PROCEEDINGS OF SCIENCE

On the conformal anomaly of k-strings

Pietro Giudice*, **Ferdinando Gliozzi, Stefano Lottini**
Dipartimento di Fisica Teorica, Università di Torino and INFN, sezione di Torino, Italy
E-mail: giudice,gliozzi,lottini@to.infn.it

We discuss the long distance behaviour of the flux tube associated to baryon vertices and argue that, if the gauge system admits stable k-strings, the conformal field theory describing this string in the IR has conformal anomaly $c = (d-2)\sigma_k/\sigma$, where σ_k is the k-string tension and σ that of the fundamental representation. We check this result in a 3D $\mathbb{Z}_4$ gauge model at finite temperature, where a string effect directly related to c can be clearly identified.

The XXV International Symposium on Lattice Field Theory
July 30 - August 4 2007
Regensburg, Germany

*Speaker.

PoS(LATTICE 2007)302

1. Introduction

The effective string description of the quark confinement predicts measurable effects on physical observables of the gauge theory, produced by the quantum fluctuations of that string [1]. The most widely known is the Lüscher correction to the inter-quark confining potential V at large distance r

$$V(r) = \sigma r + 2\mu - (d-2)\frac{\pi}{24r} + O(1/r^2)\,, \tag{1.1}$$

where σ is the string tension and μ a self-energy term. The attractive, universal Coulomb-like correction is known as the Lüscher term.

A similar universal effect has been found in the low temperature behaviour of the string tension [2]

$$\sigma(T) = \sigma - (d-2)\frac{\pi}{6}T^2 + O(T^4)\,. \tag{1.2}$$

Both effects may be summarised by saying that the infrared limit of the effective string is described by a two-dimensional conformal field theory (CFT) with conformal anomaly $c = d-2$. In this language, the (generalised) Lüscher term $-\frac{c\pi}{24r}$ is the zero-point energy of a 2D system of size r with Dirichlet boundary conditions, while the $-\frac{c\pi}{6}T^2$ term is the zero-point energy density in a very long cylinder (i.e. the string world-sheet of the Polyakov correlator) of period $L = 1/T$ [3].

We proposed a method [4] to extend these results to a more general class of confining objects of SU(N) gauge theories, the k-strings, describing the infrared behaviour of the flux tube joining sources made with k fundamental representations.

The spectrum of k-string tensions has been extensively studied in recent years, in the continuum [5, 6, 7] as well as on the lattice [8, 9]. So far, in numerical analyses one typically measured the temperature-dependent k-string tensions $\sigma_k(T)$ through the Polyakov correlators and then extrapolated to $T = 0$ using (1.2), hence assuming a free bosonic string behaviour.

From a theoretical point of view there are good reasons to expect values of c larger than $d-2$. Indeed the conformal anomaly can be thought of as counting the number of massless degrees of freedom of the k-string. Therefore the relevant degrees of freedom are not only the transverse displacements but also the splitting of the k-string into its constituent strings. If the mutual interactions were negligible, each constituent string could vibrate independently so we had $c = k(d-2)$. Thus we expect that c can vary in the range

$$d-2 \leq c \leq k(d-2)\,. \tag{1.3}$$

We studied this question in a class of 3D $\mathbb{Z}_4$ gauge models which admit a stable 2-string. At variance with the SU(N) models considered so far, our model depends on two coupling constants α and β. As a consequence, the continuum limit of the string tension ratio is not a constant, but a function $\sigma_2/\sigma = f(\lambda \equiv \alpha/\beta)$ varying continuously from $\lambda = 0$, where the system reduces to a pair of decoupled $\mathbb{Z}_2$ gauge systems, hence $\sigma_2/\sigma = 2$, to $\lambda = 1$, where it becomes the gauge dual of the 4-state Potts model with $\sigma_2/\sigma = 1$. Correspondingly also the central charge is an unknown function $c = c(\lambda)$ with $c(0) = 2$ and $c(1) = 1$. Our simulations were made at two different values of λ in the interval (0,1). There we studied the Polyakov-Polyakov correlators in the fundamental (f) and double fundamental $(f \otimes f)$ representations at temperatures in the range $T \simeq T_c/2$ where experience on other gauge systems indicates that the first universal temperature-dependent corrections show up and the expected functional form of this correlator is the Nambu-Goto two-loop expression is

$$\langle P(0)P^{\dagger}(R)\rangle_{T=1/L} \propto \frac{e^{-2\mu L-\sigma RL-\frac{\pi^2[2E_4(\tau)-E_2^2(\tau)]}{1152\sigma R^3}}}{\eta(\tau)}\,, \tag{1.4}$$

where η is the Dedekind eta function , E_j denote the Eisenstein functions and $\tau=\frac{iL}{2R}$ is the aspect ratio of the cylinder associated to the string world-sheet. While the Polyakov correlators in the fundamental representations fitted very well this formula with stable parameters in a wide range of R, the similar fit with the correlator the $f\otimes f$ sources turns out to be rather poor and the Ansatz (1.4) does not result in stable parameters [10].

On the other hand, as explained in the next Section, there is a simple scaling argument suggesting a precise value for the central charge of the k-string.

2. Scaling form of the Polyakov correlators

The string picture of confinement fixes, in the IR limit, the functional form of the vacuum expectation value of gauge invariant operators. For instance the asymptotic behaviour in the region $2R\gg L$ of the correlation function (1.4) is

$$\langle P_f(0)\,P_f^{\dagger}(R)\rangle_T \propto \frac{1}{\sqrt{R}}\exp\left[-\sigma(T)\,RL-2\mu L\right]. \tag{2.1}$$

Similar expansions are expected to be valid also for Polyakov correlators describing more specific features of SU(N) gauge theory, like those involving baryonic vertices.

A baryon vertex is a gauge-invariant coupling of N multiplets in the fundamental representation f which gives rise to configurations of finite energy, or baryonic potential, with N external sources. At finite temperature T these sources are the Polyakov lines $P_f(\vec{r}_i)$.

In the IR limit at finite temperature, i.e. $|\vec{r}_i-\vec{r}_j|\gg L\,\forall i\neq j$, we assume, in analogy with (2.1) and the $N=3$ case,

$$\langle\, P_f(\vec{r}_1)\,P_f(\vec{r}_2)\dots P_f(\vec{r}_N)\,\rangle_T = e^{-F_N}\sim \exp\left[-\sigma(T)\,L\ell(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_N)-N\mu L\right], \tag{2.2}$$

or, more explicitly,

$$F_N(\ell,L)\sim \sigma L\ell-(d-2)\frac{\pi\ell}{6L}+N\mu L\,,\;\;(|\vec{r}_i-\vec{r}_j|\gg L\,\forall i\neq j)\,, \tag{2.3}$$

where the coefficient of the ℓ/L term specifies that in this IR limit the behaviour of the baryon flux distribution is described by a CFT with conformal anomaly $d-2$ on the string world-sheet singled out by the position of the external sources.

When $N>3$, depending on the positions $\vec{r}_i$ of the sources, some fundamental strings of the baryon vertex may coalesce into k-strings [11, 6]. As a consequence, the shape of the world-sheet changes in order to balance the string tensions and ℓ becomes a weighted sum, where a k-string of length λ contributes with a term $\lambda\,\frac{\sigma_k(T)}{\sigma(T)}$, with $\sigma(T)$ given by (1.2), while

$$\sigma_k(T)=\sigma_k-c_k\frac{\pi}{6}T^2+O(T^3)\,, \tag{2.4}$$

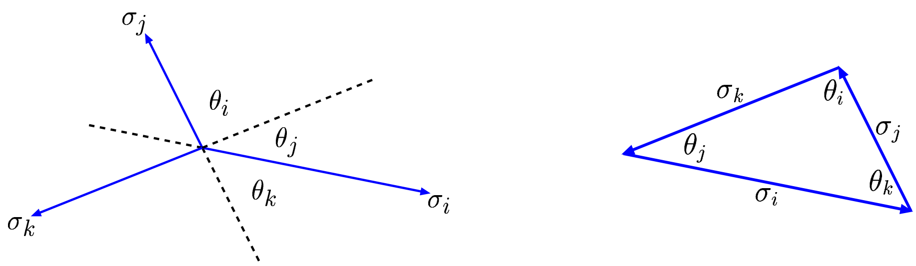

Figure 1: The balance of the string tensions.

where c_k is the conformal anomaly of the k-string.

More generally, the baryonic free energy keeps the expected asymptotic form (2.3) only if the world-sheet shape does not change while varying T. Now in a generic string configuration contributing to SU(N) baryon potential, the angles at a junction of three arbitrary k-strings are given by (see Fig.1, left)

$$\cos\theta_i = \frac{\sigma_j(T)^2 + \sigma_k(T)^2 - \sigma_i(T)^2}{2\,\sigma_j(T)\,\sigma_k(T)} \tag{2.5}$$

and others obtained by cyclic permutations of the indices i, j, k. Because by changing the temperature we can have only similar triangles (Fig.1, right), all the string tension ratios are constant. Moreover, considering an expansion of Eq. 2.5 up to T^3 terms, we can connect it to the conformal anomaly:

$$\frac{c_i}{\sigma_i} = \frac{c_j}{\sigma_j} = \frac{c_k}{\sigma_k} = \frac{(d-2)}{\sigma}, \tag{2.6}$$

which leads directly to

$$\sigma_k(T) = \sigma_k - (d-2)\frac{\pi\,\sigma_k}{6\,\sigma}T^2 + O(T^3), \tag{2.7}$$

this being the main result of our work.

3. Measurement of the conformal anomaly

It is widely believed that the IR properties of the confining string are described by a 2D CFT. This means that when the size of the string world-sheet is much larger than any other relevant length scale, there are, besides the usual area and perimeter contributions, universal shape effects which are controlled by the conformal anomaly c of the theory. However, there is no simple, general recipe to describe the way c enters into these shape effects.

For instance, if we denote with $F_{cyl}(\tau \equiv i\frac{L}{2R})$ the free energy of the CFT describing the IR behaviour of the k-string coupled to the representation $\mathscr{R}$, provided that *both* R and L are larger than $1/T_c$, which is the minimal distance at which the confining string picture applies, we can write the Polyakov correlator in the form

$$\langle P_{\mathscr{R}}(0)\,P^{\dagger}_{\mathscr{R}}(R)\rangle_{T=1/L} = e^{-2\mu L - \sigma_{\mathscr{R}} RL - F_{cyl}(\tau) + O(1/\sigma_{\mathscr{R}} RL)} \quad , \tag{3.1}$$

where the functional form form F_{cyl}, as anticipated in the Introduction, is known only in the two limits [3]

$$F_{cyl}(\tau) = -\pi\frac{L}{R}\frac{c-24h}{24} \quad , \; L \gg 2R; \tag{3.2}$$

$$F_{cyl}(\tau) = -\pi \frac{R}{L}\frac{c}{6} \quad , \; L \ll 2R; \tag{3.3}$$

h being the scaling dimension of the lowest physical state which can propagate along the periodic direction. Outside these limits, the functional form depends explicitly on the whole spectrum of the CFT. It is know that these universal terms have the same origin of the quantum Casimir effect in 3D and share with it the extreme sensitivity to the shape of the system. Thus any deformation of the boundary of the cylinder, like those considered for instance in the smearing procedure, produce modifications of the functional form which are difficult to calculate and to control.

For these reasons the standard method generally employed to measure the conformal anomaly (the one we adopt here) is based solely on the infrared limit (3.3) of a regular cylinder.

In practise we consider Polyakov correlators associated to very long cylinders in the region $2R \gg L \gg 1/T_c$ and look after the stability of the fit to (3.3).

4. Numerical test

The general scaling argument developed in Section 2 on the finite temperature corrections of the k-string tensions suggests a different behaviour with respect to the usual assumption that these corrections are those produced by a free bosonic string. Since the comparison with theoretical predictions of k-string tensions is sensitive to this behaviour, it is important to check its validity.

We addressed such a question with a lattice calculation in a 3D $\mathbb{Z}_4$ gauge theory which is perhaps the simplest gauge system where there is a stable 2-string.

Exploiting Kramers-Wannier duality and a suitable flips of the couplings of the corresponding spin model, we can insert any Polyakov correlator directly in the Boltzmann factor [12], producing results with very high precision. Because of this last property it is possible to measure in a single numerical experiment the ratio of the correlation of two Polyakov lines in the two representations: $G(r)_{ff}/G(r)_f$.

We performed our Monte Carlo simulations on the AT model, at two different points of the confining region, for which we previously measured the string tensions [10]: ($\alpha = 0.05, \beta = 0.207$) and ($\alpha = 0.07, \beta = 0.1975$). We worked on a cubic periodic lattice of size $128 \times 128 \times N_\tau$ with N_τ chosen in such a way that temperature of our simulations ranged from $T/T_c \simeq 0.5$ to $T/T_c \simeq 0.8$ and we took the averages over 10^6 configurations in each point.

The large distance behaviour of the data is well described by a purely exponential behaviour (see Fig.2)

$$G(r)_{ff}/G(r)_f = \frac{\langle P_{ff}(\vec{r}_1)\, P_{ff}(\vec{r}_2)\rangle_T}{\langle P_f(\vec{r}_1)\, P_f^\dagger(\vec{r}_2)\rangle_T} \propto e^{-\Delta\sigma r N_\tau}\,, \tag{4.1}$$

with $\Delta\sigma = \sigma_{ff} - \sigma_f$. It is important to note that the logarithmic term, which is a potential source of systematic errors when neglected in Polyakov correlators, here is cancelled in the ratio. Since (4.1) is an asymptotic expression, valid in the IR limit, we fitted the data to the exponential by progressively discarding the short distance points and taking all the values in the range $r_{min} \le r \le r_{max} = 60a$ with r_{min} varying from 15 to 40 lattice spacings a. The resulting value of $\Delta\sigma$ turns out to be very stable. The whole set of values of $\Delta\sigma(T)$ as functions of the inverse temperature $N_\tau = 1/T$ are reported in Table 1 in Ref. [4].

According to Eq.(2.7), in the low temperature limit we expect the asymptotic behaviour

$$\Delta\sigma(T) = \Delta\sigma(0)\left(1 - \frac{\pi}{6\sigma}T^2\right) + O(T^3)\,. \tag{4.2}$$

Assuming for σ the values estimated in [10], we used $\Delta\sigma(0)$ as the only fitting parameter. Neglecting one or two points too close to T_c, we got very good fits to (4.2) as shown in Figs. 3. It is important to note that $\Delta\sigma(0)$ agrees with the difference of the previous estimates [10], however the error is reduced by a factor of 25 (first set of data) and even of 50 (second set of data). The reason of this gain in precision is due to the fact that σ_2 was evaluated from a fit to (2.1), even taking in account the Next-to-Leading-Order terms, which was rather poor because the 2-string does not behave as a free bosonic string [10]. On the contrary our fits to (4.1) and (4.2) are very stable and the corresponding reduced $\chi^2/d.o.f$ are of the order of 1 or less.

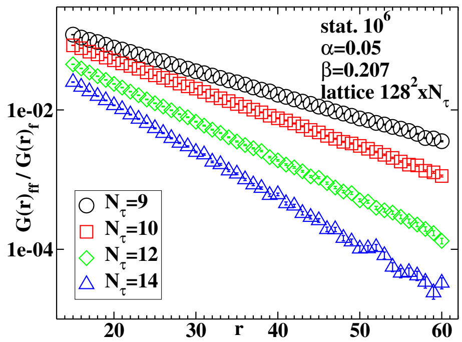

Figure 2: Plot of $G(r)_{ff}/G(r)_f$ on a log scale.

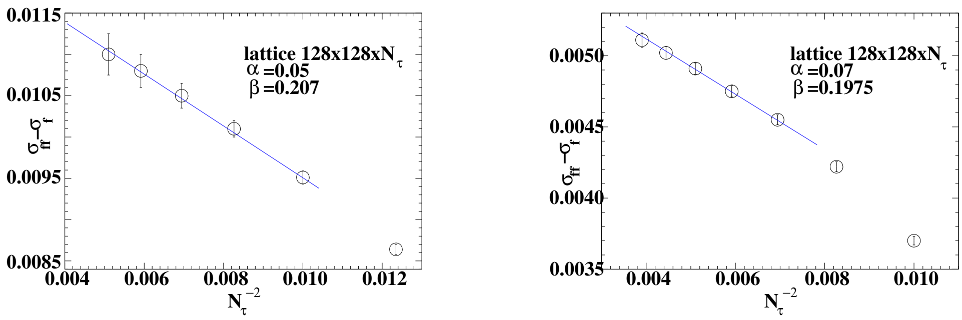

Figure 3: Values of $\Delta\sigma(T)$ versus $T^2 = 1/N_\tau^2$ for two points of the parameter space. The solid line is a one-parameter fit to Eq.(4.2).

5. Discussion

We pointed out that a simple argument on scaling properties of baryonic vertices suggested that the central charge of the CFT describing the infrared limit of a k-string is $c_k = (d-2)\frac{\sigma_k}{\sigma}$.

We checked it for $k = 2$ in two different points of the phase diagram of 3D $\mathbb{Z}_4$ gauge model, where the string tension ratio can be adjusted continuously in the interval $1 \leq \frac{\sigma_k}{\sigma} \leq 2$. In order to measure directly the central charge we used long cylindric string world-sheets with aspect ratios $2R/L \gg 1$, where R is the length of the open string (i.e. the distance between the Polyakov loops) and L the length of the closed string. The fits involved two fitting parameter and were particularly stable: when progressively discarding short distance data the fitted parameters showed always wide plateaux of more than 20 lattice spacings, with a $\chi^2/d.o.f$ of order 1 or less for more than 40 degrees of freedom for each temperature considered.

In a related work presented at this conference [13] the k-string tensions of some 3D SU(N) gauge systems have been studied with a different method and the conclusion is different in the sense that a good agreement with the free bosonic string has been reported. Of course there is no compelling reason for a 2-string of the $\mathbb{Z}_4$ system to behave as the 2-string of a $\mathrm{SU}(N)$ system, however it is possible that the discrepancy in the conclusions is due to the different methodology adopted. In [13] the whole information is extracted from the correlation matrix of vanishing transverse momentum, smeared Polyakov loops. These loops are much longer than ours, however their mutual minimal distance considered in the zero momentum projection, hence the maximal contribution in the correlation matrix, is few lattice spacings. In other terms, the world-sheets which maximally contribute to the correlation matrix are short cylinders with very small aspect ratios $2R/L \ll 1$ which is just the opposite of the limit where the central charge shows up as indicated in (3.3). It is generally believed that in the process of diagonalisation of the correlation matrix, to get the projection on the ground state, the non-universal contributions which dominate at short distance are washed out, however it is difficult to fully understand in the context of the underlying effective string theory how the random boundary deformations of short cylinders implied by the smearing process could encode the information on the conformal anomaly, which is an essentially IR quantity that can be directly observed only in long cylinders. According to the general considerations made in Section 3, it would be highly desirable to develop a similar correlation matrix analysis where the minimal loop distance taken into account in the zero momentum projection is larger than $1/T_c$, where the effective string picture starts working.

Acknowledgments

We are grateful to Michele Caselle, Paolo Grinza and Ettore Vicari for useful discussions.

References

[1] M. Lüscher, K. Symazik and P. Weisz, *Nucl. Phys.* **B 173** (1980) 365; M. Lüscher, *Nucl. Phys.* **B 180** (1981) 317.

[2] R.D. Pisarski and O. Alavarez, *Phys. Rev.* **D 26** (1982) 3735.

[3] H.W.J. Blöte, J.L. Cardy and M.O. Nightingale, *Phys. Rev. Lett.* **56** (1986) 343; I. Afflek, *Phys. Rev. Lett.* **56** (1986) 347.

[4] P. Giudice, F. Gliozzi and S. Lottini, *J. High Energy Phys.* **05** (2007) 010 [hep-th/0703153].

[5] M.R. Douglas and S.H. Shenker, *Nucl. Phys.* **B 447** (1995) 271 [hep-th/9503163].

[6] F. Gliozzi, *Phys. Rev.* **D 72** (2005) 055011 [hep-th/0504105].

[7] J. M. Ridgway, hep-th/0701079.

[8] B. Lucini and M. Teper, *Phys. Lett.* **B 501** (2001) 128 [hep-lat/0012025].

[9] L. Del Debbio, H. Panagopoulos and E. Vicari, *J. High Energy Phys.* **09** (2003) 034 [hep-lat/0308012].

[10] P. Giudice, F. Gliozzi and S. Lottini, PoS(LAT2006) 65 [hep-lat/0609055].

[11] S. A. Hartnoll and R. Portugues, *Phys. Rev.* **D 70** (2004) 066007 [hep-th/0405214].

[12] P. Giudice, F. Gliozzi and S. Lottini, *J. High Energy Phys.* **01** (2007) 084 [hep-th/0612131].

[13] B. Bringoltz and M. Teper, arXiv:0708.3447 [hep-lat].