Ninomiya theorem and thus define lattice fermions with exact chiral symmetry–the *domain wall fermions*. Shortly thereafter Narayanan and Neuberger[4] provided another realization of this idea, the *overlap* formulation. Interestingly, the relevance of the wok in ref. [1] was overlooked, perhaps because it studied the pure gauge case only.

What is noteworthy is that, in order to study chiral lattice fermions in four dimensions, it is necessary to introduce defects that live in extra dimensions. This is the first time that, in a field-theoretic context, the need for extra dimensions is required on specific physical grounds, rather than admitted *ad hoc*. It is thus interesting to ask the question, whether one should consider these extra dimensions as a computational "trick" only, or consider their implications for physics beyond the standard model. I will try to argue for the latter position. This is particularly relevant, since current computers and algorithms are starting to come to grips with various systematic effects of locality (cf., for example, [5]) so it is useful to consider sources of systematic errors more closely.

Indeed, in ref. [6] we found that, when the gauge fields become dynamical, the chiral zero mode disappears in the layered phase. This means that one cannot ignore the value of the gauge coupling in the extra dimensions, but must take care to choose it, in conjunction with the value along the layer, so as to be at a transition surface. Else, the calculated quantities, as usual, are subject to systematic lattice artifacts: if one chooses to set the lattice coupling along the extra dimension to zero, one is in the layered phase and thus the lattice propagator *must* be sensitive to lattice artifacts. If one chooses to take the two couplings equal, on is in either the bulk Coulomb phase or the bulk confining phase and, far from the transition, is also sensitive to lattice artifacts. In addition, the transition of the isotropic theory is first order, since the five-dimensional isotropic theory is non-renormalizable by power counting. The lattice artifacts are here unavoidable and large. In ref. [7] we found numerical evidence that the layered to bulk phase transitions was continuous, thus that it is, indeed, possible to simulate theories with chiral fermions and obtain a scaling limit. In this limit the coupling constant on the layer depends parametrically on the coupling constant along the extra dimension(s)-a scenario reminiscent of the proposals in ref. [8].

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One aspect of domain wall/overlap fermions that has receded in the background (no pun intended) is the precise nature of the defect that gives rise to the mass variation along the extra dimensions. It might be useful to look more closely at specific examples. The simplest case would be that of an additional scalar field, interacting through Yukawa couplings with the fermions. The classical solution of the equation of motion for the scalar–in the absence of the fermions-would be the domain wall, which is then considered as the background for the fermions. This solution carries a natural topological charge, related to the chirality of the fermionic zero modes, that live on the layer. It would be interesting to study the fluctuations of this charge, as expressed by its susceptibility, for example, by taking into account the coupling of the scalar field with the fermions, along the lines, for instance, of ref.[9]. This brings us naturally to consider scalar fields interacting with anisotropic gauge fields and this has, indeed, been done in the context of the Abelian Higgs model[10], where we mapped the phase diagram and used the susceptibility to find a continuous phase transition between the bulk and layered Higgs phases. In ref. [11] the phase diagram of the Yukawa model with domain wall fermions has been mapped and a natural next step would thus be to consider "gauged" Yukawa models, incorporating both scalar fields and fermions coupled to anisotropic gauge fields.

It has been tempting to use the extra components of the gauge field as substitutes for the Higgs field. This stems from the fact that, in continuum language, the term

$$-\frac{1}{4g'^2}F_{\mu 5}^2$$

in the action is the kinetic term of a four-dimensional scalar field, suitably rewritten[12]. I would like to argue here that appearances are misleading and that in the continuum the extra components of the gauge field decouple. The reason is the following: First of all, much of the analysis is carried out using the same value of the gauge coupling. As indicated above, this cannot be the whole story, since the isotropic theory is cutoff-dependent. Next, the rôle of the extra components of the gauge field is to trigger confinement through the area law of the Wilson loops. The lattice formulation makes this quite explicit and shows that, in the continuum limit these components decouple as fields. They survive only through the parametric dependence of the four-dimnsional coupling(s) on the extra-dimensional ones. It is noteworthy that the confinement mechanism of the chiral zero modes through scalar fields is through the domain walls these generate–the extra components of the gauge fields don't (and can't) do this.

Lastly it is necessary to stress that the layered phase exists only for compact abelian gauge fields, since only they possess both a Coulomb and a confining phase. Pure Yang-Mills theories with a simple gauge group are always confining in all dimensions and thus cannot generate a layered phase. This means that attempts to use domain wall or overlap fermions coupled to $SU(2)$ or $SU(3)$ gauge fields, for instance, are subject to systematic lattice artifacts. These were, until now, smaller than the other systematic errors, but this is changing[5]. I therefore propose that the correct way to proceed is to use $U(N)$ instead of $SU(N)$ lattice gauge fields. The reason is that $U(N) = U(1) \times SU(N)$. It is the $U(1)$ factor that will generate the layered phase and localize the fields on the layer. It is reassuring that the electroweak sector has exactly this structure, for $N = 2$!

In conclusion the layered phase of gauge theories coupled to matter fields provides, on the one hand, a solution to the technical problem of chiral lattice fermions, on the other hand provides incentive to study the effects of extra dimensions beyond the classical equations of motion currently used, for instance, in brane-world models. Indeed it *predicts* that brane world models, whose parameters correspond to bulk phases will be unstable. It sets this question within reach of concrete numerical simulations. Another open question is the precise nature of the theory along the transition lines between the bulk and layered phase(s). Recent numerical simulations[13] confirm the second order nature of the transition between the layered and the bulk Coulomb phase, so a renormalization group analysis is needed in its vicinity in order to clarify the field content and obtain concrete numbers. Might it be related to “little string theories”[14]?

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