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What's up with IR gluon and ghost propagators in Landau gauge? A puzzling answer from huge lattices

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Several analytic approaches predict for $SU(N_c)$ Yang-Mills theories in Landau gauge an enhanced ghost propagator $G(p^2)$ and a suppressed gluon propagator $D(p^2)$ at small momenta. This prediction applies to two, three and four space-time dimensions. Moreover, the gluon propagator is predicted to be null at $p = 0$. Numerical studies by several groups indeed support an enhanced ghost propagator when compared to the tree-level behavior $1/p^2$ and a finite infrared gluon propagator. However, the agreement between analytic and numerical studies is only at the qualitative level in three and in four dimensions. In particular, the infrared exponent of the ghost propagator seems to be smaller than the one predicted analytically and the gluon propagator seems to display a (finite) nonzero value at zero momentum. It has been argued that this discrepancy might go away once simulations are done on much larger lattice sizes than the ones used up to now. Here we present data in three and four space-time dimensions using huge lattices in the scaling region, i.e. up to 320^3 at $\beta = 3.0$ and up to 128^4 at $\beta = 2.2$, corresponding to $V \approx (85\,\mathrm{fm})^3$ and $V \approx (27\,\mathrm{fm})^4$. Simulations have been done on the IBM supercomputer at the University of São Paulo.

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1. Introduction

In the Gribov-Zwanziger confinement scenario for Landau gauge [1, 2] the gluon propagator $D(p^2)$ is predicted to be infrared (IR) suppressed at small momenta. In particular, one should have $D(0) = 0$, implying that reflection positivity is maximally violated. This violation of reflection positivity may be viewed as an indication of gluon confinement [3]. At the same time, the Gribov-Zwanziger [1, 4] and Kugo-Ojima [5] confinement scenarios predict (in Landau gauge) a ghost propagator $G(p^2)$ enhanced in the IR limit. This represents a long-range effect and could be related to quark confinement [1, 3, 4, 6].

Several analytic studies [7, 8, 9, 10] agree with the above scenarios predicting, for small momenta, a gluon propagator $D(p^2) \propto p^{2(a_D-1)}$ and a ghost propagator $G(p^2) \propto 1/p^{2(1+a_G)}$, with the relation $a_D = 2a_G + (4-d)/2$. Here d is the space-time dimension. Clearly, if $a_D > 1$ one has $D(0) = 0$. In the four-dimensional case, one finds [8, 9] $a_G \approx 0.59$ and $a_D = 2a_G$. Similar power behaviors have also been obtained for the various vertex functions of $SU(N_c)$ Yang-Mills theories [9, 11, 12].

These results have been confirmed at the quantitative level in the two-dimensional case, using lattices up to almost $(43\,\text{fm})^2$ [13]. In the three-dimensional case [14], one clearly sees an IR-suppressed gluon propagator. However, using lattice volumes up to about $(24\,\text{fm})^3$, it was not possible to control the extrapolation to infinite volume. In particular, the data for the rescaled gluon propagator at zero momentum $D(0)$ could be fitted [as a function of the inverse lattice side $1/L = 1/(Na)$] using the Ansatz $d + b/L^c$ both with $d = 0$ and with $d \neq 0$ [14]. Here a is the lattice spacing and N is the number of lattice points per direction. Finally, in four dimensions, the gluon propagator is clearly less divergent than in the tree-level case [15, 16, 17, 18]. On the other hand, even using lattices with a lattice side of about 10 fm, one does not see a gluon propagator decreasing at small momenta [18]. One should stress, however, that the Landau gluon propagator clearly violates reflection positivity, in two, three and four space-time dimensions [13, 19, 20]. For the ghost propagator, the IR exponent a_G obtained using analytic studies has been confirmed in 2d [13], while in the 3d [21] and in the 4d [16, 17, 22, 23] cases the exponent obtained using lattice numerical simulations is always smaller than the one predicted analytically. Let us also recall that, in Ref. [24], it was shown that gluon and ghost propagator for $SU(2)$ and $SU(3)$ Yang-Mills theories are in very good agreement from momentum $p \approx 1$ GeV to about $p \approx 10$ GeV. Similar results have been presented in [25]. These findings suggest that the IR behavior of these propagators is independent of the gauge group $SU(N_c)$, as predicted analytically [7, 8, 9].

Finally, by solving Dyson-Schwinger equations on a finite four-dimensional torus [12, 26, 27] one can show that the gluon propagator (at small momenta) seems to diverge for volumes up to about $(8\,\text{fm})^4$, develops a plateau for $V \approx (9\,\text{fm})^4$ and is IR suppressed for a lattice side larger than 10 fm. Also, the extrapolation of these results to infinite volume gives a null gluon propagator at zero momentum. At the same time, one obtains that, after eliminating (for each volume) the data corresponding to the first two non-zero momenta, the ghost propagator shows a power-law behavior and, in the infinite volume-limit, one obtains the IR exponent a_G predicted by analytic studies [7, 8, 9].

From the above results, it seems necessary to extend present numerical simulations to very large lattice volumes in order to verify if the agreement obtained between lattice data and analytic results for the two-dimensional case [13] applies to the 3d and 4d cases as well. Here we present extensive simulations in three and in four dimensions, for the $SU(2)$ case, using huge lattices. In particular, we considered lattice sides $N = 140$, 200, 240 and 320 in 3d at $\beta = 3.0$ and $N = 48$, 56, 64, 80, 96 and 128 in 4d at $\beta = 2.2$. In 3d, the number of configurations was about 630, 525, 350 and 45, respectively for the four lattice sizes, both for the gluon and for the ghost propagators. In the 4d case we have considered for the gluon propagator 168 configurations for $V = 128^4$ and about 250 configurations for the other lattice sizes. For the ghost propagator we have 21 configurations for the largest volume and about 100 in the other cases. For the inversion of the Faddeev-Popov matrix we used the so-called *point-source* method [21, 23]. Note that the lattice volumes 320^3 at $\beta = 3.0$ and 128^4 at $\beta = 2.2$ correspond, respectively, to $V \approx (85\,\mathrm{fm})^3$ and $V \approx (27\,\mathrm{fm})^4$. (See Refs. [14, 28] for details about how the physical lattice spacing a has been set in the two cases.) Also note that all our runs are in the scaling region.

These simulations have been done in the IBM supercomputer at USP. This machine has 112 blades with 2 dual-core PowerPC 970 2.5GHz CPU's, a Myrinet network and about 4.5 Tflops peak-performance (occupying position number 363 in the TOP500 list of November 2006).

2. Results

Considering the new data produced in the 3d case together with old data from Refs. [14, 19], we tried an extrapolation to infinite volume for the gluon propagator $D(0)$ as a function of the inverse lattice side $1/L$. The data for the propagator have been renormalized following Ref. [13]. Results are shown in Fig. 1. It is clear from the plot that there is no sign of a propagator going to zero as L goes to infinity. Moreover, the data show a behavior of the type $D(0) \sim 1/L$ and an infinite-volume extrapolation given by $D(0) \approx 2$ GeV^{-2}. This implies $a_D = 1$.

In the 4d case (see Fig. 2), even considering very large lattice volumes and relatively large statistics, one cannot see a clear sign of a gluon propagator $D(p^2)$ decreasing at small momenta. Similar results have been presented at this conference by other groups [25, 29]. Clearly, also in this case the data suggest $a_D = 1$. On the other hand, violation of reflection positivity is confirmed and the gluon propagator, considered as a function of the spatial separation s, becomes negative at $s \approx 1\,\mathrm{fm}$, in agreement with Ref. [30].

We have also tried to estimate the IR exponent a_G for the ghost propagator (in the 3d and 4d cases) using the Ansatz $G(p) = c/p^{2(1+a_G)}$ and considering, for each lattice volume, either the two smallest nonzero momenta or the third and fourth smallest nonzero momenta. Results are reported in Tables 1. As one can see, this IR exponent seems to go to zero as the infinite-volume limit is approached, in agreement with [16]. One should however notice that, for $p \approx 500$ MeV, the exponent a_G is about 0.3, also in agreement with Ref. [16].

3. Conclusions

The above results leave us with several open questions. From the lattice point of view, one should of course investigate if Gribov-copy effects and/or finite-volume effects could explain our results. Let us recall that an improved gauge-fixing method [31] seems capable of reducing finite-volume effects for the gluon propagator by enlarging the set of allowed gauge transformations. At

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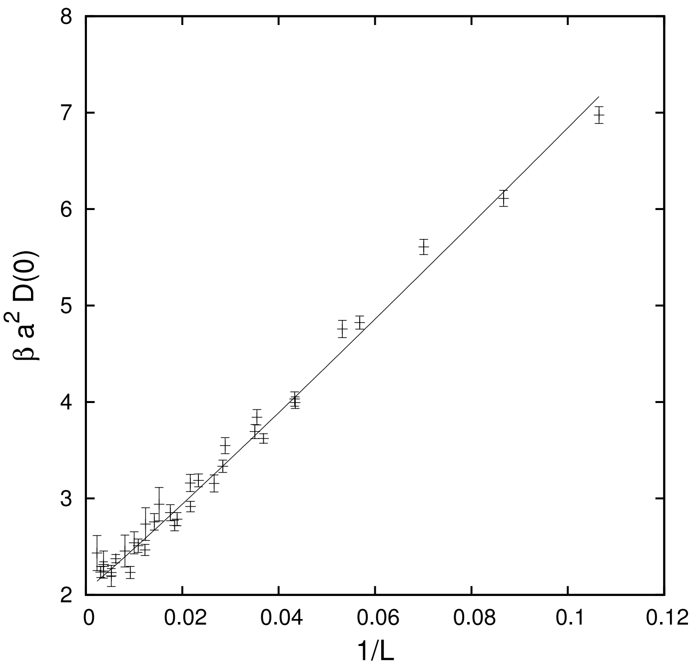

Figure 1: Renormalized gluon propagator at zero momentum $\beta a^2 D(0)$ (in GeV^{-2}) as a function of the inverse lattice side $1/L$ (in GeV) and extrapolation to infinite volume. The fit is given by $b + c/L^e$ with $e = 1.04(5)$ and $b = 2.05(5)$ GeV^{-2}.

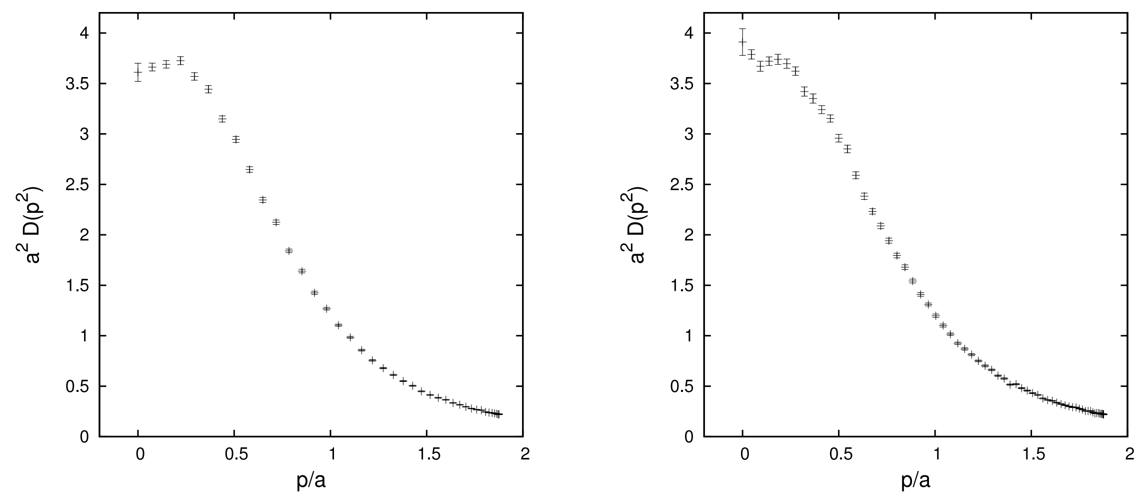

Figure 2: Unrenormalized gluon propagator $a^2 D(p^2)$ (in GeV^{-2}) as a function of the momentum p/a (in GeV) for lattice volumes $V = 80^4$ (left) and $V = 128^4$ (right) at $\beta = 2.2$.

N^3	a_G	N^4	a_G	N^3	a_G	N^4	a_G
140^3	0.073(4)	48^4	0.093(7)	140^3	0.13(2)	48^4	0.19(4)
200^3	0.051(3)	56^4	0.063(6)	200^3	0.06(2)	56^4	0.18(4)
240^3	0.003(3)	64^4	0.049(9)	240^3	0.10(2)	64^4	0.17(4)
320^3	-0.021(9)	80^4	0.052(5)	320^3	0.01(5)	80^4	0.15(2)
		112^4	0.038(6)			112^4	0.10(7)
		128^4	0.016(5)			128^4	0.06(3)

Table 1: Table for the ghost propagator IR exponent a_G, in the 3d and 4d cases, obtained using either the two smallest nonzero momenta (left) or the third and fourth smallest nonzero momenta (right).

the same time, one needs to reconcile the above results with the non-renormalizability of the ghost-gluon vertex [32] and with the suppression of $D(p^2)$ when considering simulations in the strong-coupling regime [33], in the interpolating gauge (or λ-gauge) [34] and in Coulomb gauge [35]. From the analytic point of view, it may seem necessary to reconsider partially the conventional confinement scenarios [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] discussed in the Introduction. One should of course recall that there are different solutions of Dyson-Schwinger equations for gluons and ghosts in Landau gauge [36, 37]. In particular, the results obtained in Ref. [36] support a finite non-zero gluon propagator and an essentially tree-level ghost propagator at small momenta. Similar results are obtained in Ref. [38]. Phenomenological tests [39] also seem to favor $D(0) > 0$.

We believe that a clarification of the present status of the Kugo-Ojima/Gribov-Zwanziger scenario will probably require new ideas and new methods, both for analytic and numerical studies, and that a key point will be a better understanding of the gauge interpolating between the Landau and the Coulomb gauge [40].

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