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The Landau gauge gluon and ghost propagators in 4D $SU(3)$ gluodynamics in large lattice volumes

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We present recent results on the Landau gauge gluon and ghost propagators in $SU(3)$ pure gauge theory at Wilson $\beta = 5.7$ for lattice sizes up to 80^4 corresponding to physical volumes up to $(13.2\ \mathrm{fm})^4$. In particular, we focus on finite-volume and Gribov-copy effects. We employ a gauge-fixing method that combines a simulated annealing algorithm with finalizing overrelaxation. We find the gluon propagator for the largest volumes to become flat at $q^2 \sim 0.01\ \mathrm{GeV}^2$. Although not excluded by our data, there is still no clear indication of a gluon propagator tending towards zero in the zero-momentum limit. New data for the ghost propagator are reported, too.

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1. Introduction

Presently, there is an intensive exchange of results and opinions between groups carrying out analytical and numerical studies of infrared QCD. This ongoing research focuses in particular on the infrared behavior of the Landau-gauge gluon and ghost propagators. The latter is intimately related to the confinement [1, 2, 3]. What makes analytical predictions possible also in the non-perturbative sector of the theory is the possibility to write down a hierarchy of Dyson-Schwinger equations (DSE) connecting propagators and vertices. Under rather mild assumptions the hierarchy can be truncated. However, not all assumptions have been thoroughly checked. Note that recently the full system of Landau gauge DSE has been solved without any truncations within the asymptotic infrared region with a power ansatz for all Green functions involved [4]. Numerically, the propagators can be studied from first principles in terms of Monte Carlo (MC) simulations of lattice QCD. It is worth to compare the lattice results with the asymptotic power-like behavior, and with numerical DSE solutions found in finite volumes [5]. It is interesting to see whether there remain differences as the infinite-volume limit is approached.

On the lattice we approximate the gluon propagator as the MC average

$$D^{ab}_{\mu\nu}(q) = \langle \tilde{A}^a_\mu(\hat{q})\tilde{A}^b_\nu(-\hat{q})\rangle = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z_{gl}(q^2)}{q^2} \tag{1.1}$$

with the gluon field $A_{x+\hat{\mu}/2,\mu} = (1/2iag_0)(U_{x,\mu} - U^\dagger_{x,\mu})_{\text{traceless}}$ transformed into Fourier space. The lattice momenta $\hat{k}_\mu = 2\,\pi\,k_\mu/L_\mu$ with integer $k_\mu \in (-L_\mu/2, +L_\mu/2]$ are related to their physical values by $q_\mu = (2/a)\sin(\pi\,k_\mu/L_\mu)$.

The ghost propagator in momentum space at non-zero q^2 is defined by double Fourier transformation

$$G^{ab}(q) = \sum_{x,y}\left\langle \mathrm{e}^{-i\hat{k}\cdot(x-y)}\,[M^{-1}]^{ab}_{x,y}\right\rangle = \delta^{ab}\frac{Z_{gh}(q^2)}{q^2}\,. \tag{1.2}$$

Practically, this is done by an inversion of the Faddeev-Popov (F-P) matrix $M^{ab}_{x,y}$ using a conjugate-gradient algorithm with plane waves as sources. The Faddeev-Popov operator in terms of the Landau gauge-fixed links is

$$M^{ab}_{xy} = \sum_\mu \mathfrak{Re}\,\mathrm{Tr}\left[\{T^a,T^b\}(U_{x,\mu}+U_{x-\hat{\mu},\mu})\delta_{xy} - 2T^bT^aU_{x,\mu}\delta_{x+\hat{\mu},y} - 2T^aT^bU_{x-\hat{\mu},\mu}\delta_{x-\hat{\mu},y}\right]\,, \tag{1.3}$$

with $T^a = \lambda^a/2$ (λ^a are the Gell-Mann matrices). The functions $Z_{gl}(q^2)$ and $Z_{gh}(q^2)$ are called dressing functions of the respective propagator.

In Landau gauge the gluon and ghost dressing functions are predicted to follow the simple power laws [6]

$$Z_{gh}(q^2) \propto (q^2)^{-\kappa} \quad \text{and} \quad Z_{gl}(q^2) \propto (q^2)^{2\kappa}\,. \tag{1.4}$$

in the asymptotic regime $q^2 \to 0$. Thereby, both exponents are related to some κ which, under the assumption that the ghost-gluon vertex is infrared-regular, takes a value of about $\kappa = 0.596$ [7]. That is, the gluon propagator is predicted to decrease towards lower momenta and to vanish at $q^2 = 0$. At which scale this asymptotic behavior sets in cannot be concluded from those studies, however.

A truncated system of DSE formulated on a 4D torus and numerically solved [5] predicts a specific finite-volume behavior which, at the first glance, looks quite similar to earlier lattice results obtained in particular by some of us [8]. Characteristic deviations for momenta $q \sim 1/L$ of the gluon and the ghost propagator from the momentum dependence of the respective propagators at infinite volume should be expected. In order to check the DSE predictions we decided to evaluate the gluon and the ghost propagator for increasingly large symmetric lattices. We have measured the gluon and ghost propagators for configurations generated with the Wilson gauge action at fixed $\beta = 5.7$ on 56^4, 64^4, 72^4 and 80^4 lattices. Note that the latter corresponds to a volume of about $(13.2 \text{ fm})^4$. Comparing results from either analytic or numerical approaches for varying 4-volume will hopefully allow (i) to conclude for which momenta data on both propagators are reliable, and (ii) to estimate the order of magnitude of distortion of the momentum dependence due to finite-size and Gribov-copy effects. This paper presents first results of this study.

2. Gauge fixing

To fix the Landau gauge, we apply to all links a gauge transformation $g \in G$ ($G = SU(3)$) mapping $U_{x,\mu} \rightarrow {}^{g}U_{x,\mu} = g_x U_{x,\mu} g^{\dagger}_{x+\hat{\mu}}$, with the aim to maximize a gauge functional

$$F_U[g] = \frac{1}{12V} \sum_{x,\mu} \mathfrak{Re}\ \mathrm{Tr}\ {}^{g}U_{x,\mu} \ , \tag{2.1}$$

or, more exactly, to find the global maximum of $F_U[g]$ [1]. In this work $g_x \in G$ is considered as a periodic field on all lattice sites. To find the global maximum in practice is a complicated problem which becomes exceedingly time consuming with increasing lattice volume. Starting from an initial random gauge transformation g_x one generally arrives at one of many local maxima of $F_U[g]$. The corresponding gauge-fixed configurations are called Gribov copies. They all satisfy the differential gauge condition $\partial_\mu A_\mu = 0$ together with the additional necessary condition that the Faddeev-Popov operator has a positive spectrum (apart from its 8 trivial zero modes). With increasing volume, the copies become dense with respect to the value of the functional (2.1) and the spectral density of the F-P operator near zero grows [9].

One way to suppress the effect of the Gribov ambiguity is to find N_{copy} local maxima of $F_U[g]$ and to choose among them the "best" one ("bc"), which possesses the largest value of $F_U[g]$. The underlying idea is that the maximal value of the local maxima approaches the global maximum of $F_U[g]$, and the distortion of gauge-dependent observables, computed on such copies, vanishes in the limit $N_{\text{copy}} \rightarrow \infty$. Such studies normally use the overrelaxation (OR) technique to search for the maximum of $F_U[g]$. They have been carried out in [8, 10, 11, 12] and have shown the Gribov-copy effect to become weaker with growing lattice extension L – in accordance with Zwanziger's conjecture [1]. This suggests that it is tolerable to restrict gauge-fixing computations on large lattices $(L \geq 48)$ to one gauge copy only[1]. Our simulations of the $SU(3)$ propagators at $L = 56$ were carried out using an OR algorithm with the overrelaxation parameter set to $\alpha = 1.70$. The number of gauge-fixing (GF) iterations did not exceed 10^4 in most of the cases. However, we find a considerable slowing down of the OR GF process on a 64^4 lattice. This was one of the reasons why we switched from using OR to a simulated annealing (SA) algorithm. SA, also known

[1] This is called "first copy" ("fc") in a multi-copy approach.

as a "stochastic optimization method" has been proven to be highly effective in coming close to the global maximum in various problems of different nature with multiple local maxima. It was proposed in [13, 14] and has found numerous applications in various fields of science. The idea of applying SA for gauge fixing has been first put forward and realized in the case of maximally Abelian gauge in [15]. The method is designed to keep the system long enough pending in a region of simultaneous attraction by many local maxima during a quasi-equilibrium process undergone by the "spin system" formed by g_x interacting through the fixed $\{U_{x,\mu}\}$ field with $F_U[g]$ as energy. The temperature T of the spin system is decreased by small T-steps between updates in a range of T where the penetrability of functional barriers strongly changes. Theoretically, when infinitely-slow cooling down to $T = 0$, the SA algorithm finds the global maximum with 100% probability. For complicated systems with large numbers of degrees of freedom and of functional local extrema, e.g., for GF on large lattices, we have to restrict the number of N_{iter} T-steps of cooling the system from $T_{\max}$ to $T_{\min}$ to, say, $O(10^4)$. Within these limits we can still try to attain an as high value of the functional studied (in our case, $F_U[g]$) as possible. Note that GF with SA requires a finalizing OR in order to satisfy transversality $\partial_\mu A_\mu = 0$ with a given high precision.

For the $SU(3)$ case we chose $T_{\max}$ such that it leads to a sufficiently large mobility in the functional space. The final temperature $T_{\min}$ was taken low enough that the subsequent OR was not slowed down while penetrating further functional barriers. This is witnessed by the check that the violation of the differential gauge condition, $(\partial_\mu A_\mu)^2$, monotonously decreases until the machine precision is reached (stopping criterion) in almost all cases. In practice, for $L = 56, 64, 72$ and 80 we restricted ourselves to one copy, and carried out from 5×10^3 to 15×10^3 heatbath (HB) sweeps of SA with 4 microcanonical sweeps after each HB one. We checked that the smaller T-steps are done in between, the higher the local maxima being reached. Finally, we note that a linear decrease in T seems not to be the optimal choice. T-schedules with smaller T-steps close to $T_{\max}$ and larger T-steps at the end (with N_{iter} fixed) lead to higher $F_U[g]$-values (after completing the full SA procedure).

3. Ghost propagator results

The $SU(3)$ ghost propagator at $\beta = 5.7$ is shown as a function of q^2 in Fig. 1; on the left hand side for a single 56^4 configuration, simply comparing results after either OR or SA gauge fixing, and on the right hand side as an average over 14 configurations in the case of OR, and over 7 configurations using SA gauge fixing. The influence of the gauge-fixing method, here through the emerging copy, can be seen only for the three lowest momenta at this lattice size. In general, one notices that the higher the gauge functional, the lower the estimates of the ghost propagator at the smallest momenta. This comparison (Fig. 1) demonstrates that the problem of Gribov copies does still exist for $L = 56$, resulting in maximally 10% difference of ghost propagators at lowest momenta. Note that this result cannot be directly compared to our previous studies [10, 8, 11, 12] of Gribov-copies effect, in which the "fc-bc" comparison was used to assess the Gribov ambiguity. A detailed check of Zwanziger's conjecture on the weakening of the Gribov problem with an increase of the lattice volume (using the SA vs. OR "one-copy" comparison) requires further studies both for smaller and larger lattices.

In Fig. 2 a scatter plot is shown of the ghost dressing function for a broad range of momentum, combining data obtained for 7 configurations on a 56^4 lattice, 14 configurations on a 64^4 lattice, 3

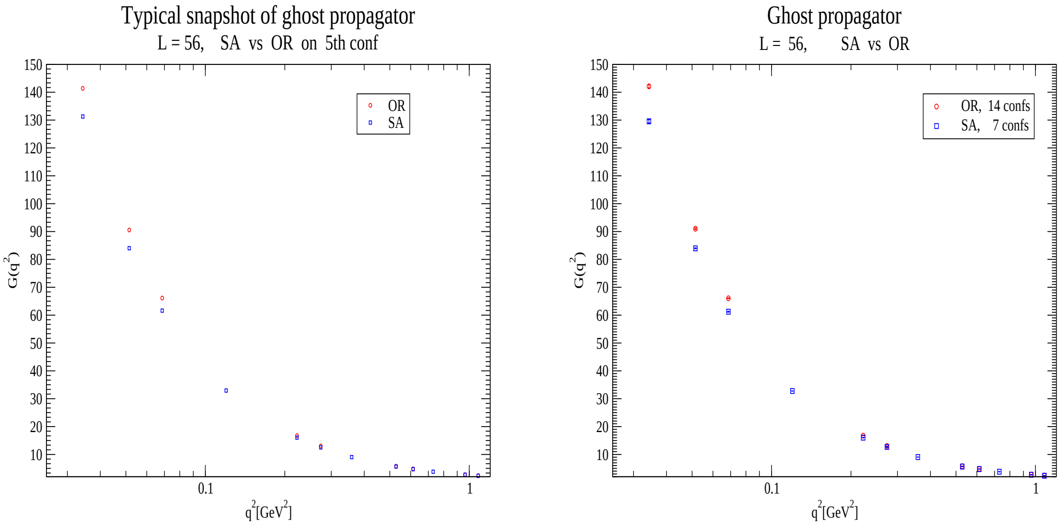

Figure 1: Ghost propagator results: the influence of the gauge fixing algorithm on the propagator, calculated for a typical single gauge field configuration (left), on the averaged propagator (right).

configurations on a 72^4 and 3 configurations on a 80^4 lattice, all thermalized at $\beta = 5.7$. The gauge field configurations were produced with a heat-bath algorithm applying O(1000) thermalization sweeps in between. We consider only momenta surviving a cylinder cut with $\Delta\hat{q} = 1$ [16]. For all lattice sizes GF has been carried out with the SA algorithm. Surprisingly, all the values for the ghost propagator fall perfectly on one universal curve (within 1% accuracy), besides those for the 2 smallest momenta. The results, especially those found at $L = 80$, show that a true IR exponent κ cannot yet be defined or does not exist at all. This is at variance with the asymptotic DSE prediction $\kappa = 0.595$ [5] and also with $\kappa = 0.2$ motivated by thermodynamic considerations in [18]. The latter estimate of κ is based on the required cancellation of gluon and ghost contributions to the pressure, that otherwise were building up a Stefan-Boltzmann law, in the confinement phase. Note that downward deviations of the data at lowest momentum for each physical box size V from the infinite-volume curve of $G(q^2)$ are predicted by the DSE approach on a finite torus [5]. However, we do not find such deviations.

4. Gluon propagator results

Fig. 3 shows data for the gluon propagator computed for three different lattice sizes at $\beta = 5.7$. There, the gauge was fixed with the SA algorithm. At the present stage, the data favor a non-vanishing gluon propagator at zero momentum, as there is no sight of a different behavior even at the largest lattice volume available to us. Also, the decrease of the zero-momentum propagator $D(0)$ upon increasing the volume seems to become less with bigger V. The 64^4 data for $D(q^2)$ resemble the pattern of overshooting deviations from an universal function of momentum known from the DSE solutions on a finite torus [5], though. If the lowest two or three momenta were removed from the plot, the picture would be less convincing in favor of an emerging plateau. Better statistics and data on even larger symmetric lattices (with $L > 80$) will make us more confident in this.

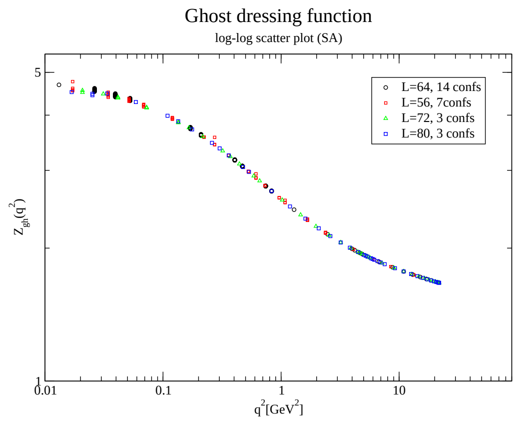

Figure 2: Scatter plot for the ghost dressing function from different lattice sizes and configurations, generated at $\beta = 5.7$ and gauge-fixed with SA.

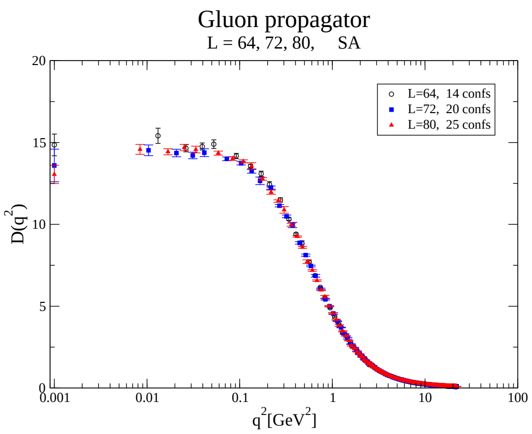

Figure 3: The gluon propagator from different lattice sizes at $\beta = 5.7$. The data points drawn at $q^2 = 0.001$ represent the zero-momentum gluon propagator $D(0)$.

5. Discussion

We conclude that using the SA technique for the purpose of gauge fixing considerably facilitates simulations of ghost and gluon propagators in Landau gauge on large lattices. We find that SA-based computations of the ghost propagator seem to be less affected by statistical fluctuations compared to other calculations where OR is used. Additionally, estimates of the ghost propagator at low momenta are systematically lower than those obtained after simple OR.

A continuous decrease in slope in the ghost dressing function below 0.4 GeV does not conform to a simple power-law ansatz. Therefore, any attempts to extract infrared exponents from lattice data seem to be premature at the present stage. Qualitatively, the same behavior is seen for the case of $SU(2)$ (see [17]) and also in the DSE solutions on a torus [5]. However, the effects of finite volumes are much less than expected from there though. The same we find for the gluon propagator which we cannot confirm to be infrared-decreasing even at volumes larger than $(13\text{fm})^4$.

Future analytical (DSE and renormalization group) studies and lattice simulations, optionally including also $\mathbb{Z}(3)$ flip operations into the GF procedure [11], at even larger volumes will hopefully help to resolve or explain the existing discrepancies between the lattice and analytical findings.

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References

[1] D. Zwanziger, *Nucl. Phys.* **B412**, 657 (1994); *Phys. Rev.* **D69**, 016002 (2004), [hep-ph/0303028].

[2] T. Kugo and I. Ojima, *Prog. Theor. Phys. Suppl.* **66** (1979) 1; T. Kugo, hep-th/9511033.

[3] R. Alkofer and L. von Smekal, Phys. Rept. **353**, 281 (2001), [hep-ph/0007355].

[4] R. Alkofer, C.S. Fischer, and F.J. Llanes-Estrada, *Phys. Lett.* **B611**, 279 (2005), [hep-th/0412330];
C.S. Fischer and J.M. Pawlowski, *Phys. Rev.* **D75**, 025012 (2007), [hep-th/0609009].

[5] C.S. Fischer, A. Maas, J.M. Pawlowski, and L. von Smekal, *Ann. Phys.* **322**, 2916 (2007), [hep-ph/0701050].

[6] L. von Smekal, R. Alkofer, and A. Hauck, *Phys. Rev. Lett.* **79**, 3591 (1997), [hep-ph/9705242].

[7] Ch. Lerche and L. von Smekal, *Phys. Rev.* **D65**, 125006 (2002), [hep-ph/0202194];
D. Zwanziger, *Phys. Rev.* **D65**, 094039 (2002), [hep-ph/0109224].

[8] A. Sternbeck, E.-M. Ilgenfritz, M. Müller-Preussker, and A. Schiller, *Phys. Rev.* **D72**, 014507 (2005), [hep-lat/0506007].

[9] A. Sternbeck, E. M. Ilgenfritz, and M. Müller-Preussker, *Phys. Rev.* **D73**, 014502 (2006), [hep-lat/0510109].

[10] T.D. Bakeev, E.-M. Ilgenfritz, V.K. Mitrjushkin, and M. Müller-Preussker, *Phys. Rev.* **D69**, 074507 (2004), [hep-lat/0311041].

[11] I.L. Bogolubsky, G. Burgio, V.K. Mitrjushkin, and M. Müller-Preussker, *Phys. Rev.* **D74**, 034503 (2006), [hep-lat/0511056]; I.L. Bogolubsky, et al., arXiv:0707.3611 [hep-lat].

[12] E.-M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, A. Schiller, and I.L. Bogolubsky, *Braz. J. Phys.* **37**, 193 (2007), [hep-lat/0609043].

[13] S. Kirkpatrick, C.D. Gelatt Jr., and M.P. Vecchi, *Science* **220**, 671 (1983).

[14] V. Cerny, *J. Opt. The. Appl.* **45**, 41 (1985).

[15] G.S. Bali, V. Bornyakov, M. Müller-Preussker, and F. Pahl, *Nucl. Phys. Proc. Suppl.* **42**, 852 (1995), [hep-lat/0303028].

[16] D.B. Leinweber, J.I. Skullerud, A.G. Williams, and C.Parinello, *Phys. Rev.* **D60**, 094507 (1999), [hep-lat/9811027].

[17] A. Sternbeck, L. von Smekal, D.B. Leinweber, and A.G. Williams, *PoS(LATTICE 2007)* 340 (2007).

[18] M.N. Chernodub, V.I. Zakharov, [hep-ph/0703167].

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