

Vortex topology and the continuum limit of lattice gauge theories

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We study the stability of \mathbb{Z}_2 topological vortex excitations in $d+1$ dimensional $SU(2)$ Yang-Mills theory on the lattice at $T = 0$. This is found to depend on d and on the coupling considered. We discuss the connection with lattice artifacts causing bulk transitions in the β_A - β_F plane and draw some conclusions regarding the continuum limit of the theory.

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1. Introduction

The rôle of topological excitations in the effective mechanism for confinement for $SU(N)$ Yang-Mills theories has long been discussed in the literature, with Abelian monopoles [1] and \mathbb{Z}_N magnetic vortices the [2] most popular candidates. For theories discretized on the lattice in the fundamental representation a large number of studies is available on this subject (see e.g. these proceedings).

On the other hand, being $SU(N)/\mathbb{Z}_N$ the actual gauge group of continuum *pure* Yang-Mills theories and resting the appearance of \mathbb{Z}_N vortices excitations indeed on such invariance [2], it is surprising how scarce the lattice literature analyzing topological mechanism of confinement in adjoint discretizations is. A partial excuse for such failing can be sought in the difficulties connected to any numerical study of $SU(N)/\mathbb{Z}_N$ on the lattice. In 3+1 dimensions the theories exhibit in the β_A - β_F phase diagram bulk transitions [3, 4] (see Fig. 1) related to the condensation of \mathbb{Z}_N magnetic monopoles σ_c and electric vortices σ_l [5, 6].

In a series of papers [7, 8, 9, 10, 11, 12, 13, 14, 15, 16] such gap was filled for $SO(3) \simeq SU(2)/\mathbb{Z}_2$, leading to two important results. On one hand the connection of bulk transitions with the stability of \mathbb{Z}_2 magnetic vortices was established, i.e. as expected [17, 18] well defined magnetic \mathbb{Z}_2 topological sectors have been found to exist only where \mathbb{Z}_2 magnetic monopoles cease to condense. On the other hand the \mathbb{Z}_2 magnetic vortex free energy was found to behave quite differently as naively expected.

Although the ultimate goal is a throughout analysis of the latter result for any N and dimension, we concentrate here at first on the former to see to what extent it can be extended to the whole β_A - β_F phase diagram in various dimensions. This is a necessary precondition for any future meaningful analysis of the \mathbb{Z}_N magnetic vortex free energy. We will show preliminary results indicating that the $T = 0$ phase diagram as obtained from \mathbb{Z}_2 magnetic and electric vortices has a richer structure than previously believed by looking at \mathbb{Z}_2 magnetic monopoles. Although a similar picture emerges for 1+1 and 3+1 dimensions and for higher N as well, we will concentrate here on $N = 2$ in 2+1 dimensions. A full analysis will appear in short time [19].

2. Action and observables

We will consider the $N = 2$ mixed fundamental-adjoint Wilson action in $d + 1$ dimensions:

$$S = \beta_A \sum_P \left(1 - \frac{1}{3} \text{Tr}_A U_P \right) + \beta_F \sum_P \left(1 - \frac{1}{2} \text{Tr}_F U_P \right); \quad \frac{1}{g^2} = \frac{1}{4} \beta_F + \frac{2}{3} \beta_A. \quad (2.1)$$

Fig. 1 shows the common picture as obtained in [4, 20] for $d = 2, 3$. Order parameters for the transitions/crossovers (except for the roughening one [21]) are \mathbb{Z}_2 magnetic monopole σ_c and electric vortices σ_l densities M and E [5, 6]:

$$M = 1 - \left\langle \frac{1}{N_c} \sum_c \sigma_c \right\rangle \quad \sigma_c = \prod_{P \in \partial c} \sigma_P \in SO(3) \quad (2.2)$$

$$E = 1 - \left\langle \frac{1}{N_l} \sum_l \sigma_l \right\rangle \quad \sigma_l = \prod_{P \in \partial l} \sigma_P \in SU(2) \quad (2.3)$$

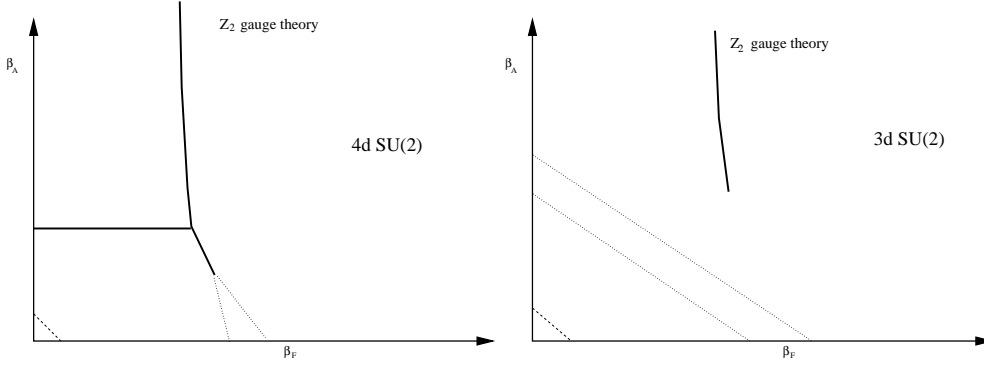


Figure 1: Phase diagram from σ_c and σ_l of the β_A - β_F plane in 3+1 (left) and 2+1 (right) dimension. Straight lines are bulk transitions, dotted lines show the crossover regions and dashed lines the roughening transitions.

Apart for the roughening transition, the $d = 1$ dimensional phase diagram is believed to be trivial, since \mathbb{Z}_2 magnetic monopoles cannot exist and the \mathbb{Z}_2 gauge theory, whose behaviour Eq. 2.3 reflects, is trivial in 1+1 dimensions.

In the continuum *pure* Yang-Mills theories are known to allow large gauge transformations linked to $\pi_1(SU(N)/\mathbb{Z}_N) = \mathbb{Z}_N$, defining topological sectors corresponding to \mathbb{Z}_N magnetic vortices [2]. These are (evolving) points and lines for $d = 2, 3$ and instanton-like objects for $d = 1$. On the lattice the fundamental discretization (quenched QCD) should only allow one sector in the continuum limit, fixed by the periodic or twisted boundary conditions. The adjoint theory with periodic boundary conditions is compatible with all sectors. In $SU(2)$ a suitable observable to measure the global \mathbb{Z}_2 magnetic flux through the $\mu\nu$ plane is given by

$$z_{\mu\nu} = \frac{1}{L^{d-1}} \sum_{\vec{p} \perp \mu\nu} \prod_{x \in \mu, \nu \text{ plane}} \text{sign}(\text{Tr}_f U_{\mu\nu}(x)) \quad (2.4)$$

Of course for the fundamental theory with periodic boundary conditions only allows $z_{\mu\nu} = 1$ regardless of d for $\beta_F \rightarrow \infty$.

\mathbb{Z}_N magnetic monopoles, which are particle like for $d = 3$ and instanton-like objects for $d = 2$, will spoil such picture, since they are source of *open* \mathbb{Z}_N magnetic vortices [17, 18]. Since to any *abelian* monopole of charge k corresponds a \mathbb{Z}_N monopole of charge $\text{mod}_N(k)$ in the continuum limit, where the latter ought not to exist, the former are only allowed charges $k \propto N$, i.e. only these are compatible with closed \mathbb{Z}_N magnetic vortices [17, 18, 22], as the case in pure $SO(3)$ approaching the continuum limit indicates [16]. This also implies that in a \mathbb{Z}_N monopole background stable vortices are submerged by the related open vortex background and $z_{\mu\nu}$ will have no well defined single value through all parallel $\mu\nu$ planes, averaging to $z_{\mu\nu} = 0$ for any MC configuration, as indeed shown in [15, 16]. Along the known bulk transition lines the order parameter

$$z = \frac{2}{d(d+1)} \sum_{\mu\nu} \langle |z_{\mu\nu}| \rangle \quad (2.5)$$

follows M , their behaviour being indistinguishable, so that in the strong coupling region $z = 0$ while as the theory approaches the continuum limit $z_{\mu\nu} \rightarrow \pm 1$ and $z \rightarrow 1$. What happens however across

the crossover regions? Does z approach its asymptotic value following M and E ? And in 1+1 dimensions, where no monopoles can appear, but the boundary conditions still dictate the global \mathbb{Z}_N magnetic flux, so that the appearance of the wrong one can still characterize the strong coupling regime? We concentrate on $\beta_A = 0$ for $d = 2$ as an example.

3. Results

Fig. 2 shows z and its susceptibility χ for increasing volume as a function of β_F at $T = 0$ in 2+1 dimensions. We remind that along the crossover M and E peak around $\beta_F = 4 - 5$. The

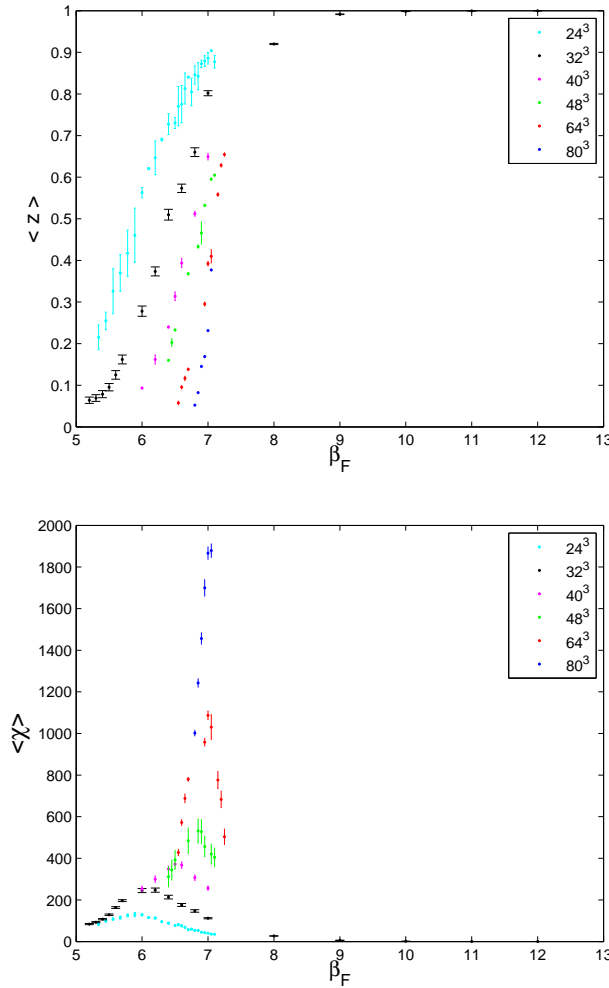


Figure 2: Order parameter z (left) and its susceptibility χ for different volumes.

critical behaviour of z at higher β_F is evident. A similar picture emerges also for $\beta_A \neq 0$, for $N = 3$ and for $d = 1$ and 3 [19]. Establishing the properties of the transition is however a hard task. Integrated autocorrelations for z show a strong critical slowing down approaching criticality. Moreover a direct investigation of the plaquette and the specific heat shows no sign of critical behaviour, excluding a 1st or standard (i.e. divergent) 2nd order transition.

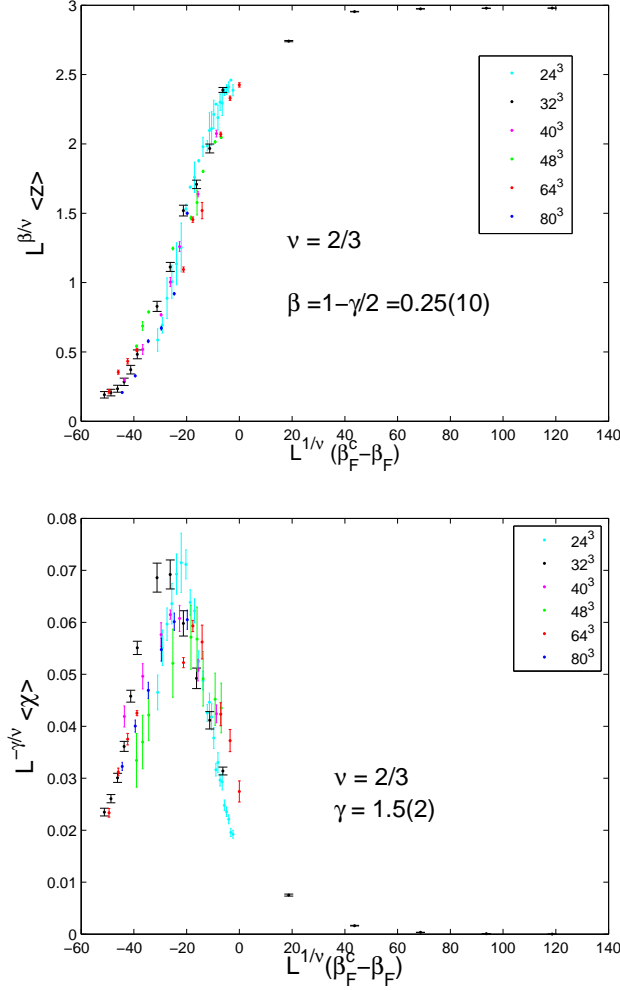


Figure 3: FSS for z (left) and its susceptibility χ for different volumes.

This leaves however room for a discontinuous 2nd order with a small gap in the specific heat or higher ($\geq 3^{\text{rd}}$) transition. Indeed this would be not unheard of, since e.g. a third order transition is known to exist in $d = 1$ in the large N limit and conjectured to extend also to higher dimensions [23]. Following standard techniques we check the consistency of FSS assuming the “critical exponents” extracted from hyperscaling relations for a 2nd order discontinuous transition [24], i.e. $\nu = 2/3$, $\gamma = 4/3$ and $\beta = 1/3$. Actually we fit $\gamma = 1.5(2)$ from $\chi_{\text{max}}(L) \simeq L^{\gamma/v}$. The curves obtained by tuning $\beta_F^c = 7.3(1)$ and using $\beta = 1 - \gamma/2 = 0.25(10)$ are shown in Fig 3. Given the high systematic errors coming from autocorrelations γ and β are in good agreement with the theoretical prediction. This FSS analysis should however be considered still tentative pending better precision in the data and alternative independent methods to establish the order of the transition. A direct analysis of Fisher zeroes [24] seems the most promising.

4. Conclusions

We have given evidence that in $2 + 1$ dimension the pure $SU(2)$ theory undergoes a bulk-like transition at $\beta_F = 7.3(1)$. The order parameter of such transition z measures the stability of \mathbb{Z}_2 magnetic vortices. A preliminary FSS analysis shows the critical exponents to be consistent with a 2nd order discontinuous transition. A similar picture emerges for $\beta_A > 0$, other spacial dimensions and $N > 3$. Fig 4 shows how e.g. the 3+1 and 2+1 β_A - β_F phase diagram would indeed look like if such results should be confirmed. In 1+1 dimensions a phase transition line would also separate

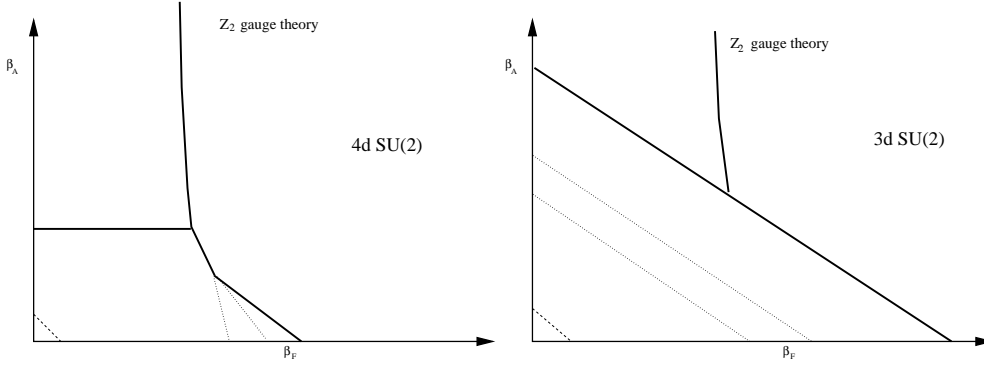


Figure 4: Phase diagram from z and σ_l of the β_A - β_F plane in 3+1 (left) and 2+1 (right) dimension. Straight lines are bulk transitions, dotted lines show the crossover regions and dashed lines the roughening transitions.

the strong from the weak coupling phase in the whole β_A - β_F plane.

The preliminary character of the results only concerns their quantitative analysis, i.e. extracting the exact position, order and critical exponents of the transitions in the β_A - β_F plane for different dimensions. To this goal a better control of autocorrelations and the use of methods alternative to those here exposed would be most welcome. In particular, studying the density of Fisher zeroes from the complexified partition function [24] should provide a direct independent method to establish β^c and the order and type of the transition. Nevertheless, the presence of a critical behaviour for z is evident from the data and needs to be addressed independently of the above caveats.

Although the presence of such transitions, being most likely of high order, might not be evident in most observable, it might affect in principle any attempt to use RG-flow methods to connect weak to strong coupling regions, especially if basing on vortex related observables [25]. To this goal it would be important to establish whether the transition line has an end point at $\beta_A < 0$ and how it does connect to the bulk lines detected through σ_l .

Moreover any result on the rôle of topological excitations for confinement, abelian monopoles and \mathbb{Z}_N magnetic vortices in particular, should be indeed rechecked above the transition lines in the region connected to the continuum theory, where both can take their correct value. Being the results found in the only case where this was done [15, 16] somehow surprising, although partly expected and partly justifiable through analysis of the Hilbert space states [26], crosschecks and better understanding would be most welcome.

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